Power Electronics
Day 5 – Dc-dc Converters; Classical Rectifiers

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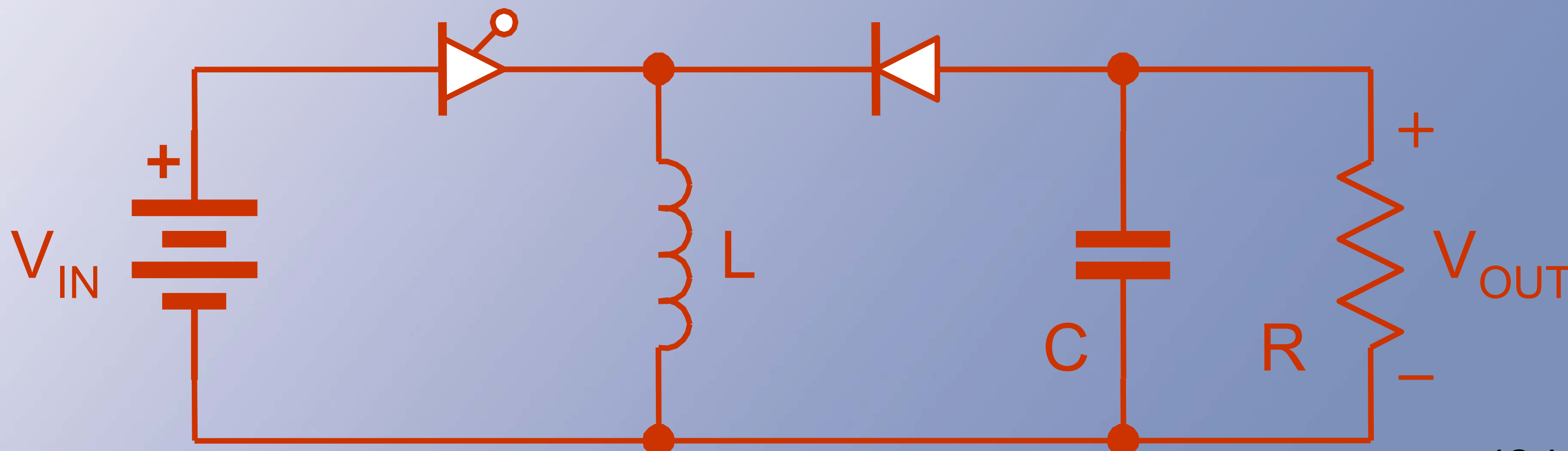
Example

- Input: +5 V to +15 V
- Output: -12 V ± 0.5%
- Power: 10 W to 20 W
- Switching: 100 kHz
- Find a circuit, and then L, C, and duty ratios to meet these needs.



Duty Ratios

- The converter gives $V_{out} = -D_1V_{in}/(1-D_1)$, when V_{out} is defined as on this drawing.
- With +5 V in and -12 V out, 12 V= D_1(5 V)/(1-D_1).
- The solutions: D_1=12/17, D_2=5/17.
- With +15 V in and -12 V out, (12 V) = D_1 (15 V)/(1-D_1).
- The solutions: D_1 = 12/27 = 4/9, D_2 = 15/27 = 5/9. They add to 1.

Currents

- To meet the need, D_1 must be adjustable from 4/9 to 12/17, or 0.444 to 0.706.

- At 10 W, the average output current is (10 W)/(12 V)= 0.833 A.
- The input current depends on duty.
- Let us allow ± 5% inductor current ripple (somewhat arbitrary).

Inductor Current

- Since $I_{out} = D_2 I_L$, the inductor current is I_{out}/D_2. For 10 W to 20 W, the output is 0.833 A to 1.67 A
- D_2 ranges from 5/17 to 5/9, so Is could be as high as 1.67/(5/17) = 5.67 A. It could be as low as 0.833/(5/9) = 1.5 A.

Inductor Value

- The $\pm$ 5% current variation limit is most restrictive with the lighter load (10 W).
- When switch #2 is on, the inductor sees -12 V, and its current falls.
- 12 V = v_L = L di/dt = L Δi/Δt.
  The time is D_2 T, with T = 10 us.
- We want (12 V)(D_2 T)/L = Δi,
  and Δi < (0.1)(0.833)/D_2.

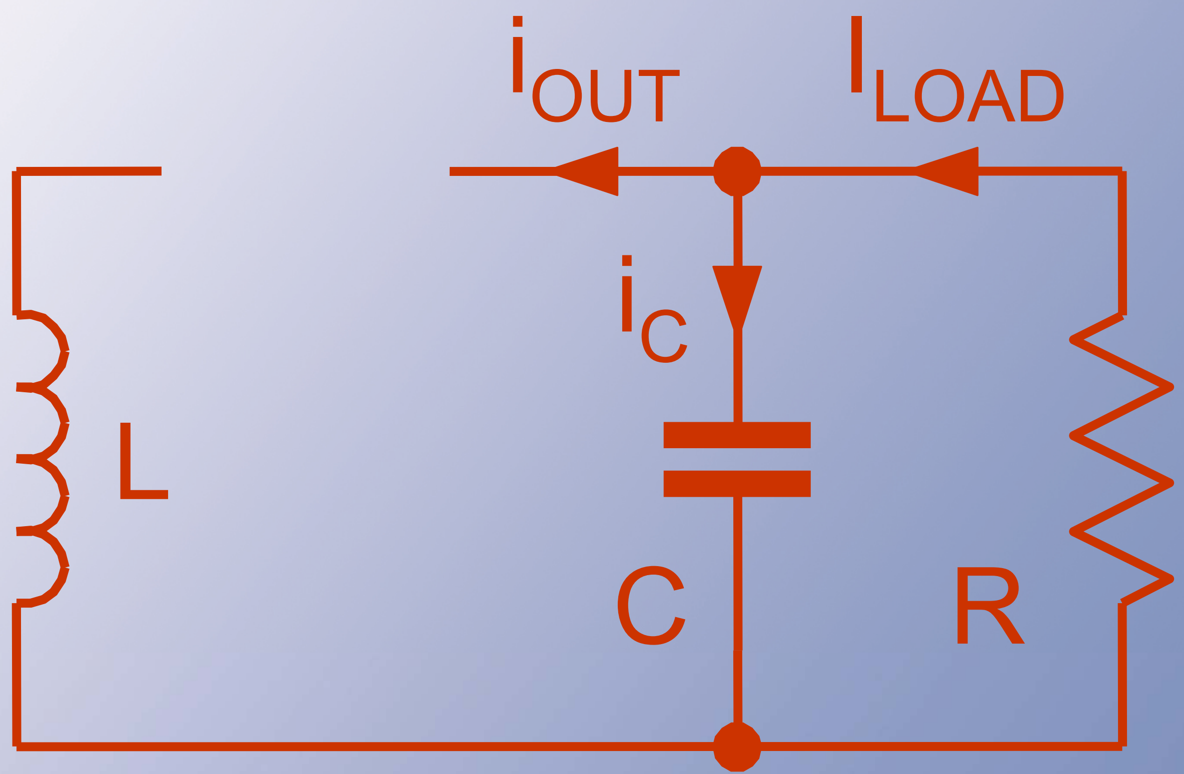
The Current Change

- This reduces to $L > 144\ D_2^2\ T$.
- We need it to work for all allowed duty values. Highest is 5/9.
- A 0.444 mH inductor should meet the requirements over the entire range.

Capacitor Value

- Output capacitor must carry the load current when switch #2 is off.
- Consider this interval:
  $i_C = I_{out}$ when #2 is off, and
  $i_C = C\, dv/dt$
  $= C\, \Delta v/\Delta t$ ← since i_C is constant during the interval when #2 is off

Capacitor Value



Capacitor

- The time when #2 is off is the same as the time with #1 on, $\Delta t = D_1 T$.
- The allowed variation of voltage is $\pm$ 0.5% of 12 V, so the total changes should not exceed 1% of 12 V peak-to-peak.

Capacitor

- Therefore, $I_{out}\,\Delta t/C = \Delta v < 0.12$ V.
- This requires $C > I_{out}\,D_1T/(0.12\text{ V})$
- The capacitor must work for any allowed values, so we need the highest value of the right side.
- This occurs at the highest load current and highest D_1 value.

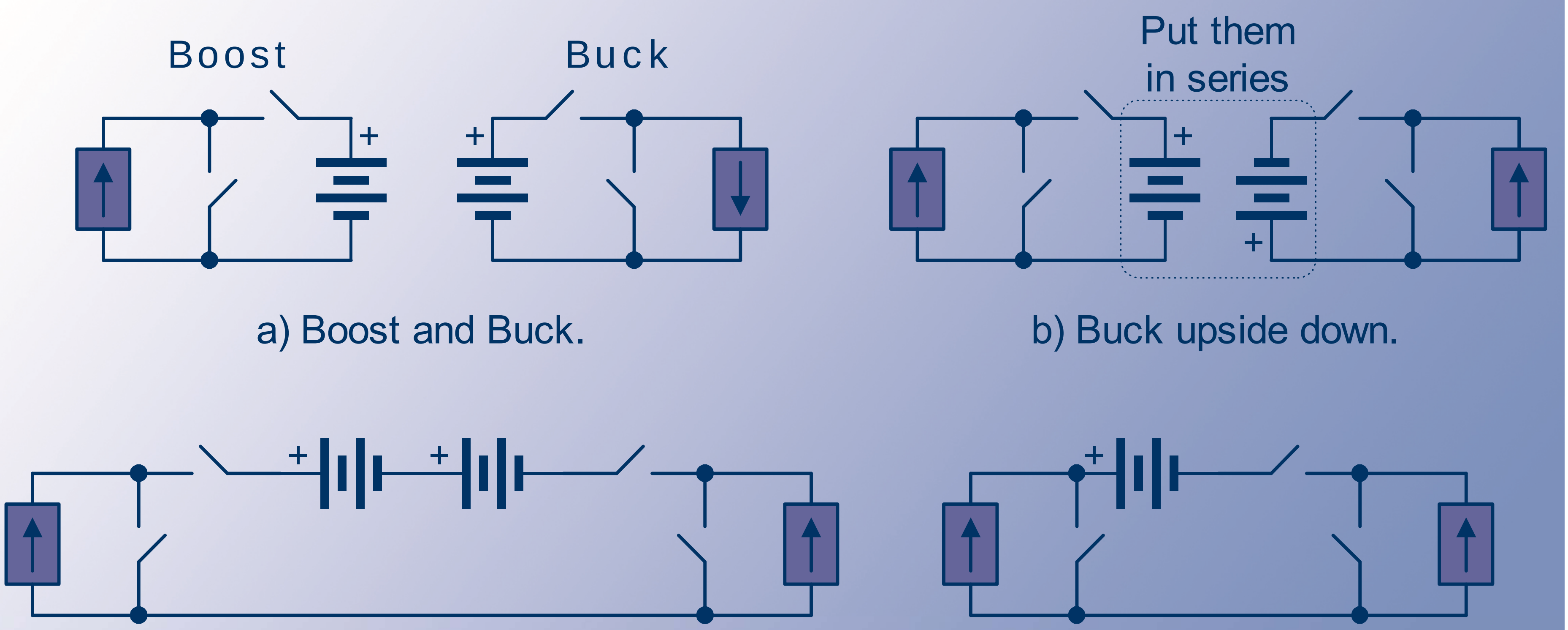
Final Result

- Then C > (1.67 A)(12/17)(10 us)/(0.12 V), C > 98.0 μF.
- In conclusion, we could use a 0.5 mH inductor, a 100 μF capacitor, and would have a duty ratio range of $0.444 < D_1 < 0.706$ for this 100 kHz frequency selection.

More Indirect Converters

- We could use a boost as the input to a buck.
- This again ought to allow any level of output.
- Will there be a polarity change?

Boost-Buck Development



a) Boost and Buck.

b) Buck upside down.

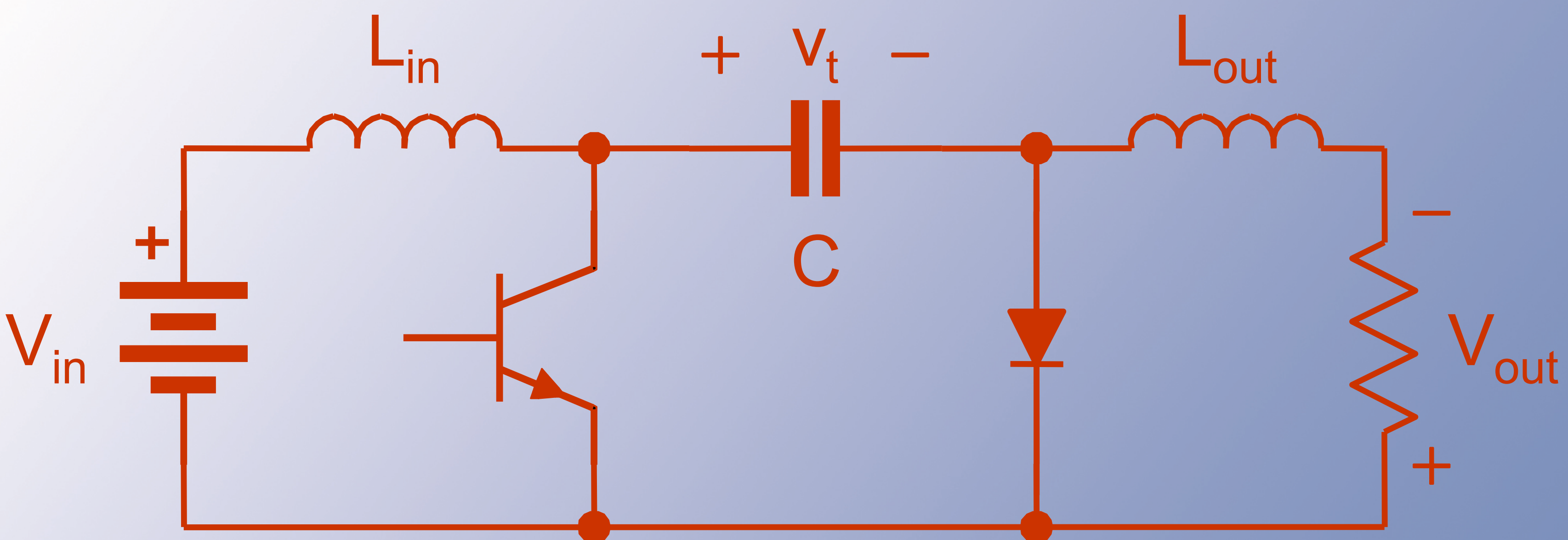
c) Voltages in series.

d) Remove redundancy.

Final Simplification

- The switch in series with the voltage source is not necessary for KCL.
- Try removing it.
- The voltage source is a transfer source.

Boost-Buck Converter



- Left switch is FCFB.
- Right switch is FCRB.

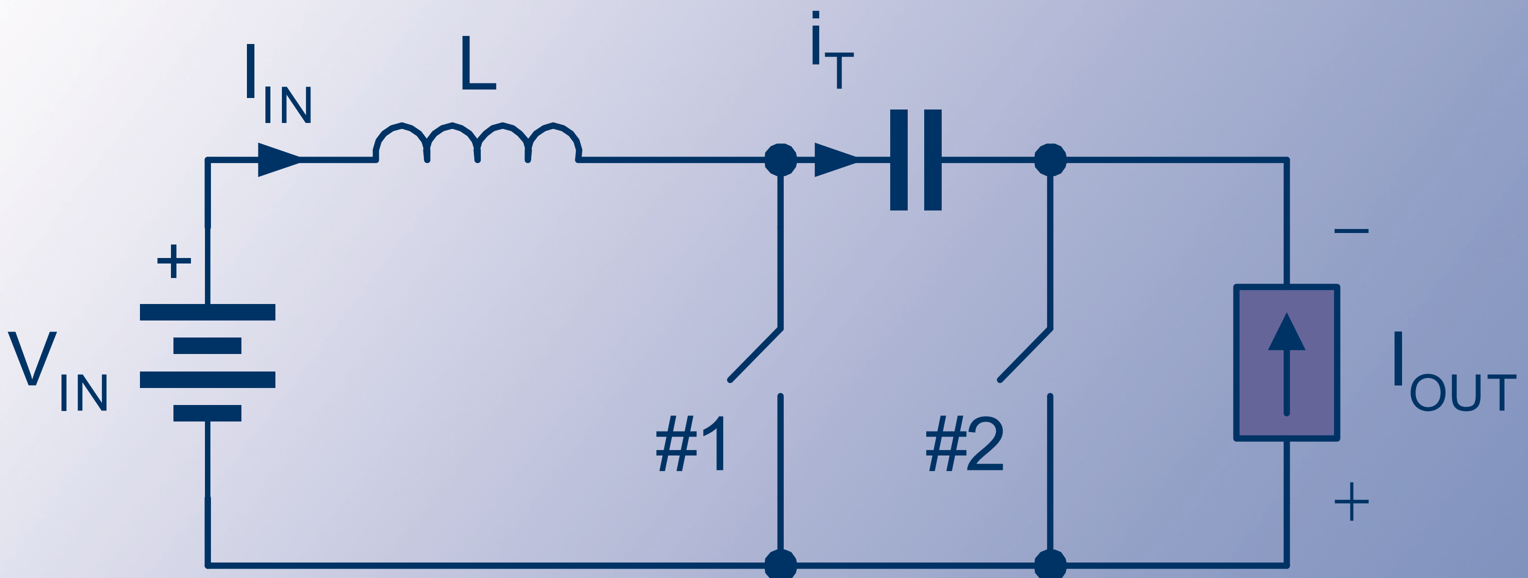
Relationships

- To meet KVL and KCL, $q_1 + q_2 = 1$.
- There are really two matrices now. Let us consider the transfer source.
- Transfer current is subject to control.
- Transfer current $i_t = q_2 I_{in} - q_1 I_{out}$.
- Transfer source power is $i_t V_s = q_2 I_{in} V_s - q_1 I_{out} V_s$ ← Want 0 average!

Relationships

- This can be done if $D_2 I_{in} = D_1 I_{out}$.
- Since $D_1 + D_2 = 1$, we have
  $$D_1 I_{out} = (1 - D_1) I_{in}.$$
- This becomes $V_{out} = D_1 V_{in}/(1-D_1)$, based on conservation of energy.
- The polarity reversal comes from the cascade process.

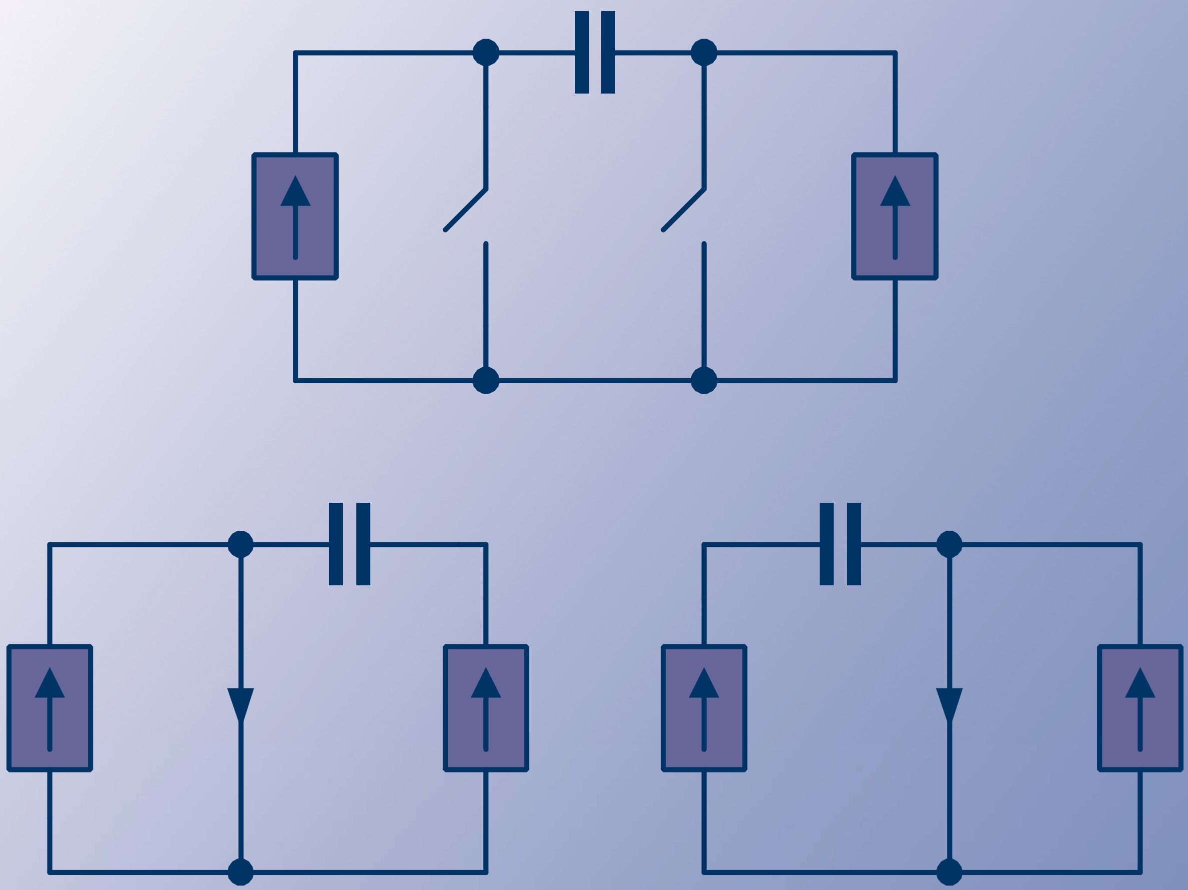
Relationships



$$D_1 I_{out} = (1 - D_1) I_{in}$$

$$D_1 V_{in} = (1 - D_1) V_{out} \quad \text{(Same as for the buck-boost)}$$

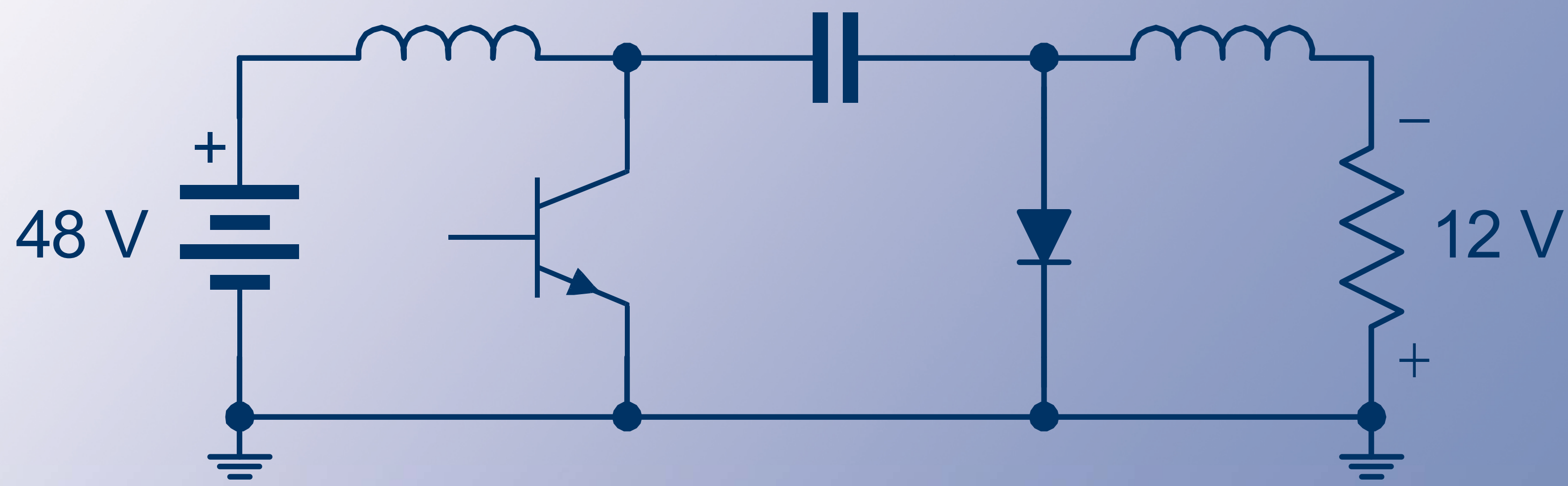
Relationships



The switches must carry both I_{in} and I_{out}.

