Power Electronics

Day 4 – Equivalent Sources, "Power Filtering" Analysis, Dc Conversion

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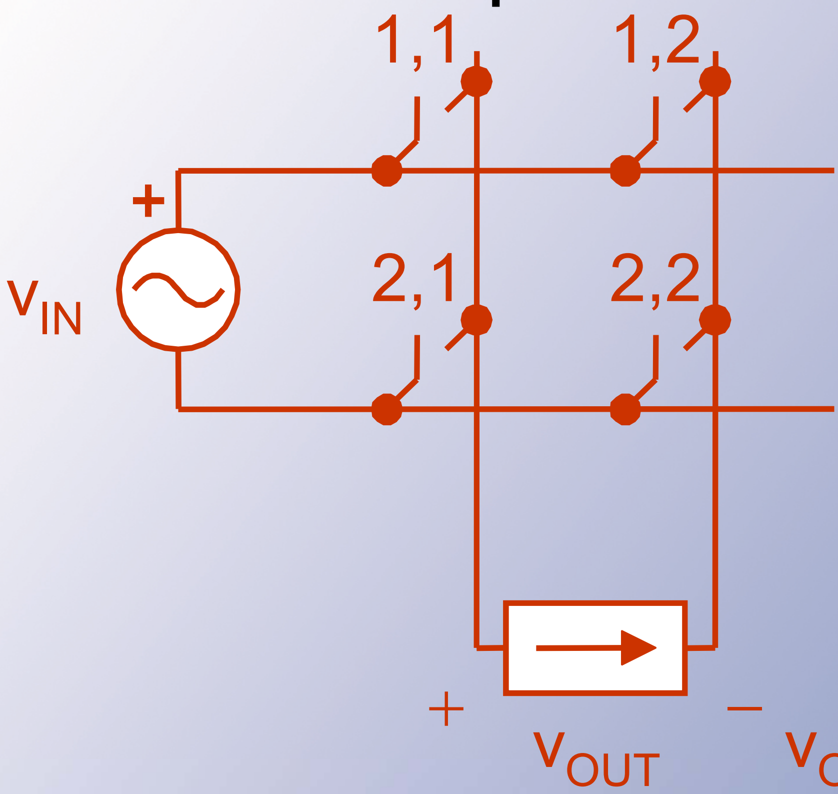
Equivalent Sources

When a switch matrix operates to satisfy KVL and KCL, many of the waveforms become well defined.

Example: Matrix 2x2 ac voltage to dc current converter. The output must be $+v_{in}$, $-v_{in}$, or zero.

Equivalent Sources



$$V_{OUT} = \begin{cases} +v_{in} & 1{,}1 + 2{,}2 \text{ on} \\ -v_{in} & 2{,}1 + 1{,}2 \text{ on} \\ 0 & \begin{cases} 1{,}1 + 1{,}2 \text{ on} \\ 2{,}1 + 2{,}2 \text{ on} \end{cases} \end{cases}$$