Boost-Buck

The circuit is often called the "Cuk" converter, after the original patent holder (now expired).



Cuk

Relationships

- The boost-buck (Cuk) also has a polarity reversal, and generally produces the same relationships as the buck-boost .
- Each switch must carry $I_{in} + I_{out}$ and must block $V_s = V_{in} + V_{out}$.
- Transfer source: a capacitor.

What About Voltages?

- The input voltage: $v_{in} = q_2 V_s$,
- The output voltage in a negative direction : $v_{out} = q_1 V_s$,
- Average input: $V_{in} = D_2 V_s$,
- Average output: $V_{out} = D_1 V_s$.
- Add to get $V_{in}+V_{out} = (D_1+D_2)V_s = V_s$.

Example

- Input: +15 V
- Output: -15 V ± 1%
- Power: 15 W
- Switching: 150 kHz
- We need to find L, C, and duty ratios to meet these needs.

Duty Ratios

- The converter gives $V_{out} = -D_1V_{in}/(1-D_1)$.
- With +15 V in and -15 V out,
  $(15\text{ V}) = D_1\ (15\text{ V})/(1-D_1)$.
- The solutions: $D_1=1/2$, $D_2=1/2$
- At 15 W, the average output current is 1 A. So is the average input current.
- The output inductor is allowed $\pm 1\%$ current ripple to enforce the target output voltage variation.

Inductor Current

- When the diode is on, the output inductor sees 15 V = L di/dt.
- The duration of the diode-on interval is D_2T = 3.33 us.
- Simplify to 15 V = L $\Delta i/\Delta t$, with $\Delta t = D_2T$, and $\Delta i < (0.02)\text{x}(1\text{ A})$.
- Therefore, $L > (15\text{ V})(3.33\text{ us})/(0.02\text{ A})$.

  $L >$ 2.5 mH (output inductor).

Transfer Capacitor

- Rather arbitrarily, allow $\pm$ 10% variation in transfer voltage.
- The average voltage is $V_{in} + V_{out} = 30$ V. Allowed variation: 6 V (a total of 20%).
- With switch #2 on, $i_C = I_{in}$

$$= 1 \text{ A.}$$

$$i_C = C\, dv/dt$$

$$= C\, \Delta v/\Delta t,$$

$$\Delta t = 3.33 \text{ us.}$$

Capacitor Result

With $\Delta v < 6$ V,

$C > (1\ \text{A})(3.33\ \text{us})/(6\ \text{V})$,

$C > 0.56\ \text{uF}$.

The transfer capacitor in this converter typically carries substantial ripple current.

Polarity Issue

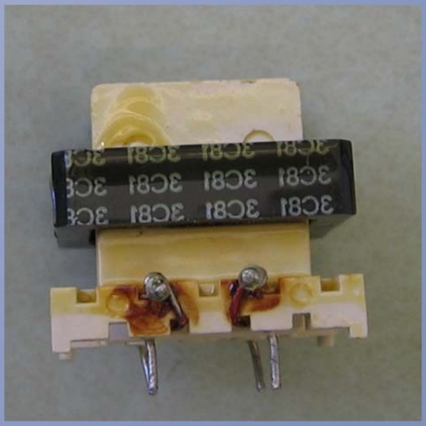
- We have any allowed output value -- except that the values are negative.
- This is not always convenient.
- Options: Cascade some more, e.g., boost-buck-boost.
- Other option: check inductor.

Coupled Inductors

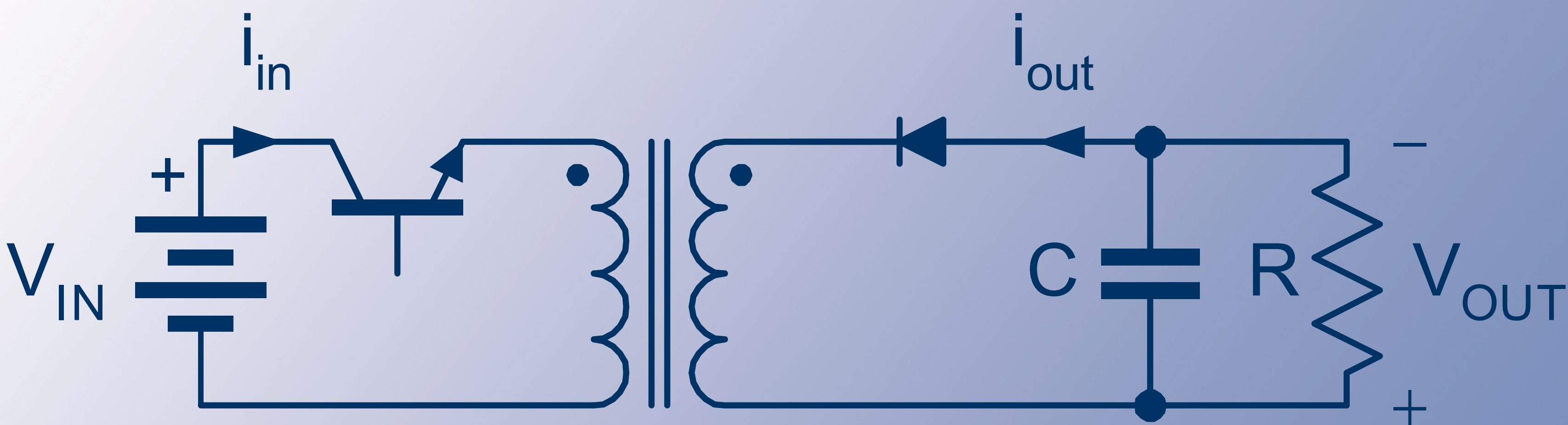
- The buck-boost converter uses a transfer inductor.
- The energy is stored in a magnetic field, with $W_L = 1/2\ Li^2$.
- The inductor is built as a coil on a magnetic core.

Coupled Inductors

- What if we use two (or more) core windings? Then the stored energy is a sum for individual windings:
- $W_L = \Sigma \frac{1}{2} Li^2$.
- We could have i = 0 in one winding -- if another carries current.



Buck-Boost, Coupled L

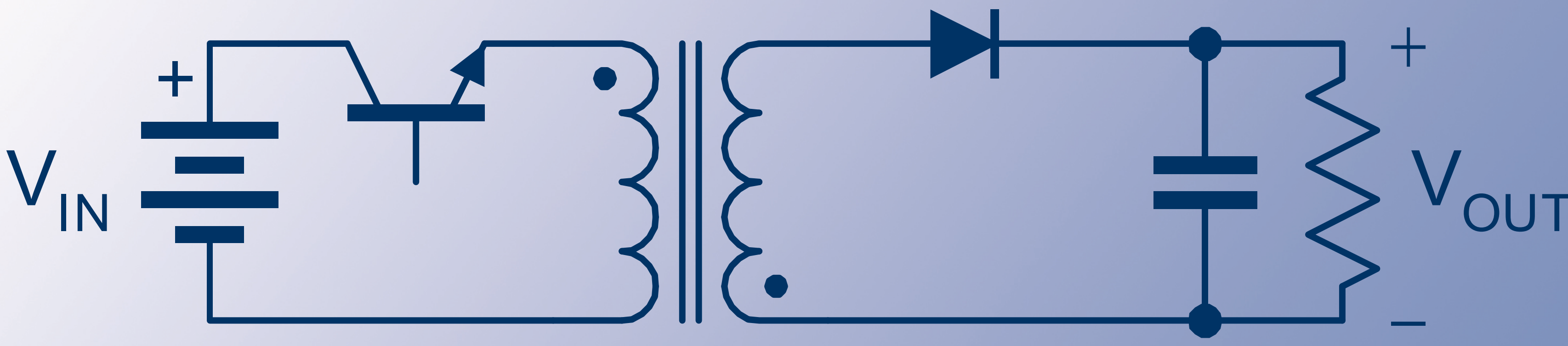


- When switch #1 turns off, the other coil provides a current path.
- We meet KCL, based on magnetics.

What About This?

- This converter (which is still a buck-boost, really) adds isolation.
- We can connect the output in either polarity!
- The result allows either polarity, and also any output.
- This is called a *flyback* converter.

Flyback Converter



The basis for most switching power supplies up to about 100 W.

Flyback Converter

- We also have the option of providing a turns ratio.
- This is very helpful when high conversion ratios are desired.
- The flyback converter is a “true” dc transformer.

Analysis

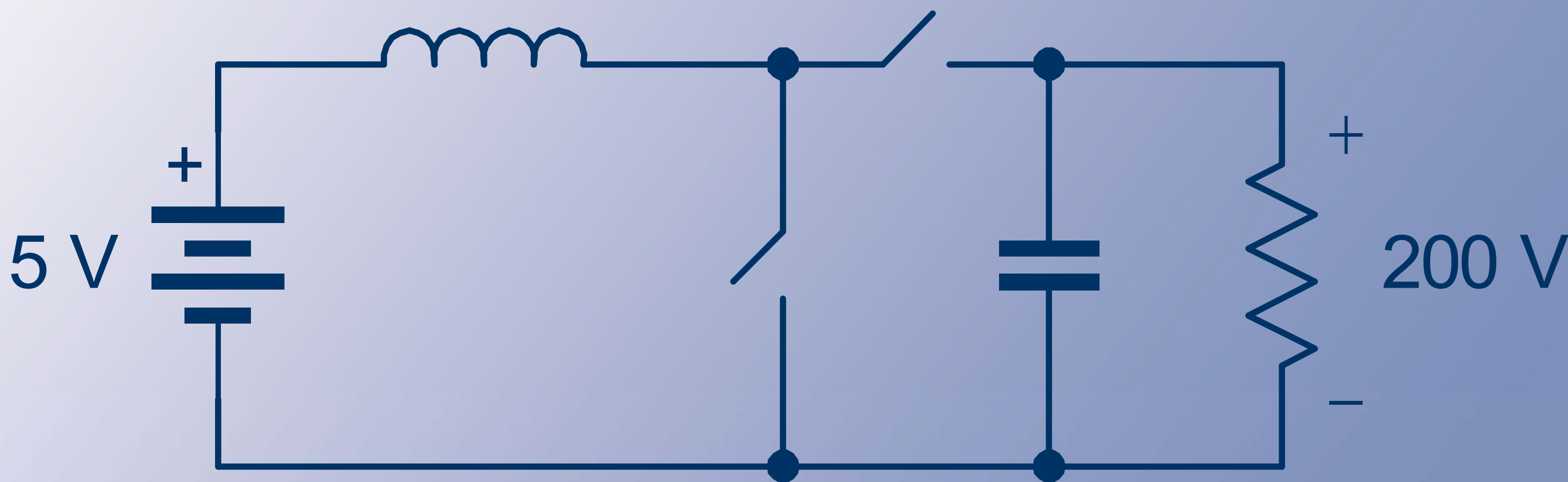
- What if there is a turns ratio?
  - We know how to analyze it at 1:1, since that is a buck-boost circuit.
  - Do a conversion to get 1:1, then analyze.
- Example: 200 V to 5 V. If we use a 200:5 turns ratio, D = 1/2.

Analysis

- Why? The 200 V input to a 200-turn winding is equivalent to a 5 V input on a 5-turn winding.
- In general, V/N is a constant, so a simple ratio can give an equivalent.
- The inductance depends on N^2.

Boost Alternative

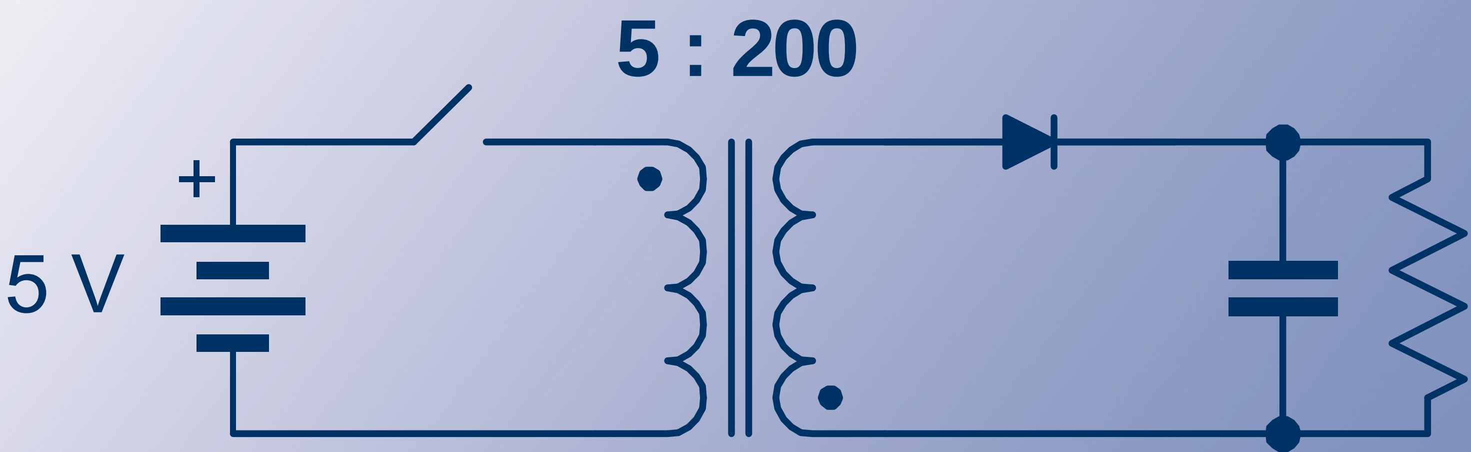
If V_{in} is very different from V_{out}, one of the switches needs to be on for a very short time. Time errors will be more important.



$$D_1 = {}^{39}\!/_{40}$$

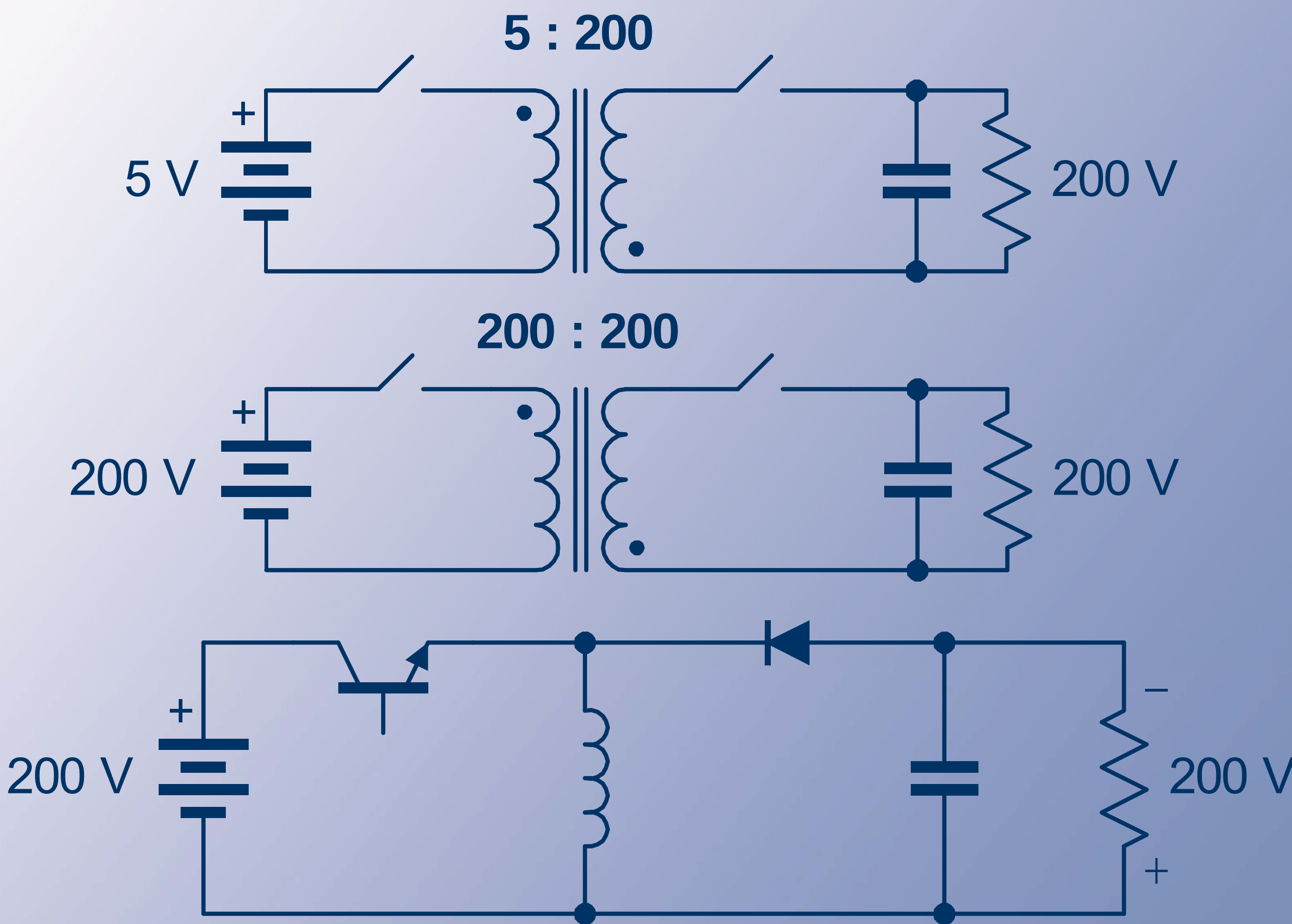
Flyback Converter

More practical: Take advantage of the turns ratio of the transformer.



$D_1 = \frac{1}{2}$ Much easier to achieve, and not so sensitive.

Flyback Converter: Equivalent Buck-Boost Converter



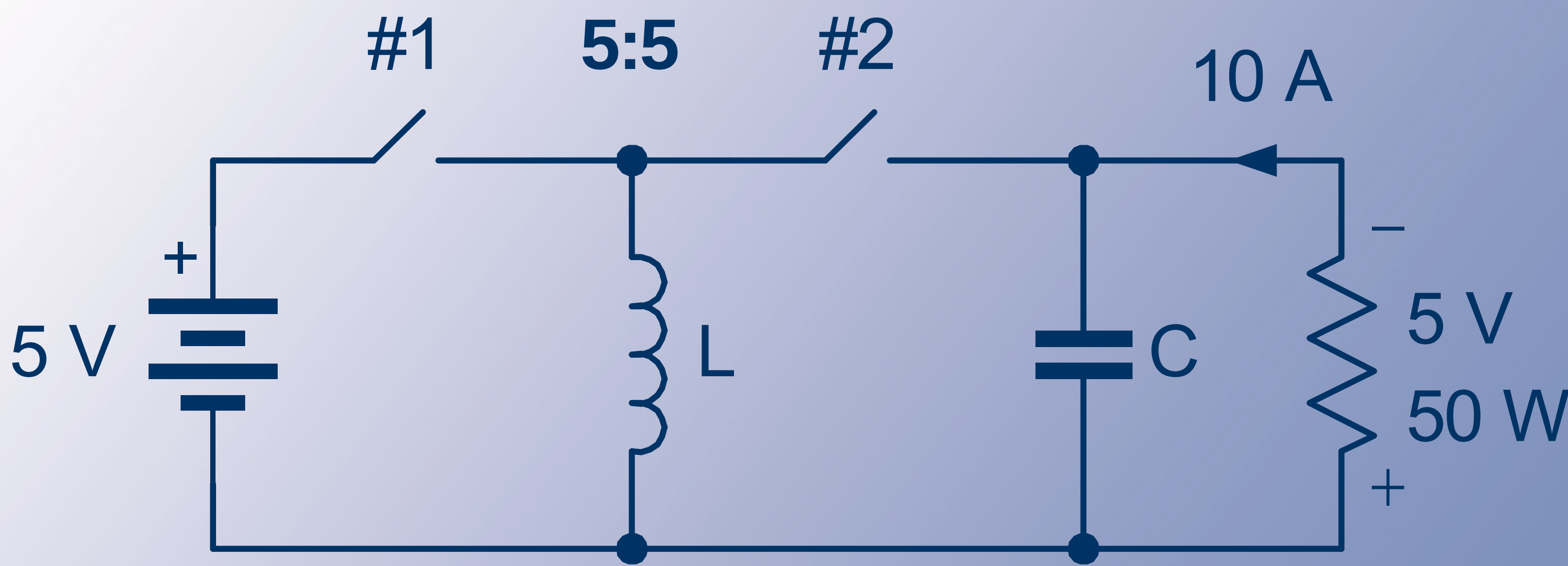
A Pointer

- It is usually helpful to have a duty ratio D_1 in the 0.3 to 0.5 range.
- We often select a turns ratio for a target duty ratio of about 40%.

Example

- 200 V to 5 V converter, 50 W, 100 kHz switching. Want ±1%.
- Let us select a turns ratio of 200:5. Then a +5 V to -5 V, 50 W, buck-boost converter can be the basis for design.

Example



$$D_1 = D_2 = \frac{1}{2}$$

Example

- The duty ratio is 50%. The average output current is 10 A. The inductor current must be $(10A)/D_2$ = 20 A.
- The capacitor carries 10 A when the diode is off, and $i_C = C\,dv/dt$.
- For Δt = 5us, C > 500 uF.

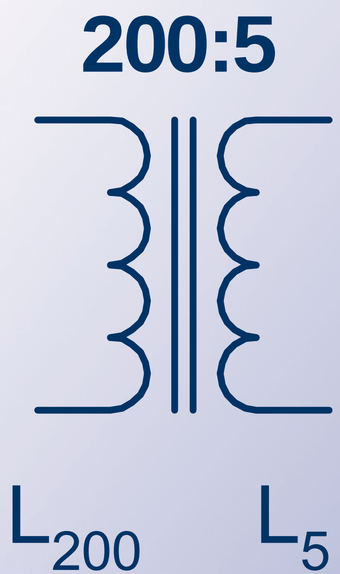
Inductor Value

- Consider a $\pm$5% current ripple (but what does this mean in a flyback?).
- The inductor sees 5 V when the diode is on, 5 V = L Δi/(5 us).
- With Δi < 2 A, L > 12.5 uH.

The Flyback

- For the flyback, the coupled inductor should measure 12.5 uH from the 5 V side, and that coil will carry 20 A when on.
- On the other side, at 200:5 ratio, there are 40 times as many turns.

The Flyback



$$\frac{L_{200}}{L_5} = \left(\frac{200}{5}\right)^2 = 1600$$

The Flyback

- The inductance measured at the input is 1600 times higher, or 20 mH.
- The input coil carries (20 A)/40 = 1/2 A.
- The input switch must carry 1/2 A and block 400 V (why?)
- The output switch must carry 20A and block 10 V.
- It is not the inductor current that stays nearly constant, but rather the magnetic flux.

Major Indirect Converters

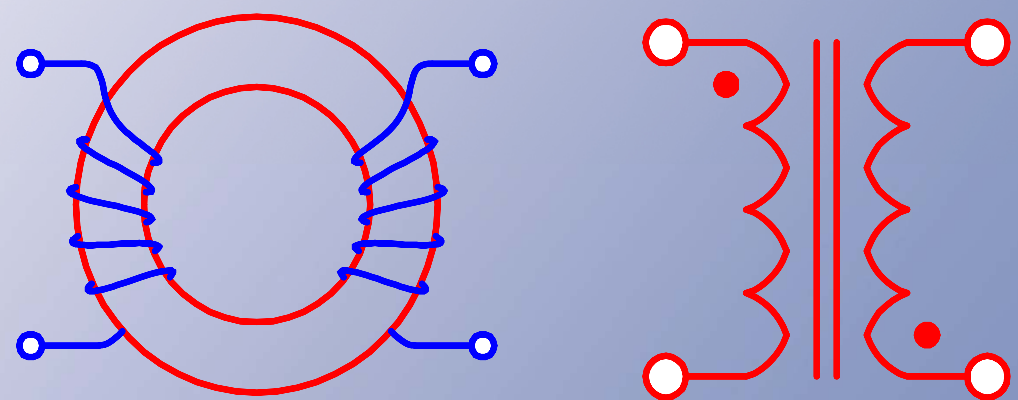
- Buck-boost
- Boost-buck (Ćuk)
- SEPIC (single-ended primary inductor converter) = boost-buck-boost
- Zeta = buck-boost-buck
- These are all "two switch" converters
- There are a few others.
- Some "four switch" versions exist, but are less common.

Ratios

- SEPIC and Zeta same ratio as buck-boost and boost-buck, $V_{out}/V_{in} = D_1/D_2$, *except:*
- No polarity reversal.
- Others: boost-buck-boost-buck… buck-boost-buck-boost…
- Two switch versions just add more filter elements.
- Notice: current-voltage-current...

Isolation needs

- The flyback circuit (derived from buck-boost) uses a "coupled inductor" for isolation.
- This part is not the same as a magnetic transformer. It stores energy.



Isolation needs

- A true transformer has

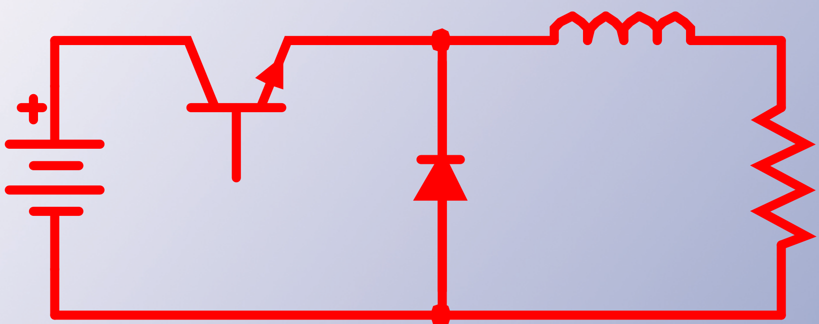
  $p_{in} = p_{out}$
  $i_{out}/i_{in} = 1/a$
  $v_{out}/v_{in} = a$
  isolation
- We want a dc transformer. A flyback does this, up to ~100 W.

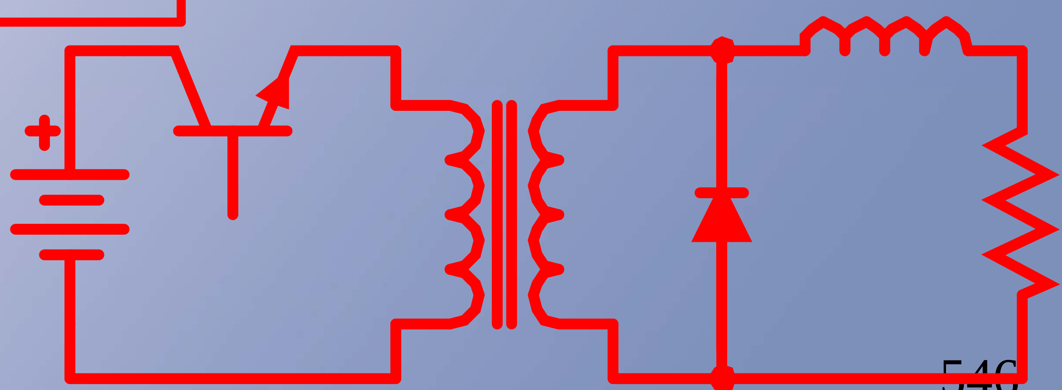
Magnetic Transformers

- Can we insert a magnetic transformer into a converter?
- To answer this, we need to consider ac issues in a magnetic transformer.
- In a true transformer, voltages and currents are related by a ratio.
- We cannot turn one winding “off” and then draw current from another.

Magnetic Transformers

- Example: insert a magnetic transformer into a buck converter.
- No. This is a KCL problem.





Magnetic Transformers

- We need to analyze this to understand:
  - Distinctions between coupled inductors and magnetic transformers
  - How and when a magnetic transformer can be used in a dc-dc converter.

Real Transformers

$\lambda \rightarrow$ magnetic flux linkage

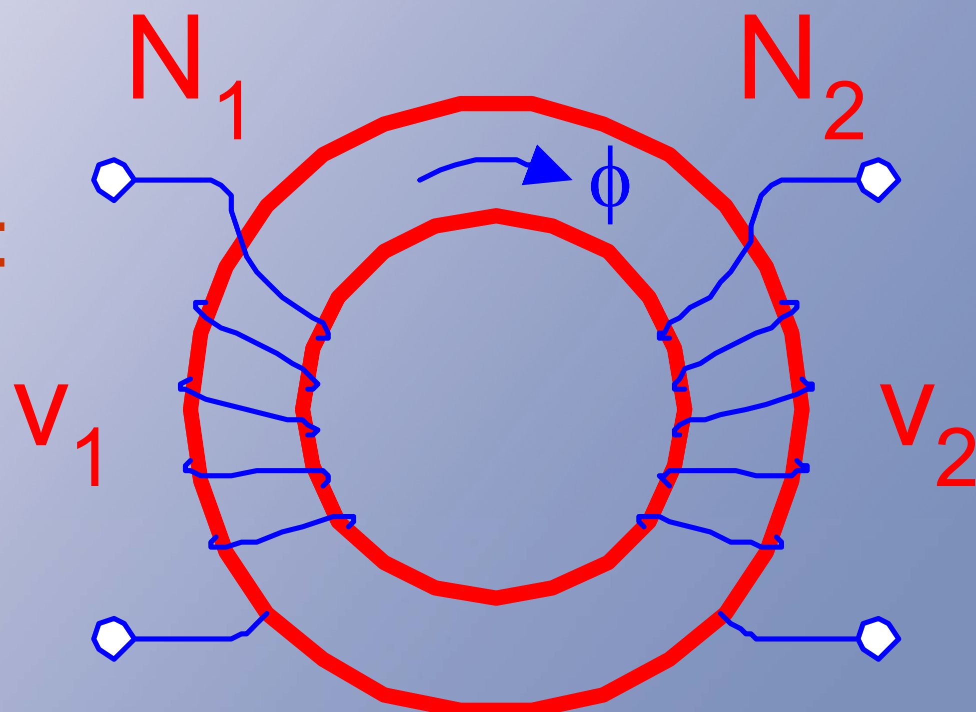
$\lambda_1 = N_1 \rightarrow$ magnetic flux linkage

$\lambda_2 = N_2\ \phi$

Faraday's Law:

$v_1 = d\lambda_1/dt$

$v_2 = d\lambda_2/dt$



Faraday's Law: Implications

$$v_1 = N_1 \frac{d\phi}{dt} \qquad \lambda_1 = N_1 \phi$$

$\div$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad if \quad \frac{d\phi}{dt} \neq 0$$

This is IDEAL CASE.

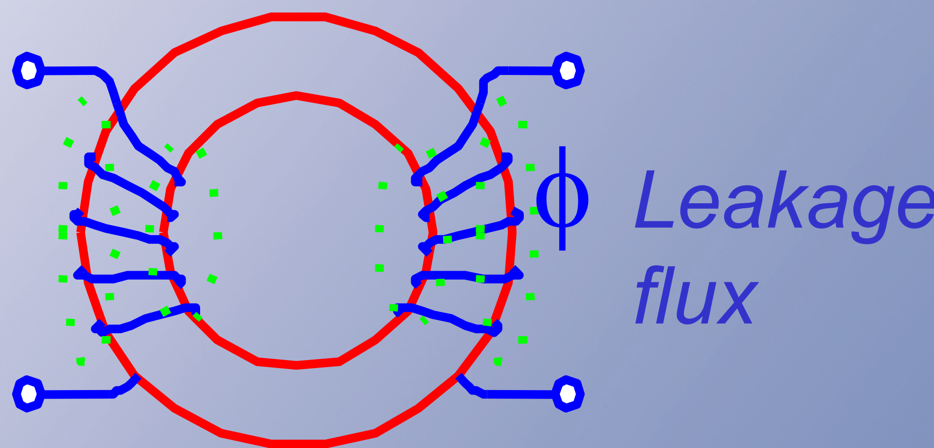
Real Transformer

IF $v = d\lambda/dt$,

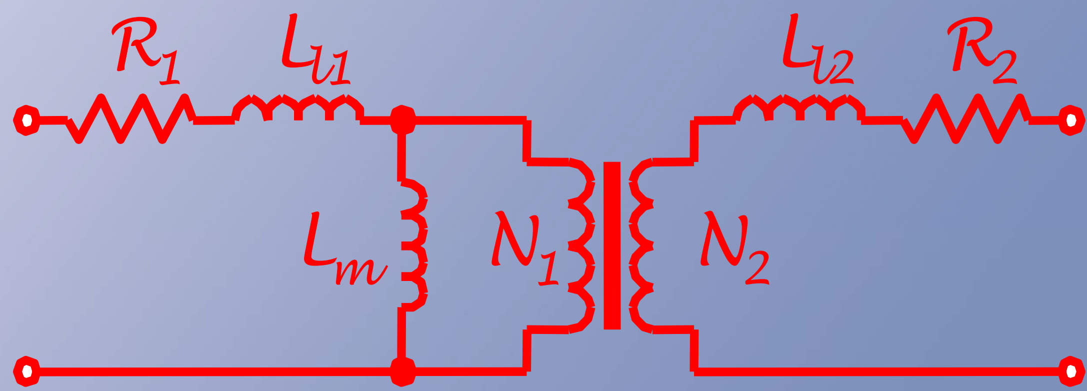
$\phi = Ni/\mathscr{R}$

$v = N^2/\mathscr{R}\ di/dt$

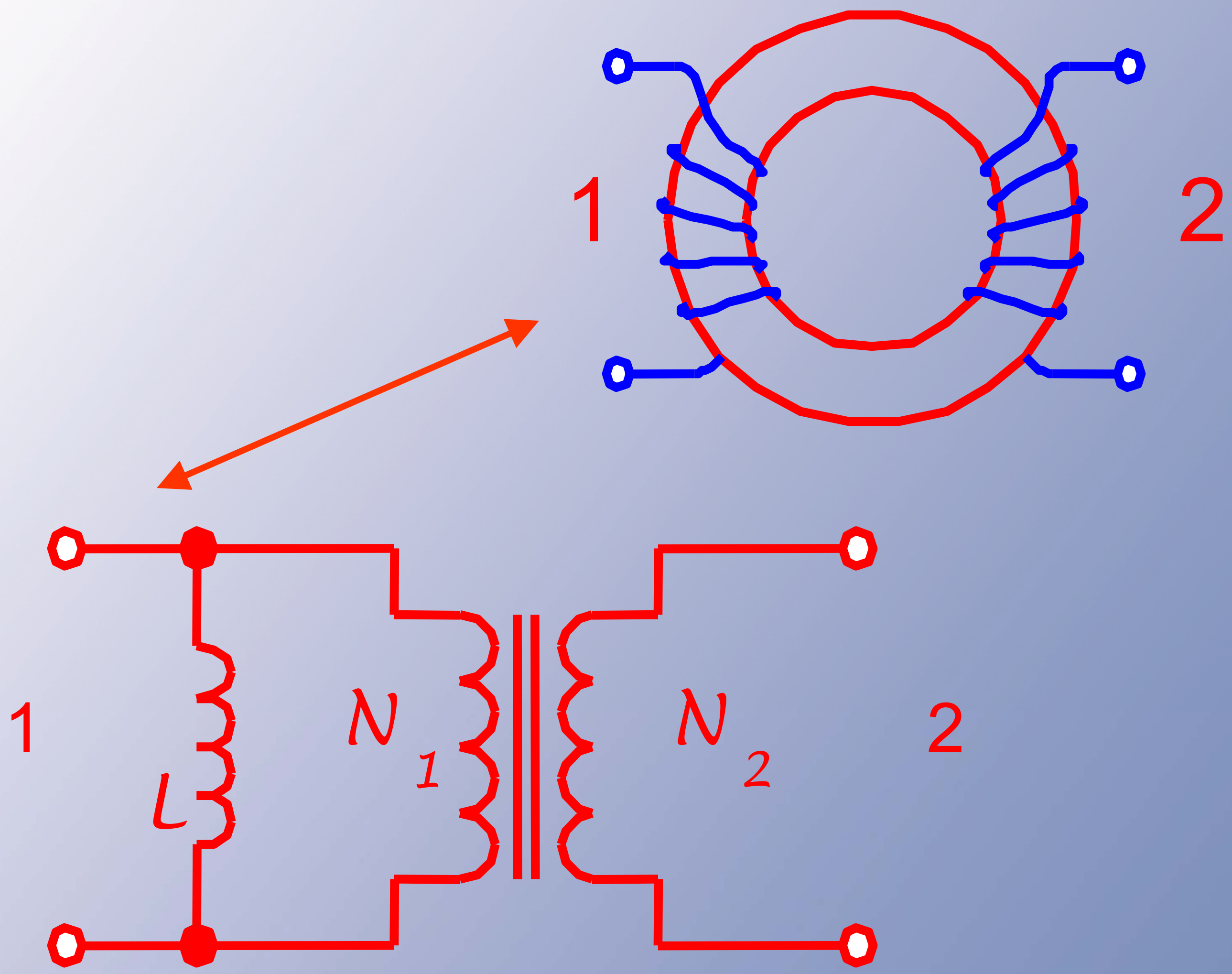
$= L\ di/dt$



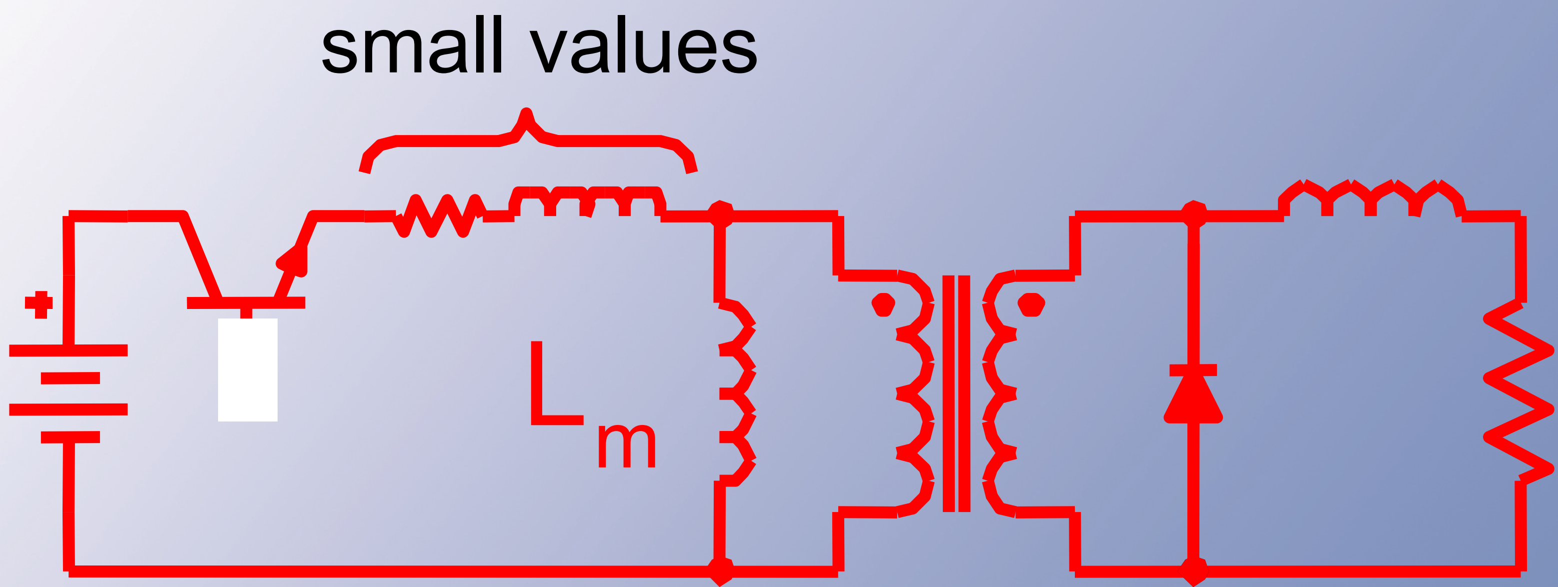
Circuit Model:



Real Transformer



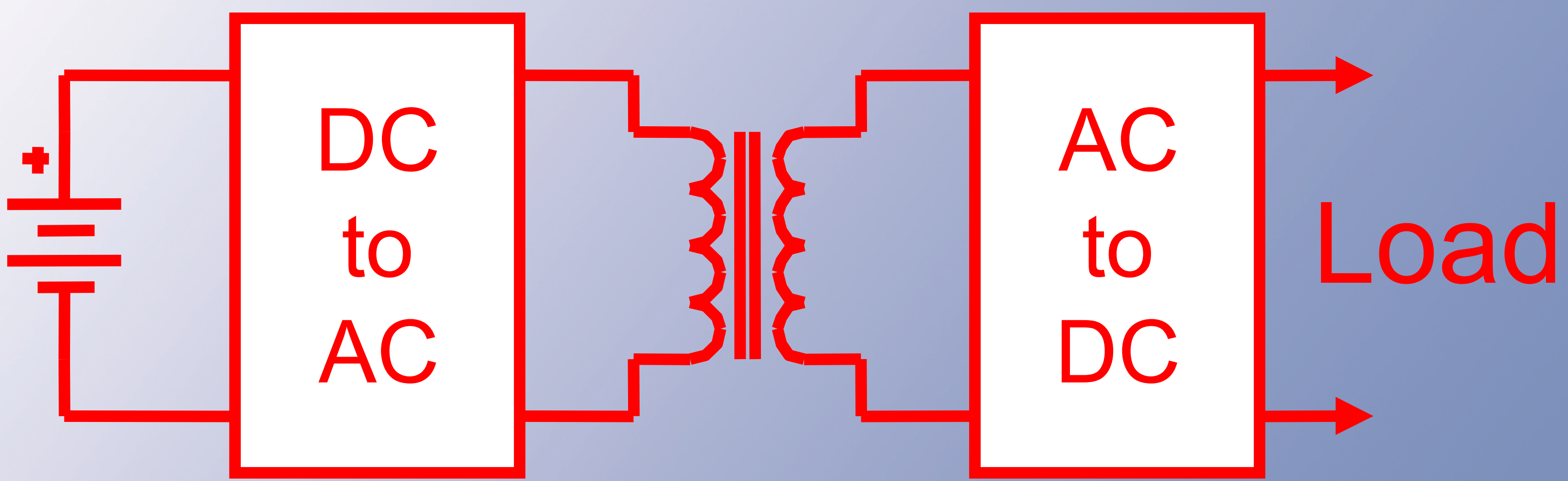
Real Transformer



Does not work! Average voltage across L_m is not zero.

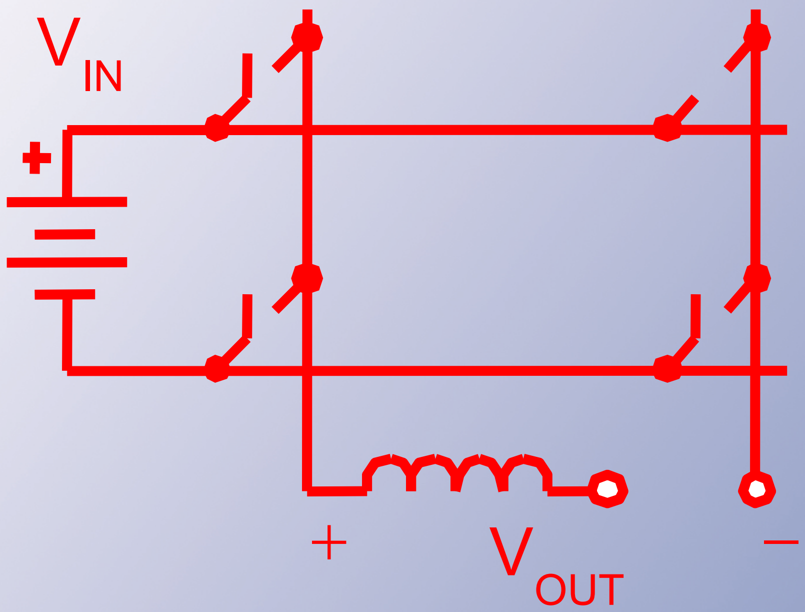
How to insert a transformer

Need an "ac" node

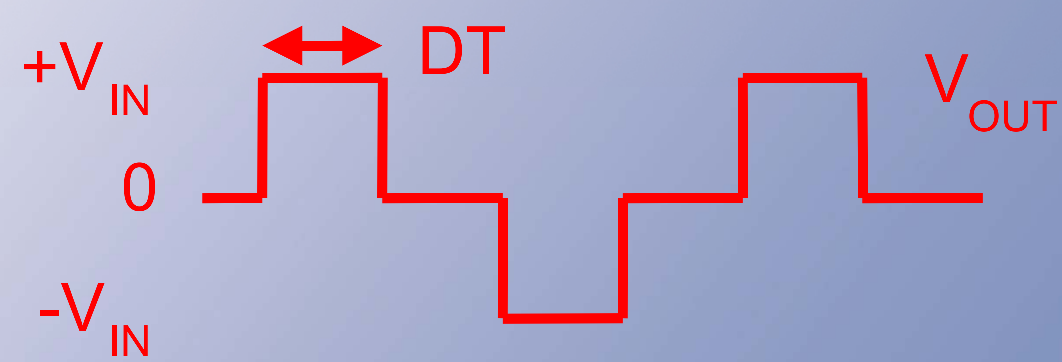


ac link converter

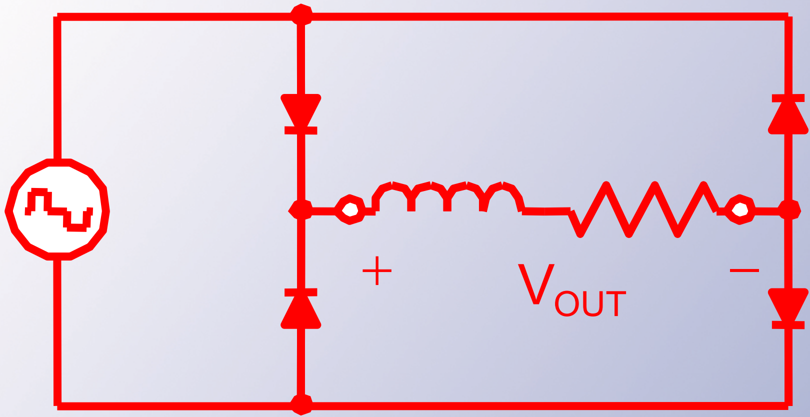
How to insert a transformer

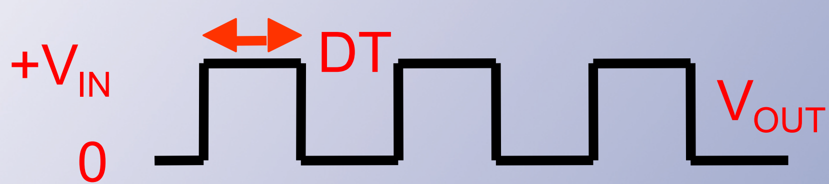


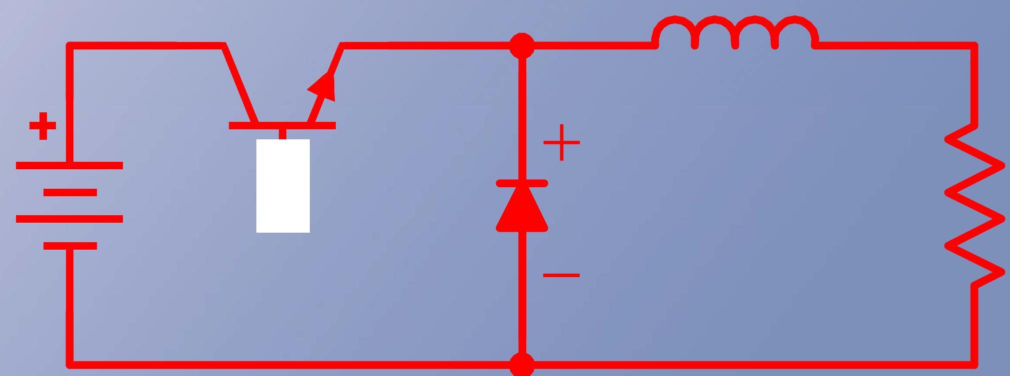
$V_{OUT} = \begin{cases} +V_{IN} \\ 0 \\ -V_{IN} \end{cases}$



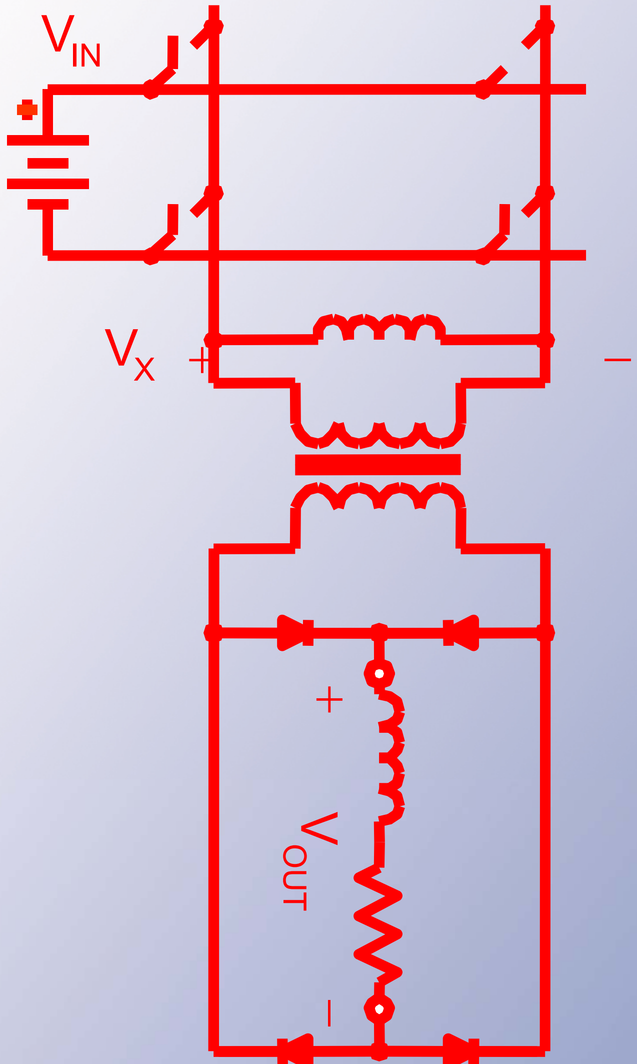
How to insert a transformer

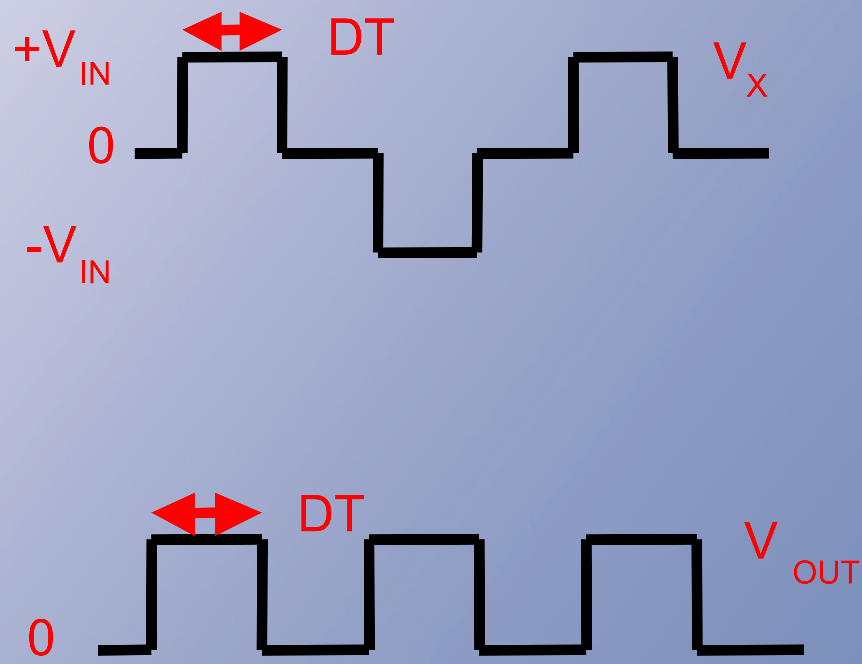




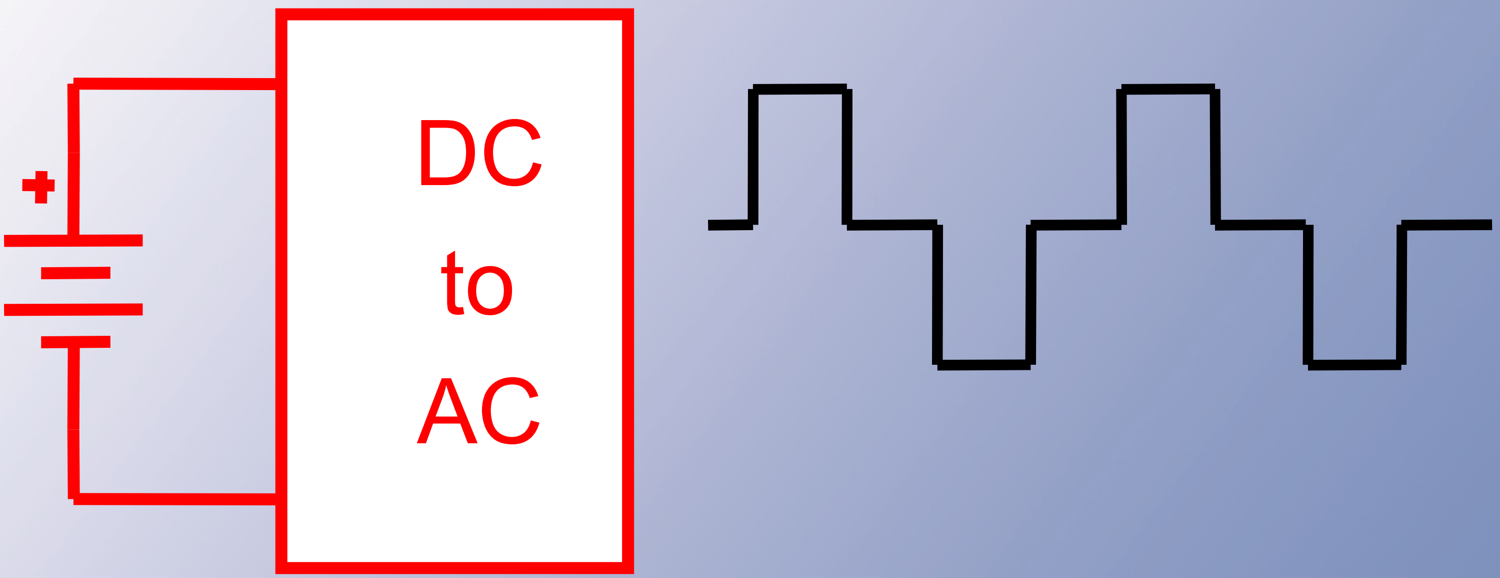


How to insert a transformer





Full-Bridge Forward Converters
Buck “Matrix” Converter

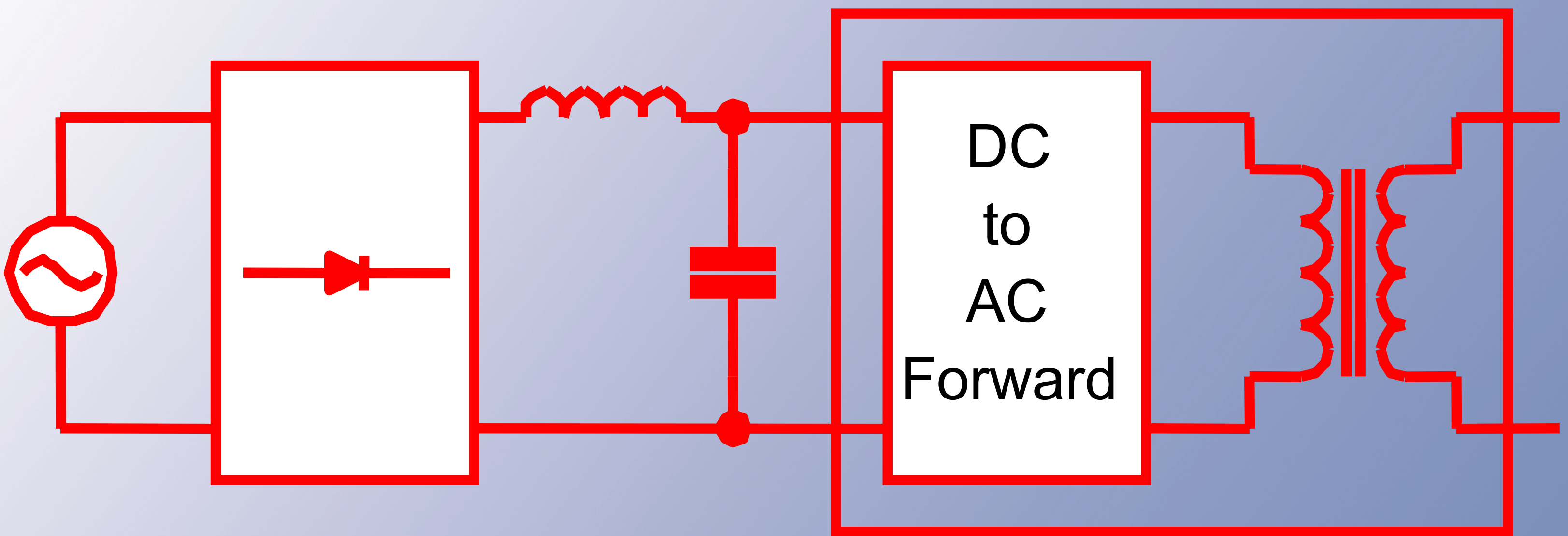


500 W or more

Forward Converters

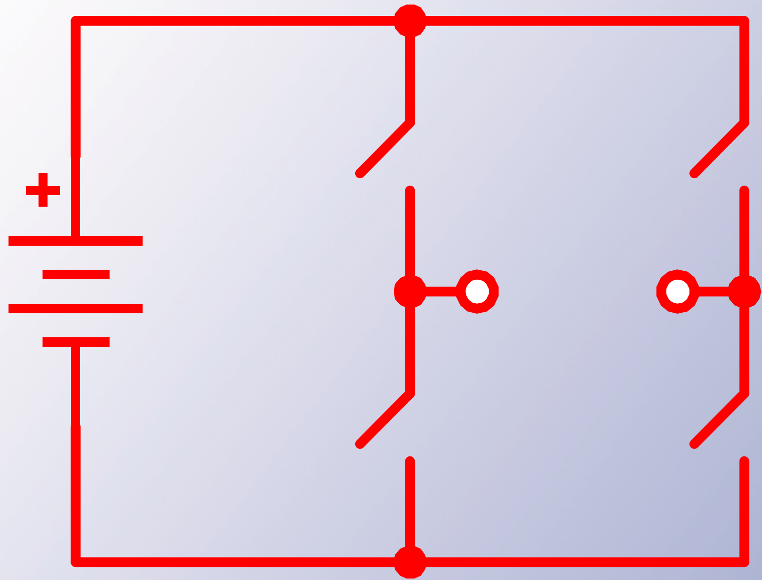
- When a magnetic transformer is inserted into a dc-dc converter, the resulting structure is called a *forward converter*.
- There are two general types:
  - Ac link converters
  - Flux reset converters.

Full-Bridge Forward Converters



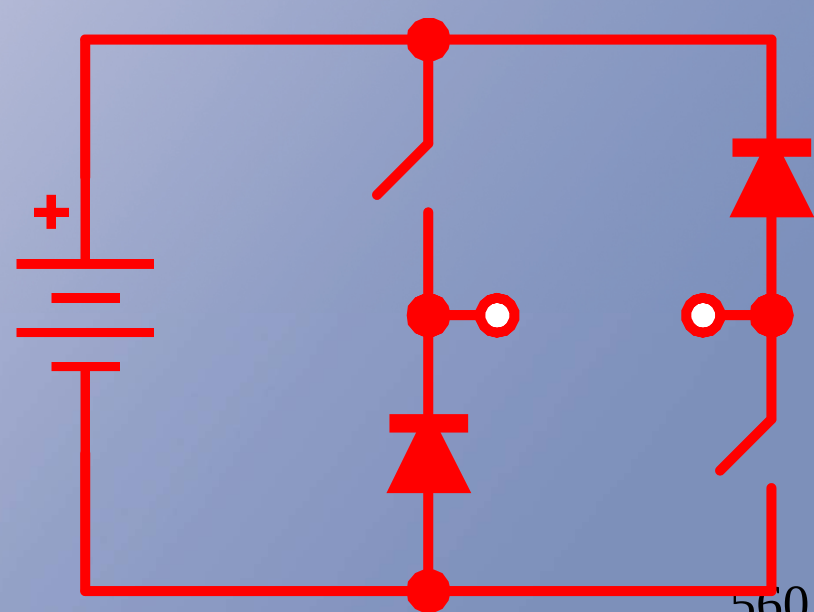
Matrix inverter

Full-Bridge & Single-Ended

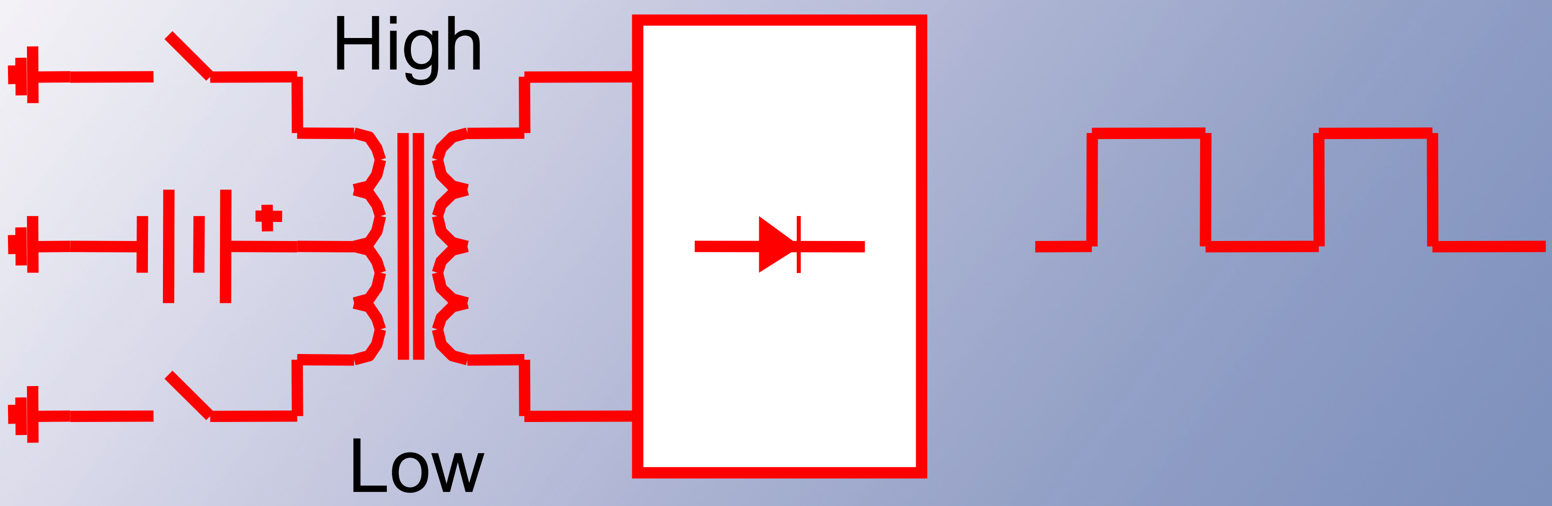


Full-bridge forward

"Single-ended" forward converter



Push-Pull Forward Converter



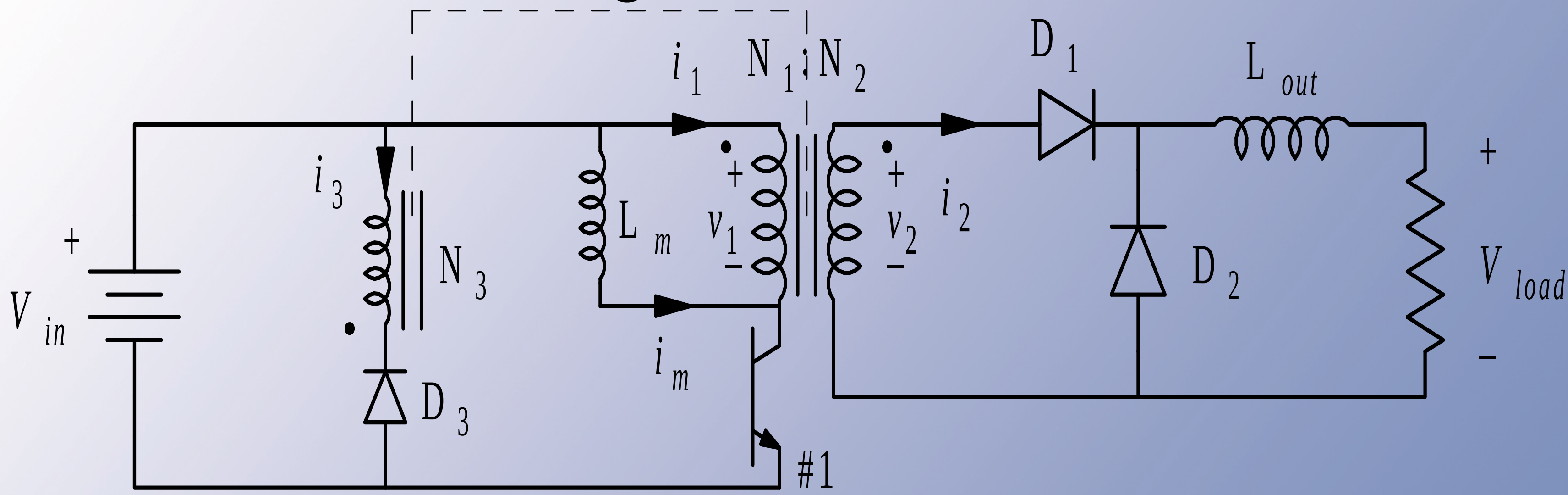
Forward Converters

- The converters so far are all ac link converters.
- They are based on the buck converter, and are called “buck-derived forward converters.”
- Boost-derived converters are just as feasible, and use an input current source.

Forward Converters

- Bridge-type forward converters are used at high power levels.
- Common at 1 kW and up.
- Flux reset converters tend to be simpler, and sometimes appear in place of flyback converters.
- The idea is to provide a current path with some other winding.

Catch-Winding Forward Converter

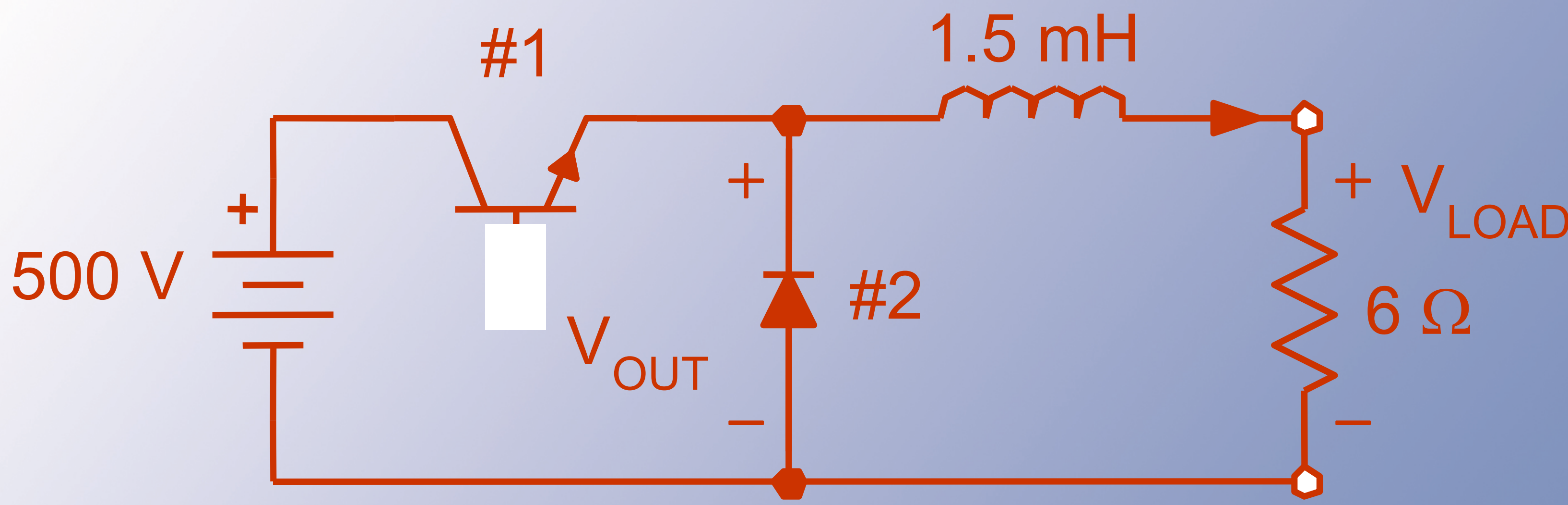


- In this circuit, a third winding acts like a flyback converter.
- The average voltage across L_m can be zero now, if duty ratio is limited.

Buck converter -- Filter

- A buck converter with 6 ohm load, 500 V input, 10 kHz switching, 1.5 mH output inductor.
- Duty is 10%.

Converter Analysis



$$f_{switch} = 10\ kHz,\ D_1 = 10\%$$

$$v_{out} = q_1 V_{in} + q_2\,(0)$$

$$\langle v_{out} \rangle = D_1 V_{in}$$

$$= \langle v_{load} \rangle$$

Converter Analysis

- Average output: D_1 Vin = 50 V
- Inductor current (average): (50V)/(6 ohms) = 8.33 A
- Variation: with the diode on, the inductor ideally sees -50 V. This lasts 90% of a period, or 90 us.

Filter Analysis

$\langle v_{LOAD} \rangle = 50\ V$

$$I_L = {}^{50V}/_{6\Omega}$$

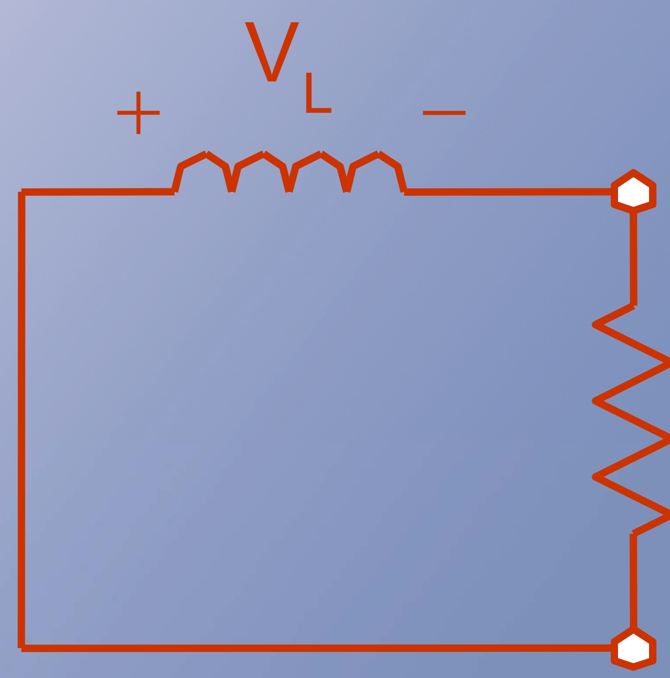
$$= 8\ {}^1/_3\ A$$

#2 ON

$v_L = -50\ V$

$= L\ di/dt$

$= L - \Delta i / \Delta t$



Filter Analysis

$$50\ V = L\ di/dt$$

$$\Delta t = 90\%\ \text{of a period}$$

$$T = 100\mu s$$

$$\Delta t = 90\mu s$$

$$(50\ V)\ 90\mu s\ /\ 1.50\ mH = \Delta i$$

$$= 3\ A$$

Filter Analysis

- 50 V = L di/dt. Since 50 V/L is intended to be constant, this is nearly a slope, 50V = L Δi/Δt.
- (50 V)(90 us)/(1.5 mH) = 3 A.
- This translates to an output change of $\pm$ 18 V, hardly fixed.

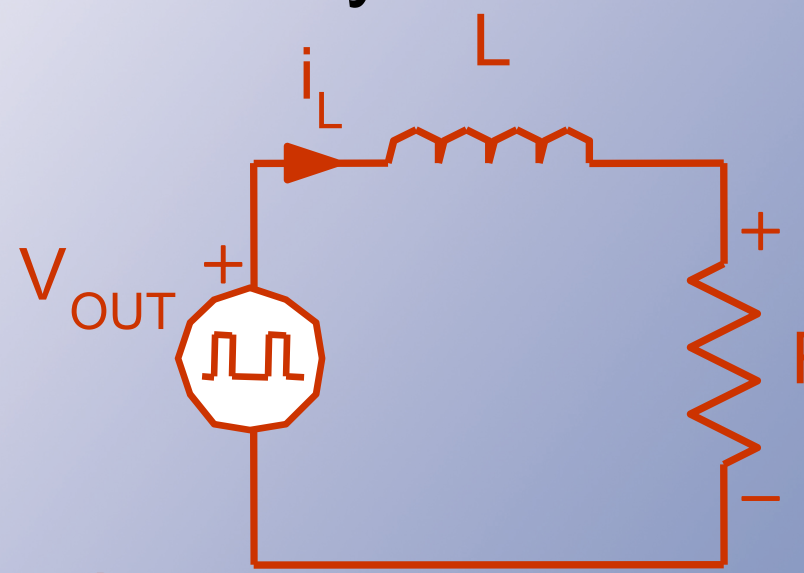
Check Ideal Action

- Can the ideal action assumption still be used?
- For the actual exponential action, the average output is still 50 V.
- The actual Δi value is 2.996 A.
- Ideal action overestimates by 0.12%.

Filter Analysis

$\Delta v_{LOAD} = 18\ V$

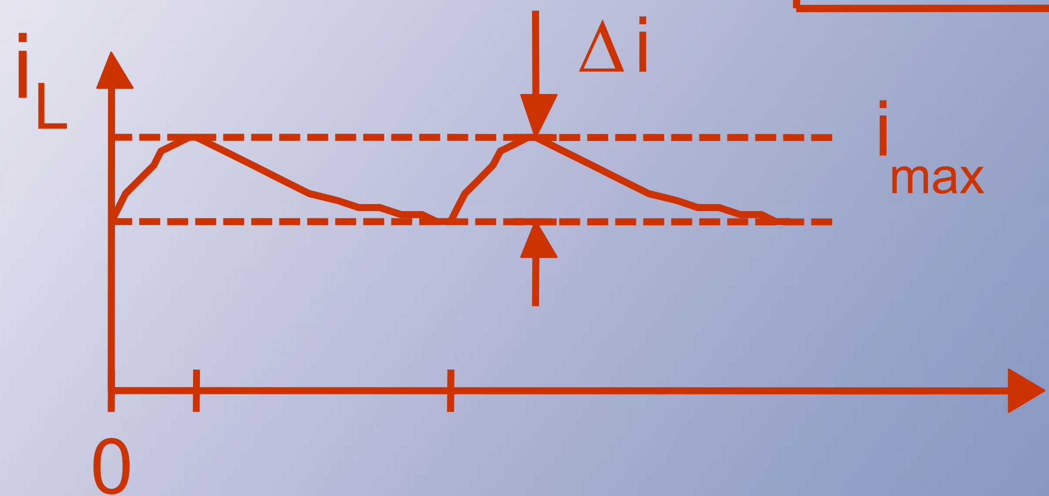
$\langle v_{LOAD} \rangle = 50\ V$



$-t/\tau$

$/$

$\tau = {}^{L}/_{R}$



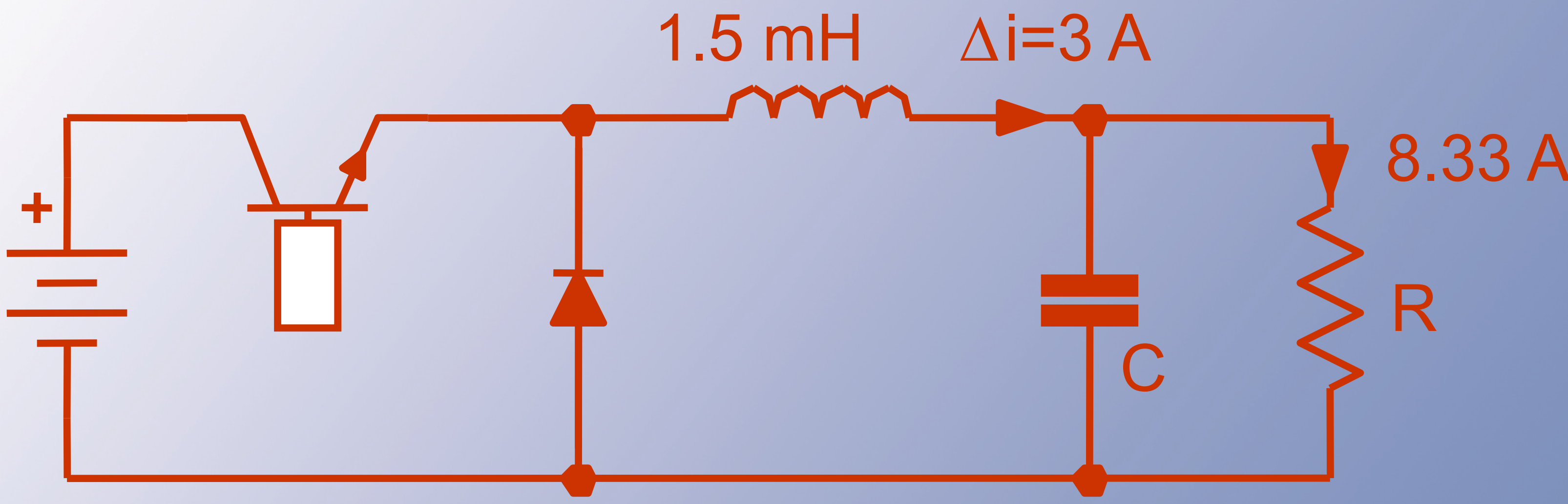
$\Delta i_L = 2.996\ A$

$= 3.00\ A$

Filter Analysis

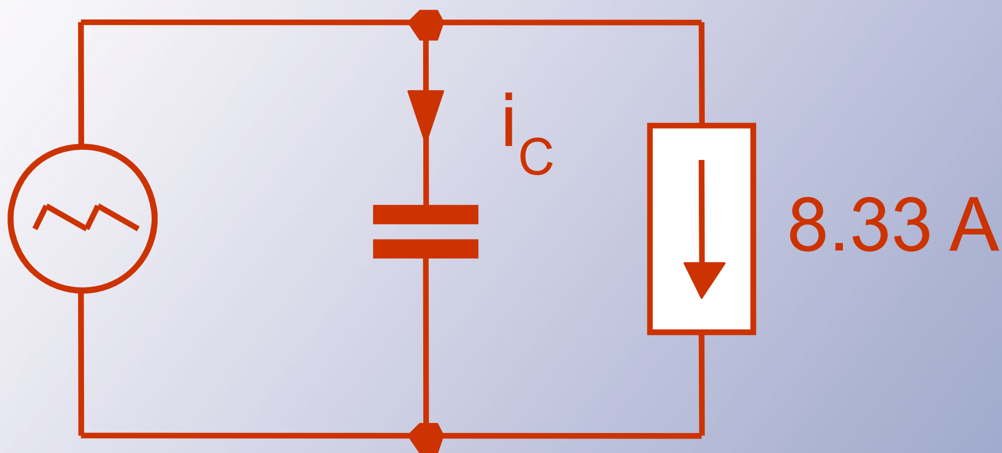
- Actual Δi is 0.12% lower than the 3A from "Ideal Action," even though Δi is ~ 35% of $< I >$.
- This is a conservative estimate. (Ideal action overestimates the ripple.)

Capacitive Filter



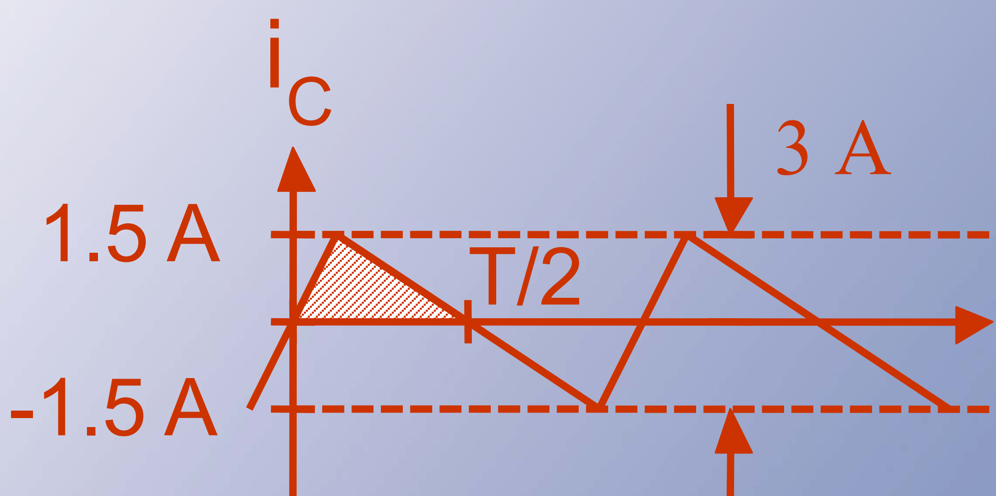
Add C to make Δ v_{out} < 1% peak-to-peak.

Capacitive Filter



$$i_c = C \, {dv}/{dt}$$

$$\neq C \frac{\Delta v}{\Delta t}$$



$$\frac{1}{c}\int i_c \, dt = v$$

Capacitive Filter

$$\frac{1}{C}\int_{t_0}^{t_1} i_c\, dt = v(t_1) - v(t_0)$$

- If we choose the right times, this gives Δv.
- The integral is the area under a triangle.

Capacitance Value

$$\frac{1}{C}\int_0^{T/2} i_C(t)\,dt = \Delta v_{peak-to-peak}$$

- The area integral: a triangle, ½ x base x height, or (1/2) x (T/2) x (Δi/2).
- Therefore:

$$\Delta v_{peak-to-peak} = \frac{1}{C}\frac{1}{2}\frac{T}{2}\frac{\Delta i}{2} = \frac{T}{8C}\Delta i$$

Capacitance Value

- We want $\Delta v < 0.5$ V.
- Di/2 = 1.5 A.
- $(1/C) \times (25\ \mu s)(1.5\ A) = \Delta v < 0.5$ V.
- This requires C > 75 uF.
- Might use 100 uF.

Converter Example

- Input: +6 V to +15 V.
- Output: +12 V ± 1%, 24 W.
- Common ground, input and output.
- This cannot be met with buck, boost, buck-boost, or boost-buck.
- Need flyback, SEPIC, or Zeta.
- Example: Flyback design.

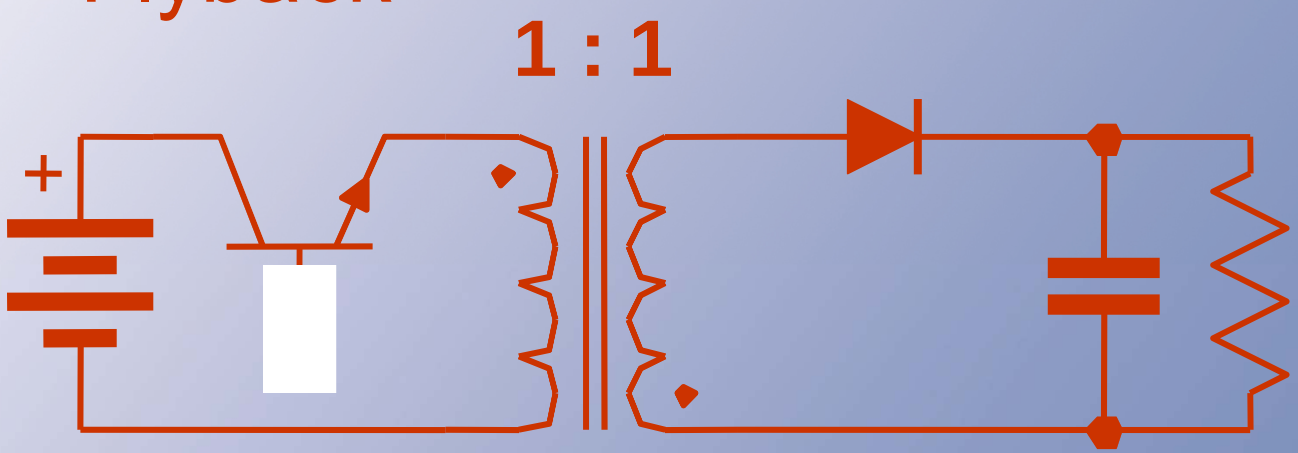
Design:

Input: +6 V to +15 V

Output: +12 V ± 1%, 24 W

Common ground

Flyback

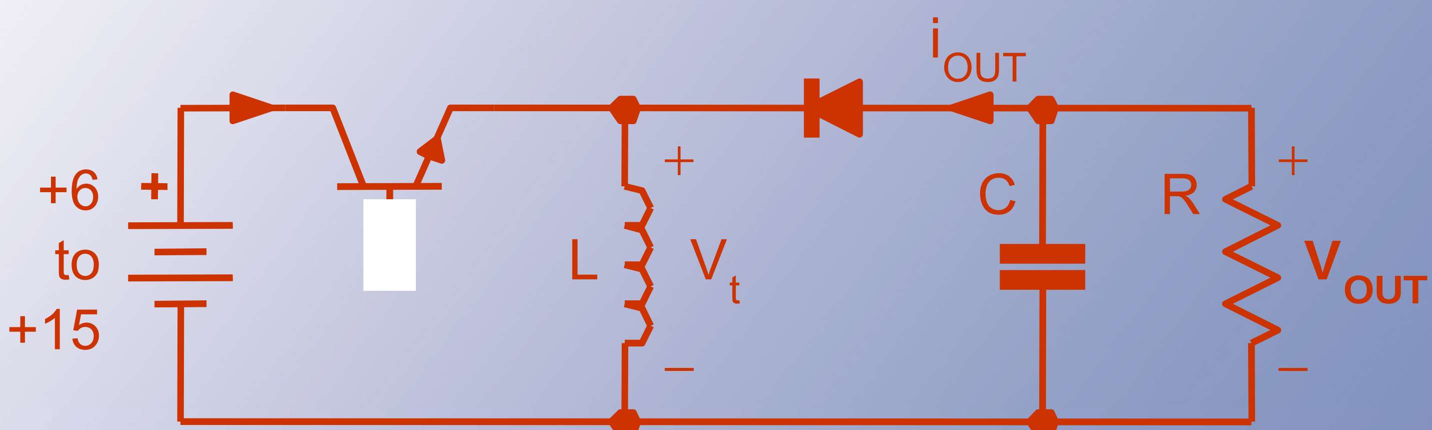


Equivalent Buck-Boost

Devices: MOSFET & Diode

f_{switch} ~ 50 kHz to 200 kHz

24 W, 12 V output. Equivalent buck-boost:



V_{out} = -12 V

Analysis

- Let f_{switch} = 200 kHz. Then T = 5 us.
- Transfer source: $v_t = q_1 V_{in} + q_2 V_{out}$.
- Since $\langle v_t \rangle = 0$, we have $D_1 V_{in} = D_2 |V_{out}|$
- Duty ratios: $(D_1/D_2) \times V_{in} = |V_{out}|$.

Duty Ratios

- For +6 V in, $D_1/D_2 = 2$, $D_1 = 2/3$, $D_2 = 1/3$.
- For +15 V in, $D_1/D_2 = 12/15$, $D_1 = 4/9$, $D_2 = 5/9$.
- Range: $4/9 < D_1 < 2/3$, $1/3 < D_2 < 5/9$.

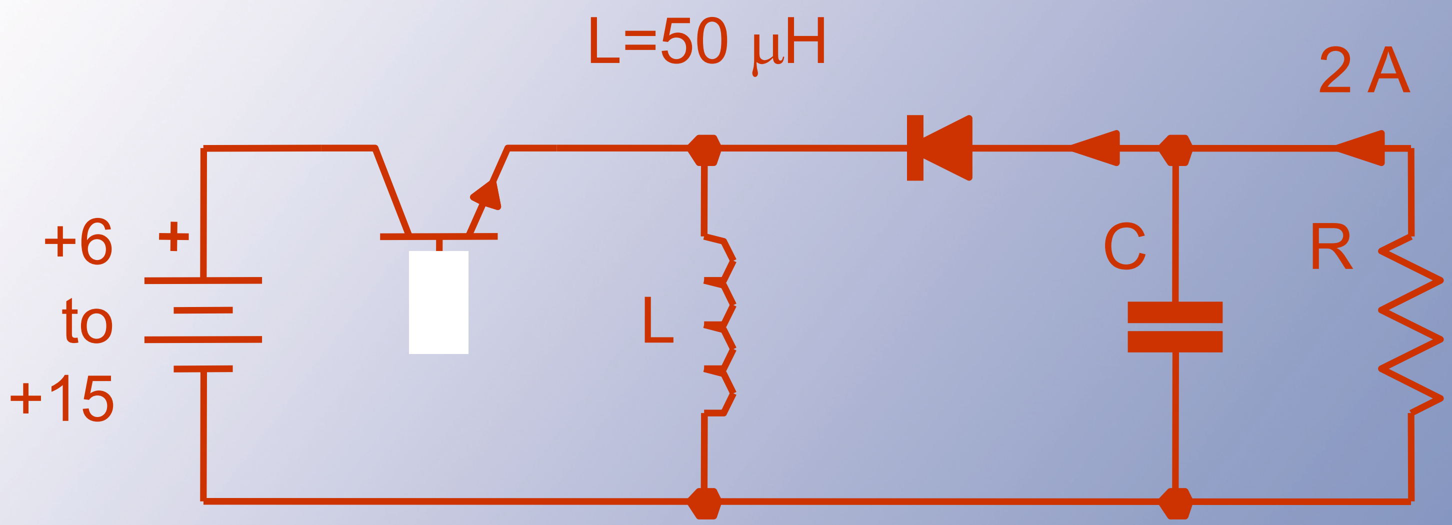
Currents

- I_{out} = (24 W)/(12 V) = 2 A.
- $q_2 \, I_L = i_{out}$
- $D_2 \, I_L = \langle i_{out} \rangle = I_{out}$ = 2 A.
- I_L = (2 A)/D_2
- I_L range is 3.6 A to 6 A.
- For design, might let $\Delta i_L = \pm 10\%$, or 20% peak to peak.
- Requires $\Delta i_L < (0.2)\text{x}(2\text{ A})/D_2$.

Inductor Voltage

- $v_L = L\, di/dt$.
- When the diode is on, $v_L = -12$ V, a constant. During that time, then, we have $v_L = L\, \Delta i/\Delta t$, with $\Delta t = D_2 T$.
- $(12\text{ V})\,(\Delta t)/L = \Delta i < (0.2)(2\text{ A})/D_2$.
- Simplifies to $L > 30\, D_2^2\, T$.
- Need an L that works in all cases.

Equivalent Buck-Boost



Now, output voltage ripple.

Diode off: $i_c = 2A$ $\Delta t = D, T$

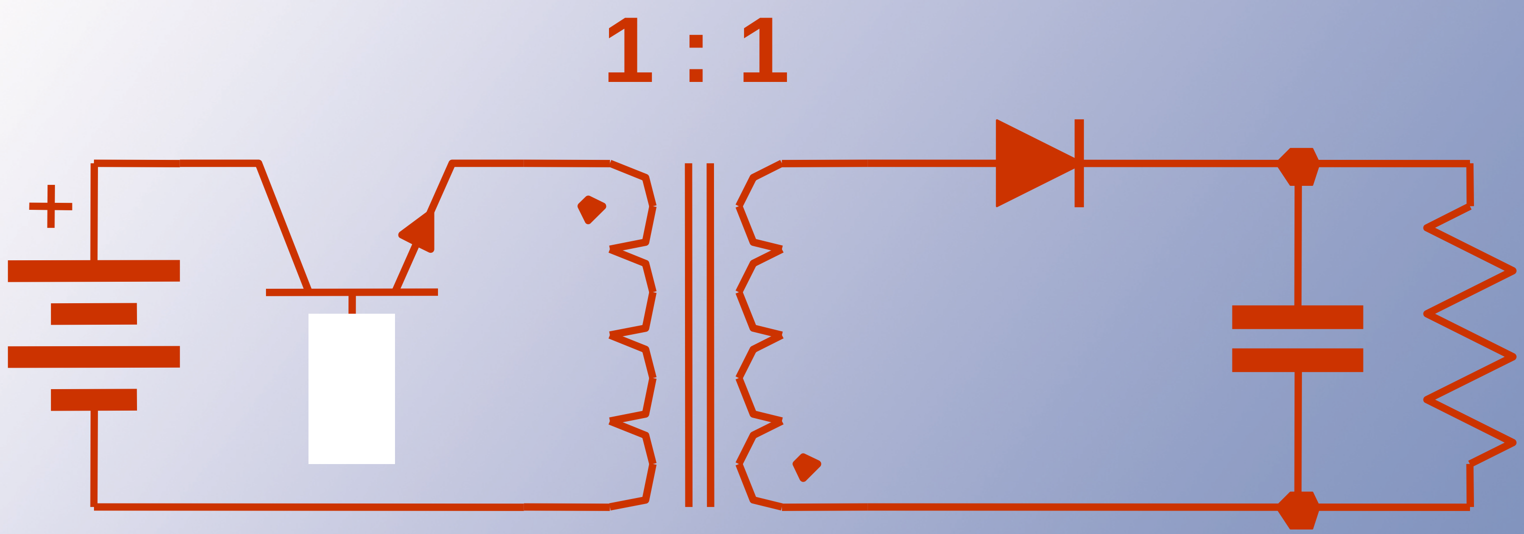
$= C\,{}^{dv}/_{dt}$ $\Delta v < (0.02)12$

$= C\,{}^{\Delta v}/_{\Delta t}$ $< 0.24\ V$

Output Voltage Ripple

- We know the capacitor current when the diode is off.
- Can find C.

Final Result



R → 6 Ω

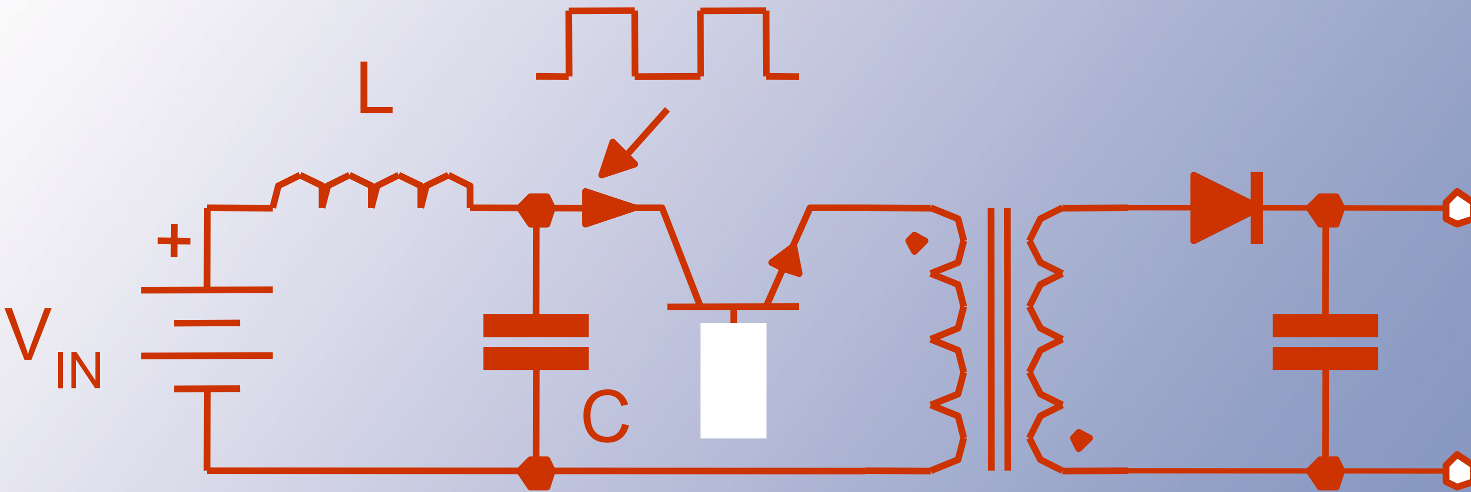
C = 30 μF

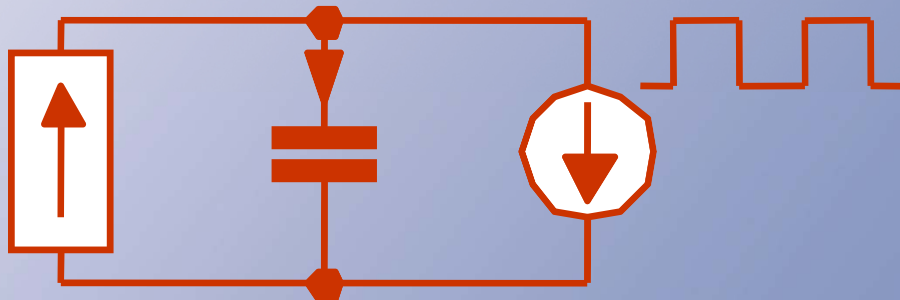
Coupled L:

Either side: = 50 μH

Same number of turns

Input filter?

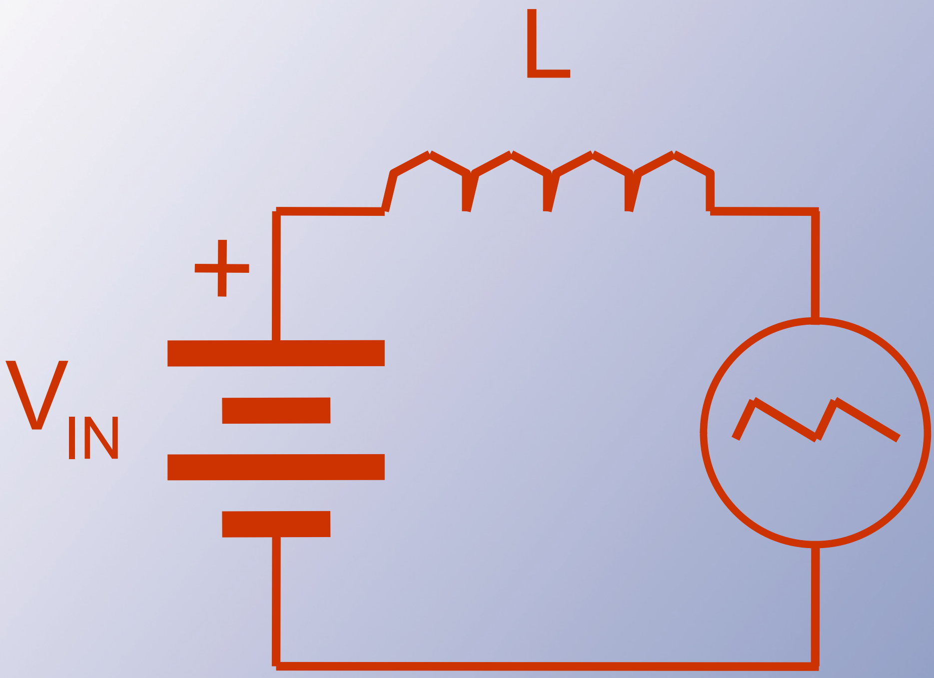


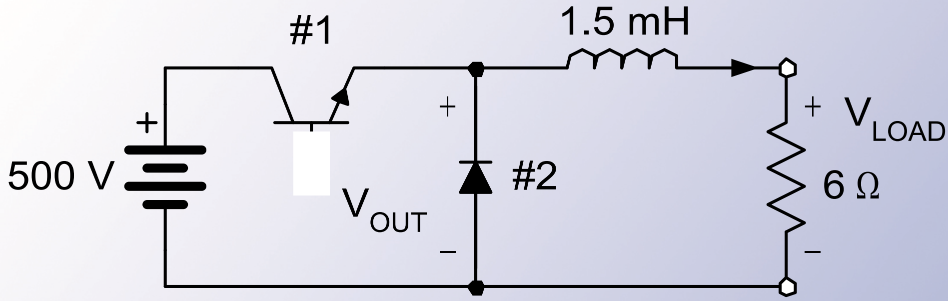


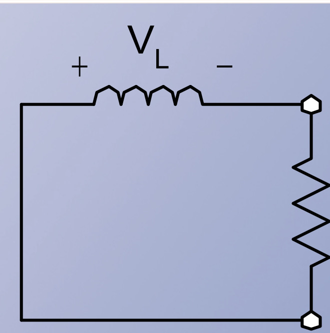
$\Delta V_{out} \rightarrow \pm 1\%$

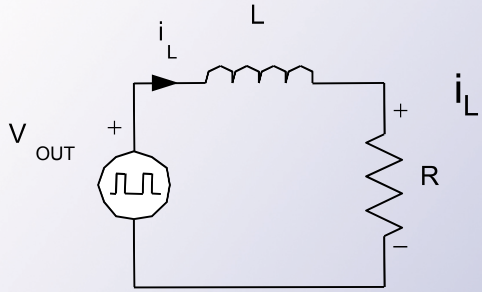
$\Delta I_{in} \rightarrow \pm 5\%$

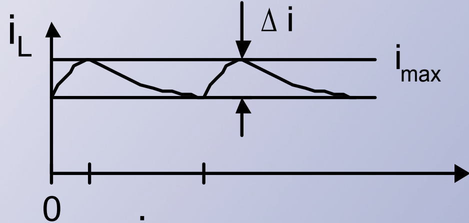
Ideal Action to Find Input

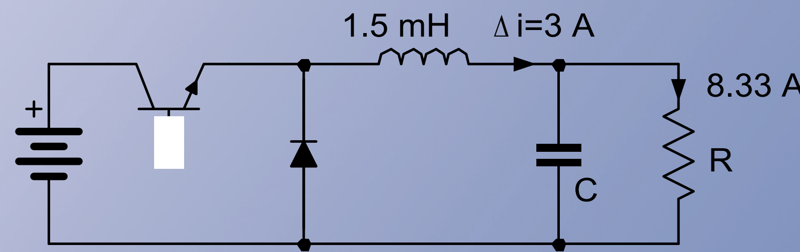


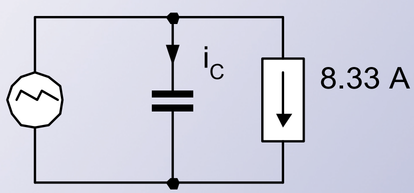

#1
1.5 mH
500 V
#2
V_{OUT}
V_{LOAD}
6 Ω

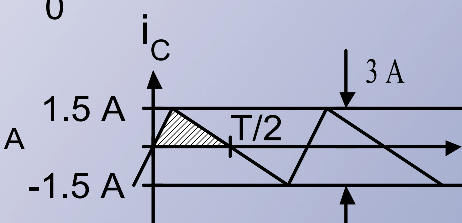

V_L

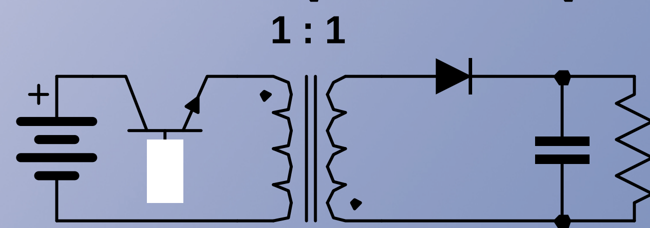

i_L
L
V_{OUT}
R

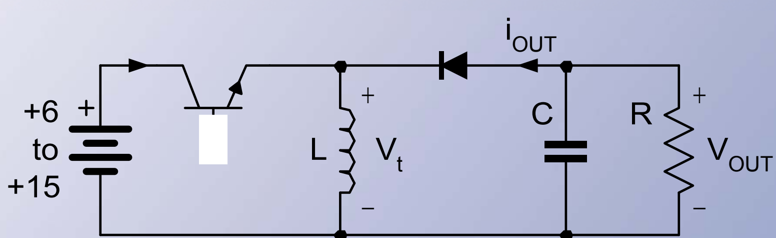

i_L
Δ i
i_{max}
0

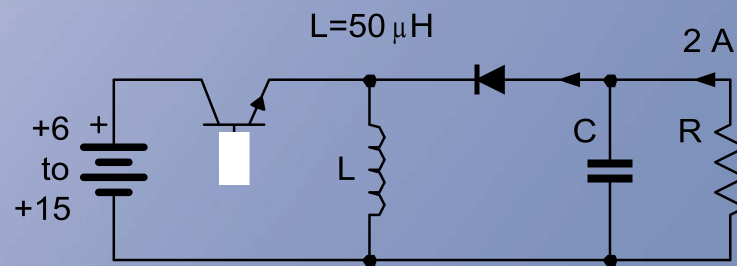

1.5 mH
Δ i=3 A
8.33 A
C
R

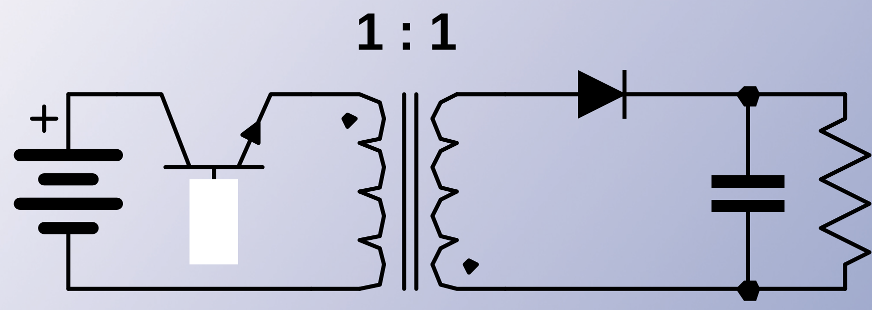

i_C
8.33 A

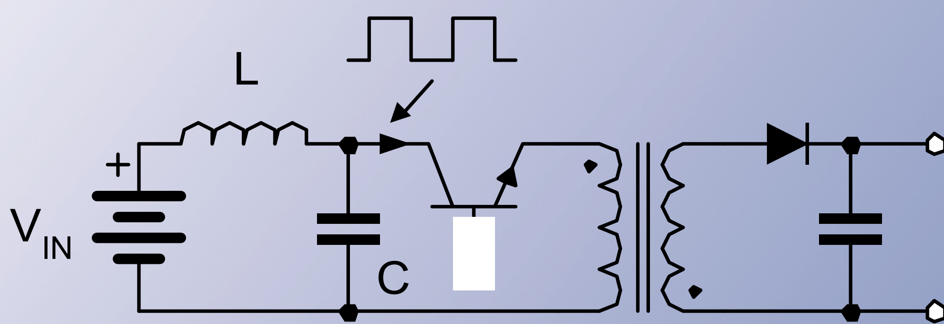

i_C
3 A
1.5 A
T/2
-1.5 A

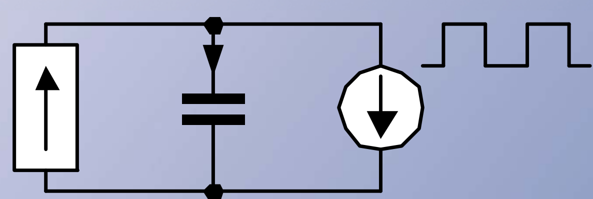

1 : 1

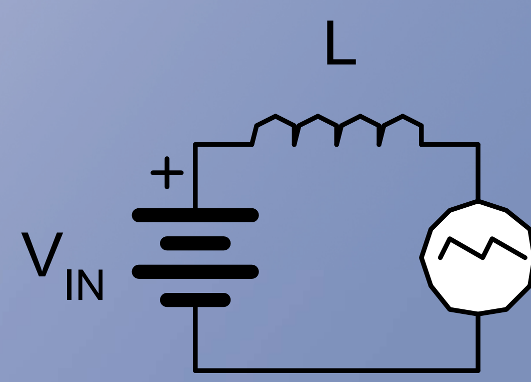

i_{OUT}
+6
to
+15
L
V_t
C
R
V_{OUT}


L=50 μH
2 A
+6
to
+15
L
C
R


1 : 1
+


L
+
VIN
C




L
+
VIN

Introduction to Rectifiers

- Rectifier is a general term for ac-dc conversion.
- Usually the term implies converters with ac voltage source input.
- In principle, an ac current could also be used.

The Basics

- Consider direct ac voltage to dc current conversion -- a 2 x 2 matrix.
- The switches should be FCBB (forward conducting, bidirectional blocking).
- SCRs and GTOs are appropriate.
- But we know that diodes can be used, too (but no control is possible!)

DAY 6 Start Frequency Matching

- The input frequency is the same as the ac source.
  - Low frequency mains (50 Hz, 60 Hz)
  - Higher frequencies if an ac link is involved
- The output frequency is 0 Hz.
- If $v_{out} = q\ v_{in}$, with $v_{in} = V_0 \cos(\omega t)$, the product trig identities for $q\ v_{in}$ give $\cos[(n\omega_{switch} \pm \omega)t]$.

Frequency Matching

- We want 0 Hz output.
- If switching is performed at the input frequency, the n=1 term gives rise to dc and to $2\omega_{in}$ at the output.
- Thus 50 Hz in → 50 Hz switching
- This gives both 0 Hz output and 100 Hz ripple, plus harmonics.

Reality Issues

- To provide a current source, we need to keep current nearly constant under a large $2\omega_{in}$ ripple voltage.
- Consider 120 V_{RMS} input (170 V peak) at 60 Hz, with a 12 W load.
- The average current is ~ 0.1 A.

Filter Realities

- With 170 V peak input, if the inductor is large, the output could be $<|v_{in}|>$, which is $2V_0/\pi$.
- The output is 108 V dc.
- What if the current ripple does not exceed $\pm$5%, or 0.01 A peak-to-peak?

Filter Realities

- To estimate the inductor size, we could formally integrate the voltage waveform.
- Instead, let us get a quick estimate.
  What L is needed for a 60 V signal lasting 1/240 s to give a change of less than 0.01 A?

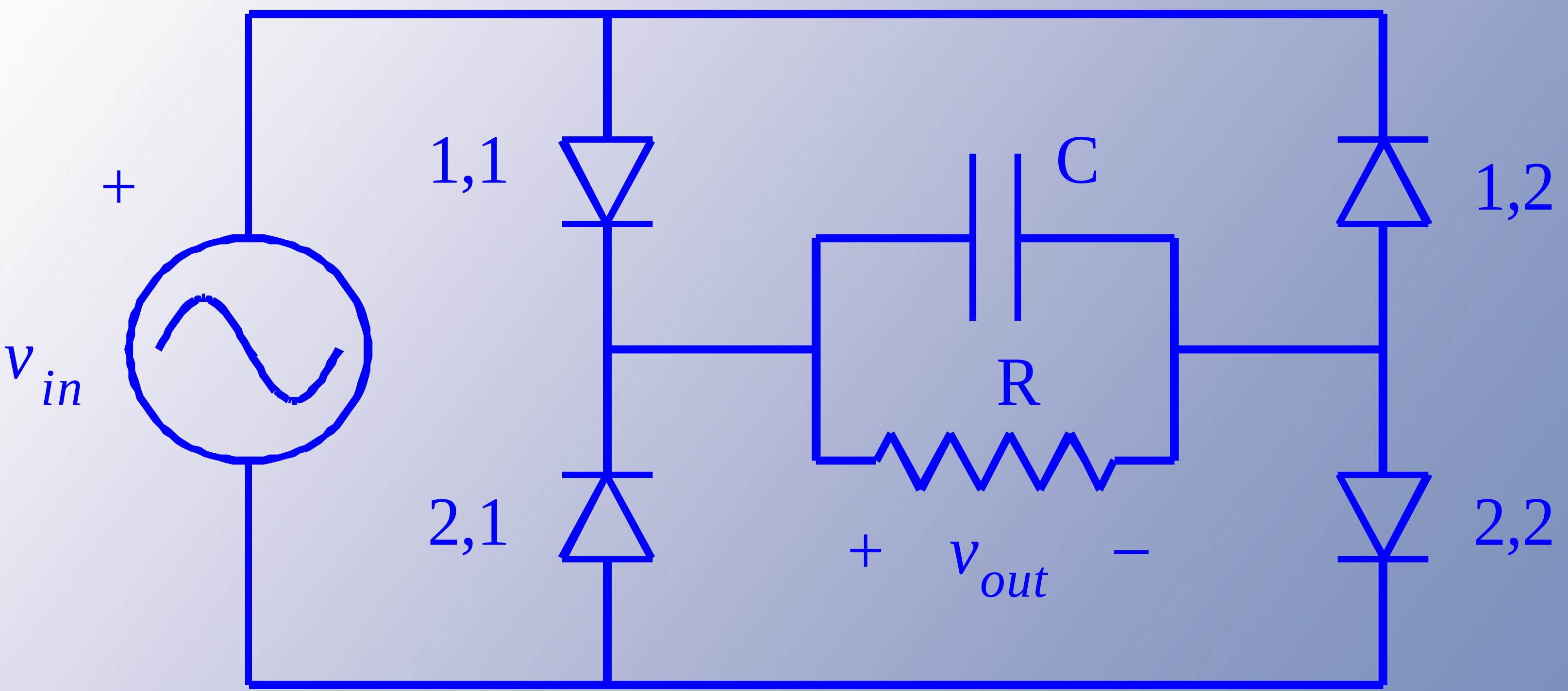
Filter Realities

- 60V= L di/dt ~ L Δi/Δt, Δt=1/240 s.
- Δi < 0.01 A requires L > 25 H.
- These excessive inductor values are typical for low-power rectifiers.
- Can we dispense with the inductor entirely?

The Classical Rectifier

- The classical rectifier is a diode full-wave or half-wave circuit operating into a capacitive filter.
- This might be expected to have KVL problems, and it does!
- At low power levels, simplicity sometimes outweighs problems.
- Even so, circuit are disappearing in favor of switching converters.

Classical Rectifier

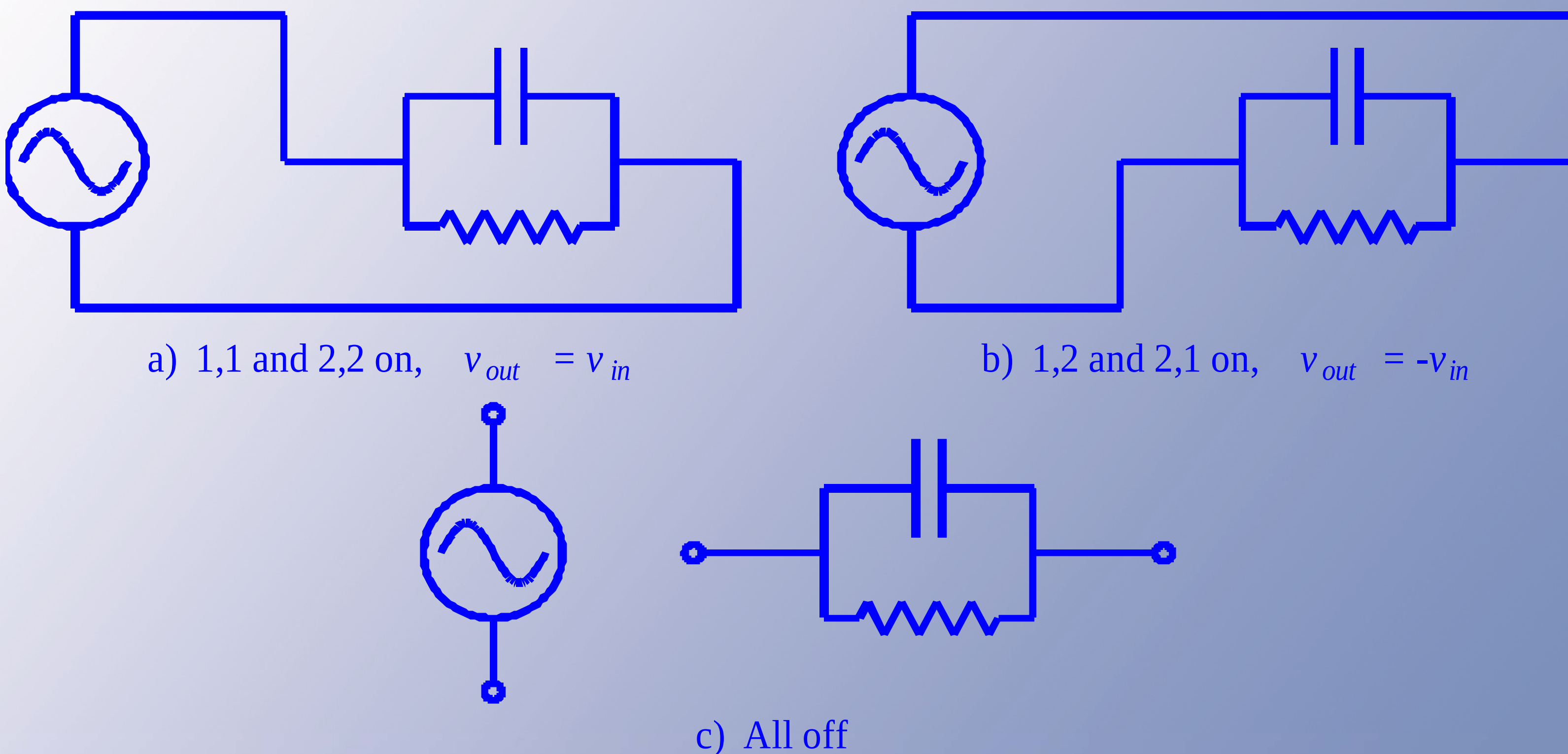


Notice the "voltage to voltage" arrangement.

Trial Method

- Three configurations are allowed :
  - 1,1 and 2,2 on is consistent when the input current is positive.
  - 1,2 and 2,1 on is consistent when the input current is negative.
  - All off is consistent when C keeps the output about $|v_{in}|$.

Trial Method



In fact, “all off” is active almost all the time!

The Transitions

- Start at the input peak, time t=0.
- Let us set 1,1 and 2,2 on (trial method).
- The input current is V_0/R + C dv/dt, but dv/dt = 0 at t = 0.
- Input current > 0 -- consistent.
  Off devices have v < 0 -- consistent.

The Transitions

- Slightly later, $i_{in} = v_{in}/R + C\, dv/dt$, and dv/dt is negative.
- After a time, the negative capacitor current plus the positive resistor current add to zero.
- At that moment, all diodes turn off.

The Transitions

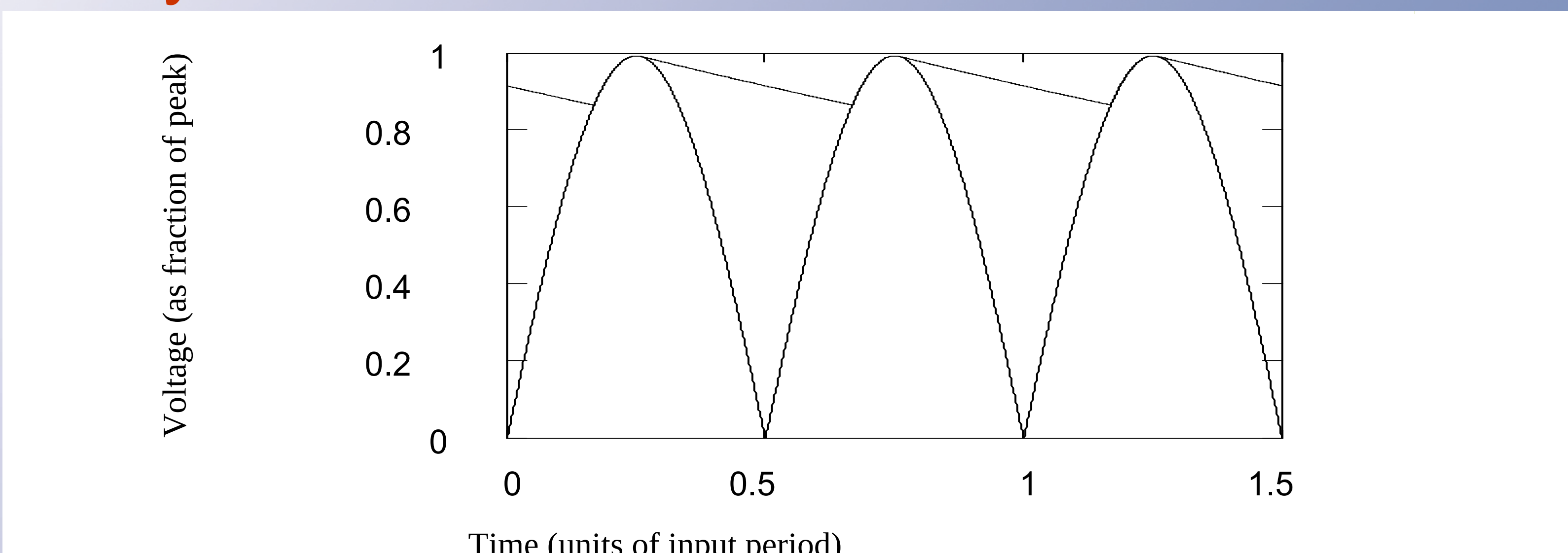
- This happens when $-\omega C V_0 \sin(\omega t) + V_0 \cos(\omega t)/R = 0$, or
- $\tan(\omega t) = 1/(\omega RC)$.
- The resistor represents the load.
- Once the diodes are off, the output voltage decays exponentially.

The Exponential

- During the decay, $v_{out} = V_{max}\, e^{-t/\tau}$.
- We can keep the decay small (and the ripple small) with a long time-constant.
- How long? For any x, e^x is
  $e^x = 1 + x + x^2/2! + x^3/3! + \ldots$
- For small x, $e^x \approx 1 + x$. Linear.

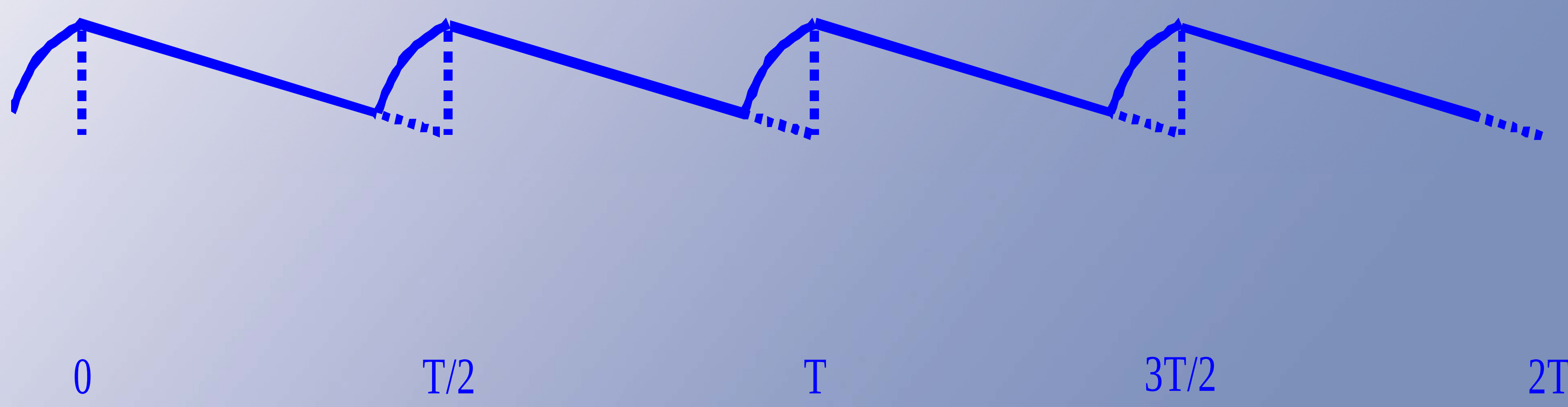
Analysis

- The decay will continue until the exponential fall hits the rising voltage waveform, $V_{max}e^{-t/\tau} = |V_0 \cos(\omega t)|$.
- We get a short sinusoidal piece attached to a nearly linear fall.



Worst-Case Ripple

- Worst-case ripple is easy to estimate.
- What if the fall is truly linear?
  It cannot last longer than the time between voltage peaks.
- The time is T/2 in the full-wave case.



Worst-Case Ripple

- The fall is overestimated by the triangle, $V_0 e^{-(T/2)/\tau} \approx V_0 (1 - T/(2\tau))$.
- With the RC time constant, the actual fall is approximately $T/(2RC)$, with $T = 1/f_{in}$.
- Thus $\Delta v/V_0 \approx 1/(2 f_{in} RC)$.
- Half-wave case has no 2.

Design

- The load current $I_{load} \approx V_0/R$.
- Therefore, $\Delta v = I_{load}/(2fC)$.
- Example: 12 V, 1 W supply with 1% ripple.
- $\Delta v = 0.12$ V, $I_{load} = 0.083$ A.
- We need C = 5800 uF.

Design Example

- Example: 230 V rms input, 50 Hz.
- Want 5 V output at 10 W.
- Ripple should not exceed ±0.5%.
- First, a peak value of 5 V is needed at the output. The input peak is 325 V, so a 65:1 transformer is needed.
- Second, $\Delta v = I/(2fC)$, and $\Delta v < 0.05$ V.
  - This gives C > 400000 uF.

Diode Timing

- If ripple is 1% peak to peak, the voltage falls just 1% before it hits the input ac waveform again.
- The inverse cosine of 0.99 suggests that the diodes are on for about 8° on the angular time scale.
- This is a duty ratio of 8/180 = 4.4%.
- Each diode is on less than 5% of a cycle!

Current

- If the diodes are on just 5% of the time, the input current waveform has a 5% duty ratio, too.
- To deliver energy, the input current must flow in brief, high spikes.
- Notice that as C → ∞, the current must increase without bound.
- This is because we have a KVL problem!

Finding the Current

- The current includes a capacitance part C dv/dt and a resistance part v/R.
- When a diode is on, $v = v_{in} = V_0 \cos(wt)$.
- Then $C\, dv/dt = -wC\, V_0 \sin(wt)$.
- This is highest at the moment of diode turn-on, perhaps 8° before the peak.
- Thus 1% ripple gives a peak current of more than 50 C V_0 at 60 Hz.

Current Points

- The extreme current is almost all delivered to the capacitor.
- Current flows in short, high spikes.
- Ideally, the spikes are inversely proportional to the square root of the ripple spec.
- This means that the spike with 1% ripple is about 40% higher than that for 2% ripple.
- In reality, the current is limited mainly by line and stray transformer leakage inductance.

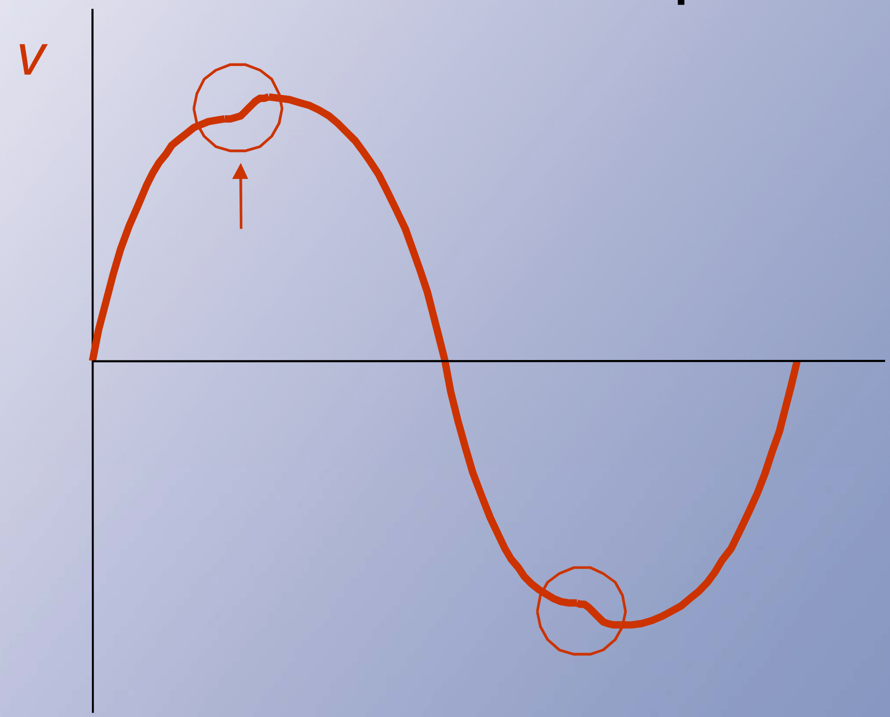
Current

- The power factor is especially poor.
- For 1% ripple we get ~ 0.26 power factor.
- Classical rectifiers are a major source of "power quality" problems.

Peak Input Current

- Low frequency → large C, transformer
- "KVL violation"

  Current flows in brief spikes



Regulation

- A classical rectifier has no line regulation: the output is proportional to the input voltage.
- The load regulation is half the ripple level (think about why this is).

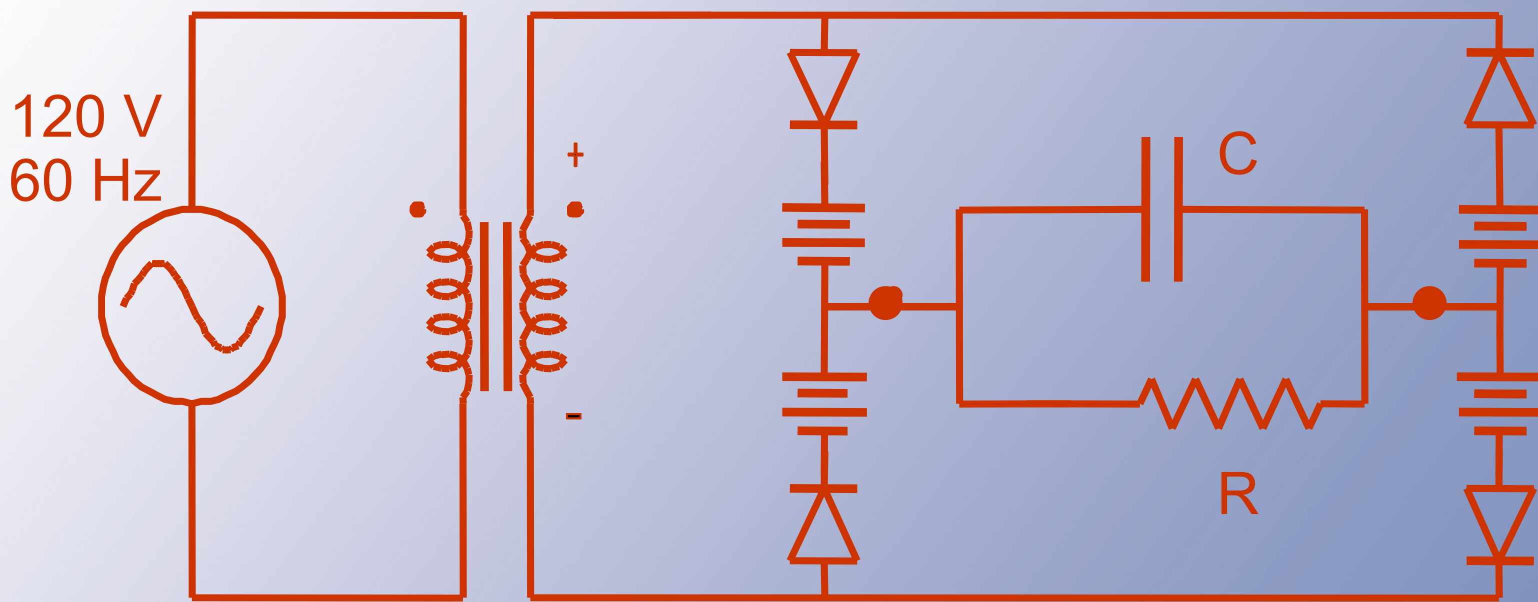
Inductive Filtering

- We would rather use a series inductor to avoid the KVL problem.
- The inductor can be placed at either the input or the output, since the diodes will not turn off until the current → 0.
- We can use an equivalent source method to estimate ripple when an inductor is present.

Design Example

- Example: 15 V, 0.2 A (3 W) for a computer network node.
- Set ripple of 1% peak-to-peak maximum.
- Compare approximate C with a precise result.

Design Example



If diodes are on, there is a 2V drop.

Want 15 V out → 17 V peak

Design Example

- There are four diodes in the bridge, with two on at a time.
- We should account for the 1 V diode drops, since they are a large fraction of the output.
- To get 15 V out, we need 17 V peak.

Design Example

- 17 V peak corresponds to (17 V)/√2 = 12 V RMS.
- For 120 V, 60 Hz input, we should buy a 120 V/12 V transformer.

Design Example

- The capacitor value: We want the voltage change to be no more than 0.15 V with a 0.2 A load.
- $\Delta v_{out} = I_{out}/(2fC)$, f = 60 Hz
- With the change less than 0.15 V, this gives C > 11111 uF.
- Here R = 75 Ω, so RC = 0.83 s.

Design Example

$$\Delta V_{OUT} = \frac{I_{OUT}}{2fC}$$

$$\Delta V_{OUT} < 0.15V$$

$$C > \frac{I_{OUT}}{2fC}$$

$$C > 11111 \quad uF$$

Design Example

$$RC = 0.833\ s$$

$$^{\tau}/_{T}\ ,\ T = {}^{1}/_{60\ s}$$

$$^{\tau}/_{T} = 50$$

$$^{\tau}/_{(T/2)} = 100$$

Assumed $V_{max} = V_{peak}$

Assumed turn-off → peak

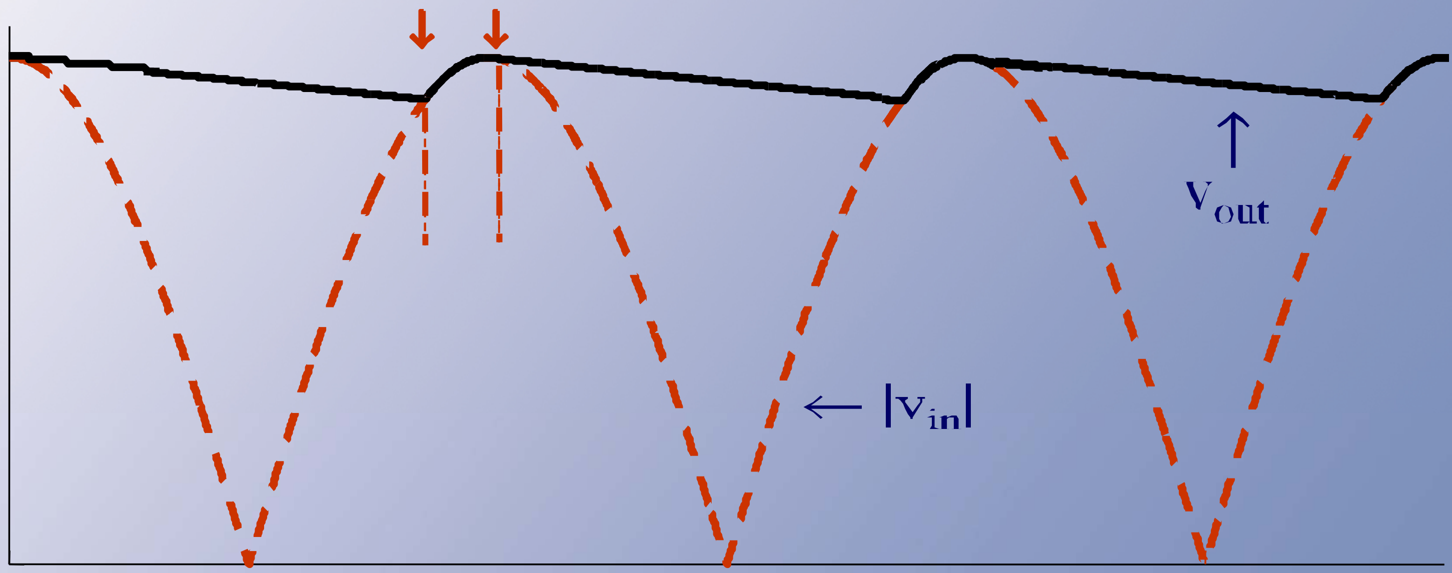
Assumed linear decay, lasting $^{1}/_{20\ s}$

Design Example

"Exact" 10.5 mF

vs. "Approximate" 11.1 mF ← ±10%

12000 μF



Peak Current

- For the current peak, we need the turn-on point.
- This is where a 15 V sine wave rises to 14.85 V.
- This occurs 8.11° before the peak.

Peak Current

Given:

V_o

$v = 15 \cos(wt)$

$t_{on} = -8.11°$

$0°$

$i_c = -w\,C\,(15)\sin(wt)$

wt_{on}

$i_{cmax} = w\,C\,V_o \sin(8.11°)$

$i_c = C\,{}^{dv}/_{dt}$

Peak Current

- The peak current is $\omega C V_0 \sin(8.11°)$ = 8.9 A by estimate.
- Actual value is 8.46 A.
- The RMS input is nearly 1 A.
- The transformer rating is 12 VA – for a 3 W load.

Peak Input Current

- Classical rectifiers → Common
- Advantages:
  - Simple
  - Easy design
  - Few parts
- Disadvantages:
  - No line regulation
  - Harmonics
  - Large, heavy
- Being phased out in favor of small switchers

Peak Input Current

- We are asking the utility to supply a distorted (spike) current.
- Useful work for only a portion of each cycle

$120V_{RMS}$: $5V_{RMS}$ transformer

$5V_{RMS}$: → 200W
170 A RMS

Input from utility : 170 A /24

Input ~ 7.1 A

Peak Input Current (cont.)

120 V, 15 A outlet

Two of these *at most.*

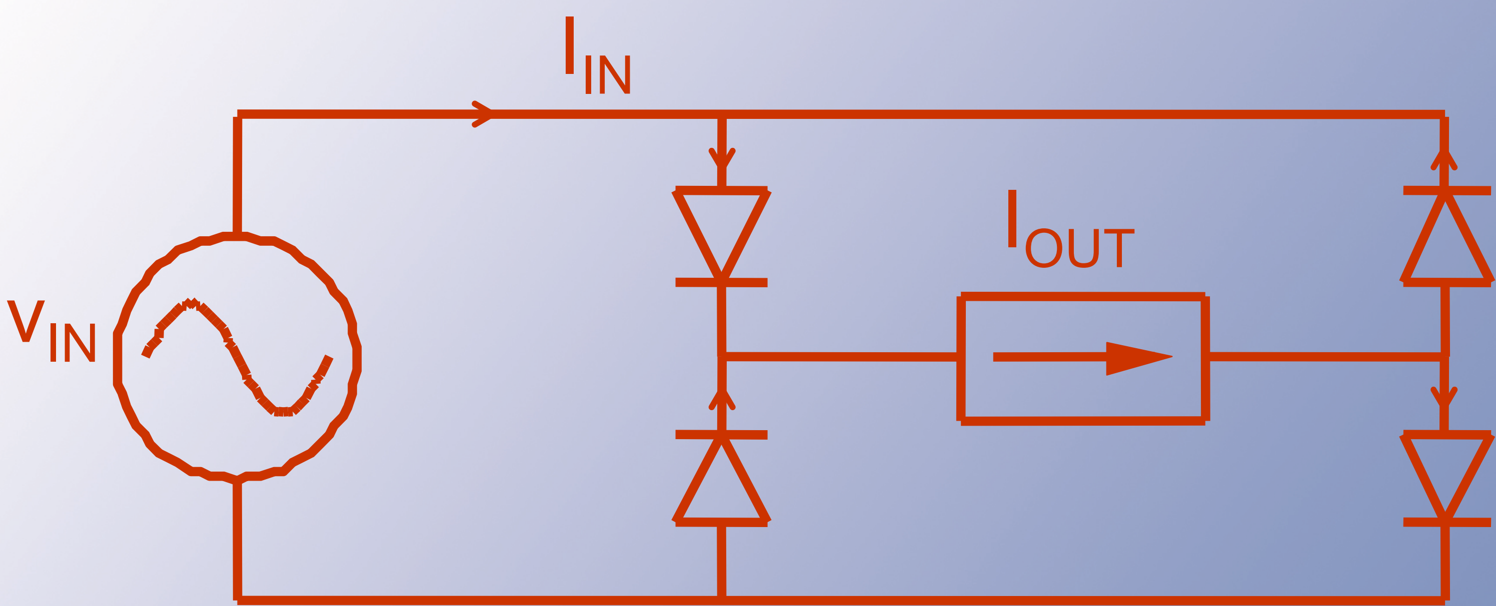
400 W → 1700 VA

- This poor power factor gives very poor system utilization.
- If pf → 1, we could support nine units on a circuit instead of two.

Basic Target

- When a very large inductor is used, the output will look like a dc current source.
- If this is true, each diode will carry I_{out} when on.
- Each diode will have 50% duty.

Diode currents



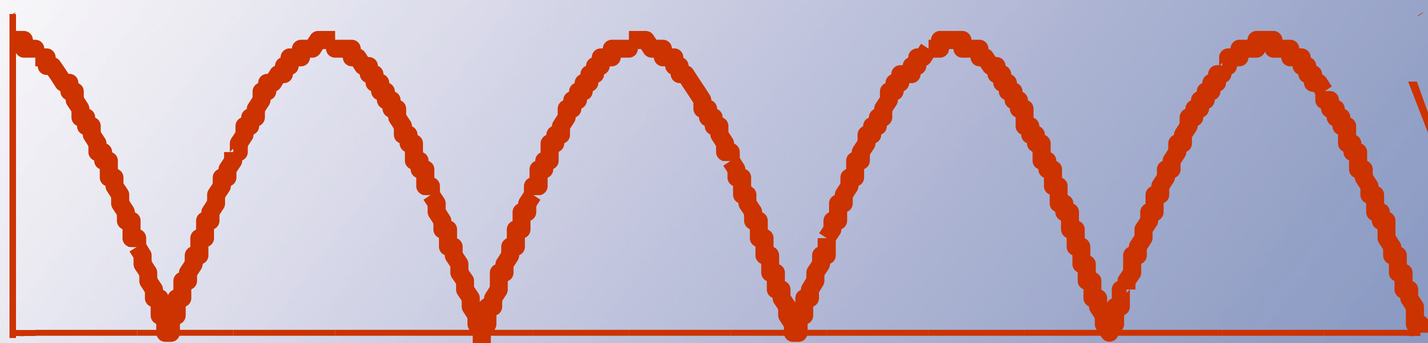
$$I_{IN} = \begin{cases} +I_{OUT} \\ -I_{OUT} \end{cases}$$

Diode currents: I_{OUT}, D=50%

Output Voltage

- In the classical rectifier, the output is close to V_{peak} -- IF the capacitor is large.
- The voltage is more nearly the peak if the load is lighter.
- Good load regulation, up to a limit on the load.

Output Voltage



$V_{OUT} = |V_{IN}|$

Classical case: $V_{OUT} \approx V_{IN_{peak}}$

Output Voltage
Classical case, large C

Up to a load limit, then

$\langle V_{OUT} \rangle \approx V_{peak}$; little change.

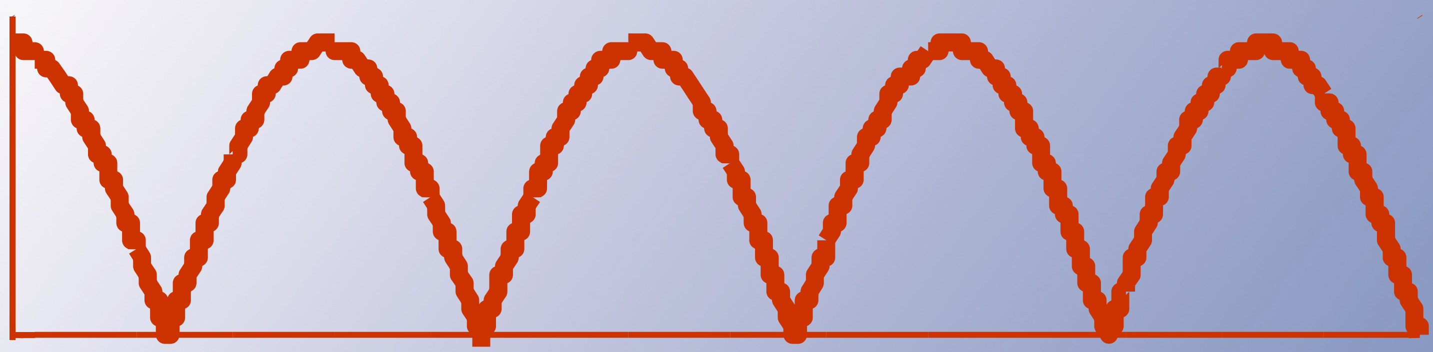
→ Good load regulation,
(not perfect) depends on C.

Output Voltage

- With large L, the output voltage from the diode bridge is a full-wave sinusoid.
- The load sees the average of this waveform, $\langle|V_0 \cos(\theta)|\rangle$.
- Compute this. The result is $2V_0/\pi$, less any diode drops.

Output Voltage

Large L



Waveform does not change, provided L is big enough.

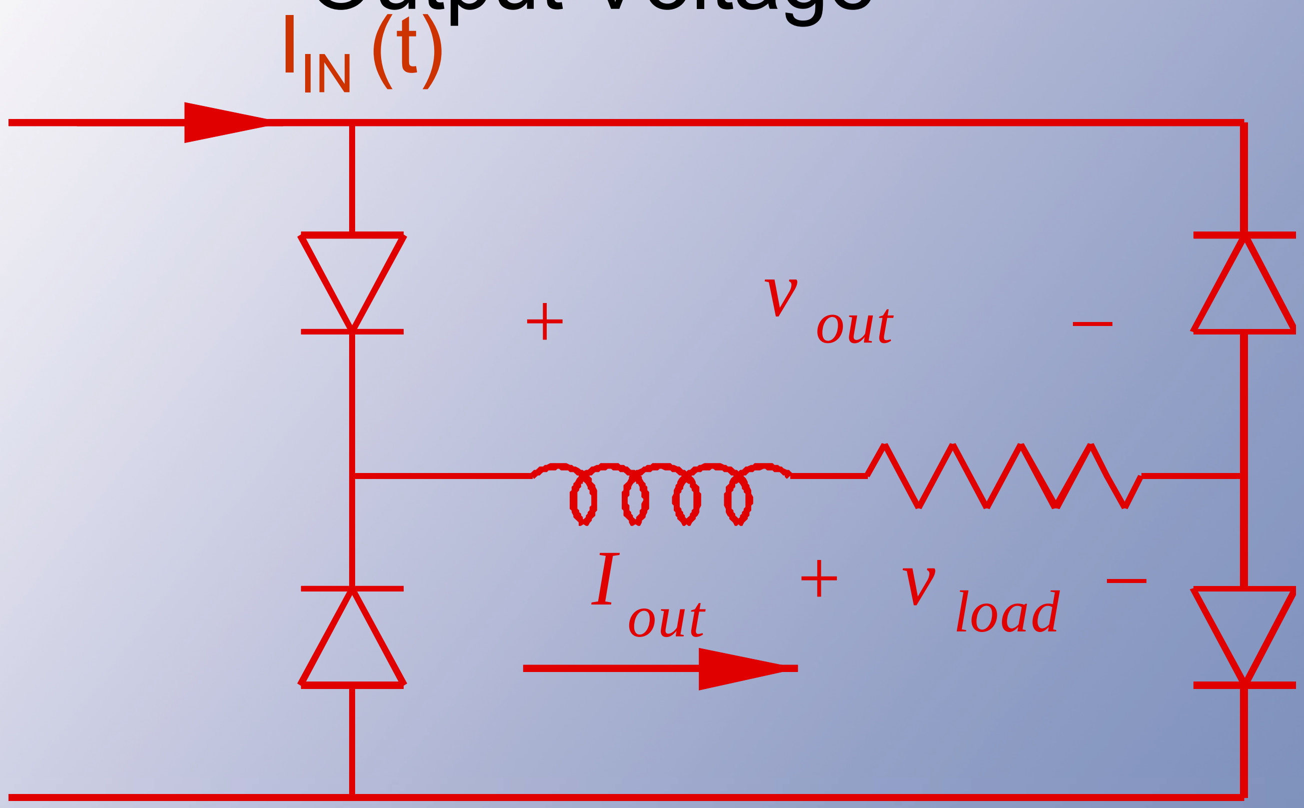
Load regulation – perfect.

No line regulation.

Output Voltage

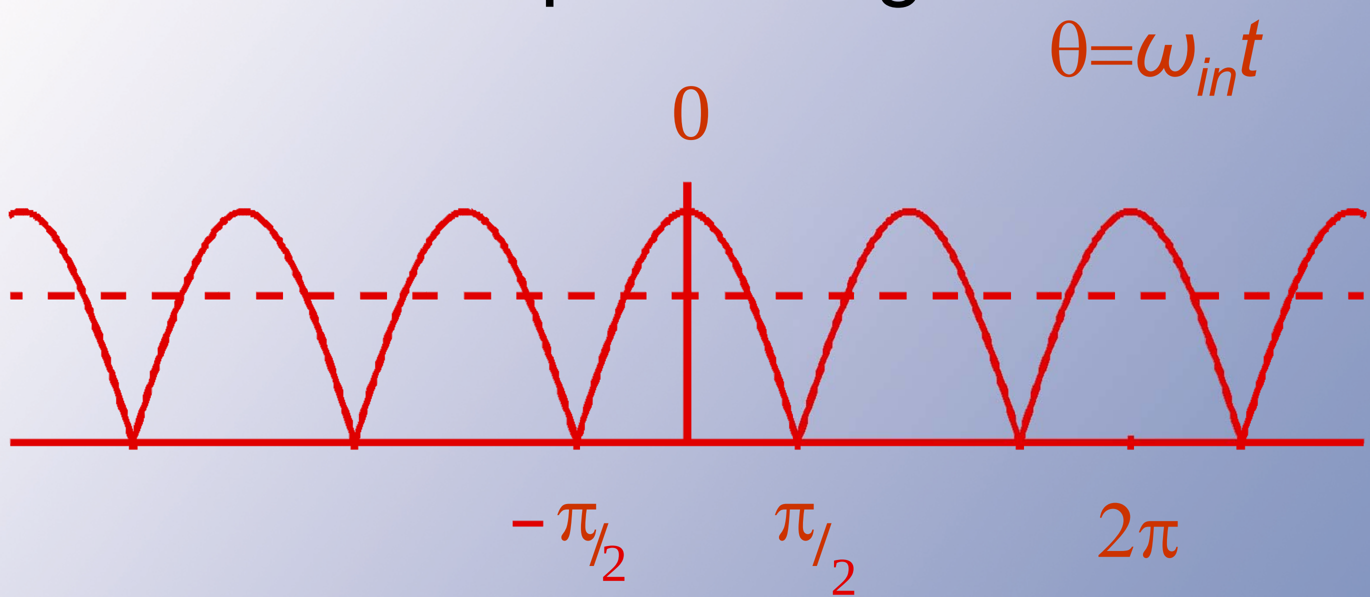
- This output is independent of load as long as L is large enough.
- We are still at the mercy of line variation.

Output Voltage



$$<V_{OUT}> = <|V_{IN}|>$$
$$= <|V_0 \cos(wt)|>$$

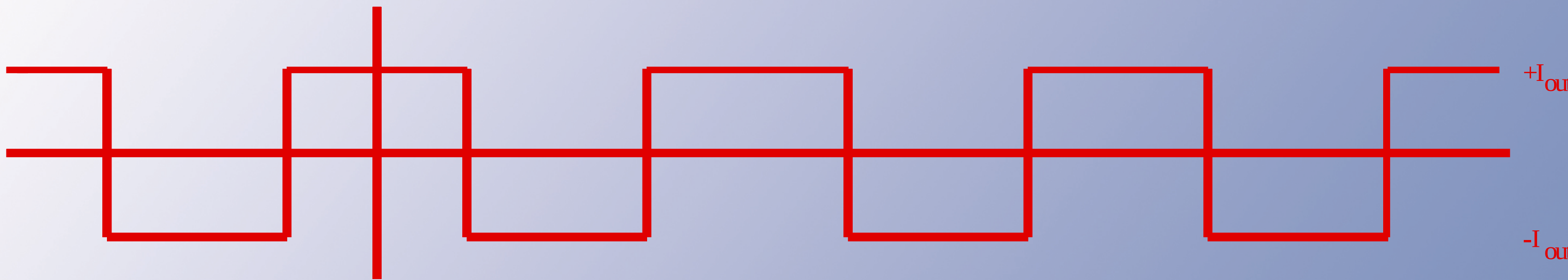
Output Voltage



$$\langle|V_{IN}|\rangle = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} V_0 \cos\theta \, d\theta$$

$$= \frac{2V_0}{\pi}$$ Lower than classical case

Output Voltage



- With large L, input current is a square wave rather than short spikes.
- Much better power factor and lower current distortion.

Input Current

- The diodes ensure that the input current is either $+I_{out}$ or $-I_{out}$.
- The input current must be a square wave with peak value I_{out} and duty ratio of 50%.
- This is much less distorted than in the classical case.

Power Factor

- We can compute a power factor. The average power is $V_{out}I_{out} = (2V_0/\pi)(I_{out})$.
- The input RMS current is just I_{out}.
- The input apparent power $S = V_{RMS}I_{RMS}$ is $(V_0/\sqrt{2})(I_{out})$.
- $pf = P/S = (2/\pi)\sqrt{2} = 0.900$

Comments

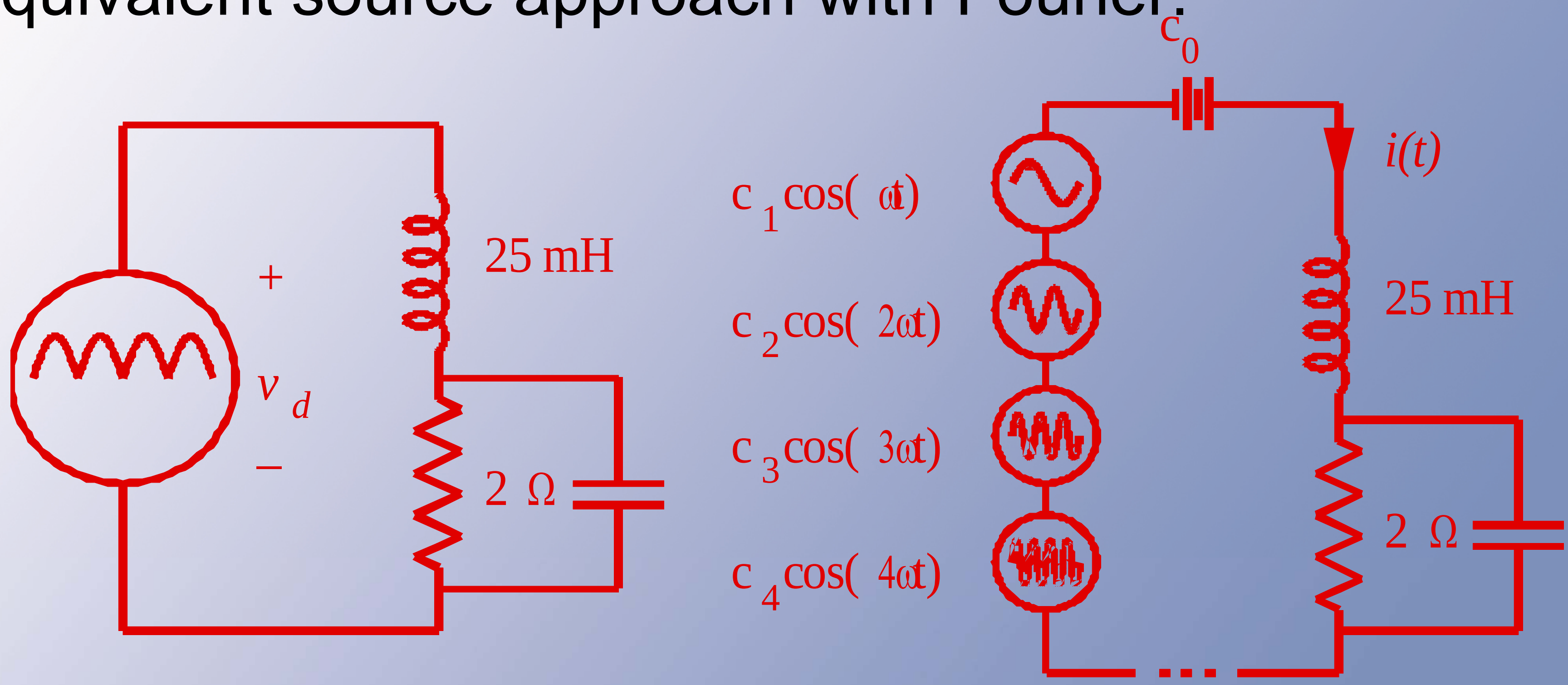
- It is interesting that for large L the power factor does not depend on load, ripple, or anything else.
- A power factor of 90% is far better than for a classical case, but at low power levels L can be excessive.

Choosing L

- We can choose L by using the fundamental Fourier component for a ripple estimate.
- For a full-wave voltage, $c_1 = 4V_0/3\pi$ (see p. 175). The frequency is twice that of the input.

Inductor Value

Equivalent source approach with Fourier.



Inductor Value

- Focus on the first harmonic as a basis for estimation.
- We have a divider, R || C in series with L.
- The impedances are those at the main ripple frequency.
- Find the resistor current to estimate output ripple.
- Please see p. 176 for the details.

Choosing L

- This is a voltage and current divider.
- The ripple current in the resistor is given by

$$\frac{2\sqrt{2}V_0}{3\pi(R + j\omega L - \omega^2 RLC)}$$

Choosing L

- With this relation, a poor choice of L could actually increase the ripple.
- Good results require that the resonant frequency $1/\sqrt{(LC)}$ is much less than the ripple frequency.

Choosing L

- When both L and C are present, it is possible to get good results without excessive values of either.
- Even if L is not large, it will improve power factor and the input current waveform.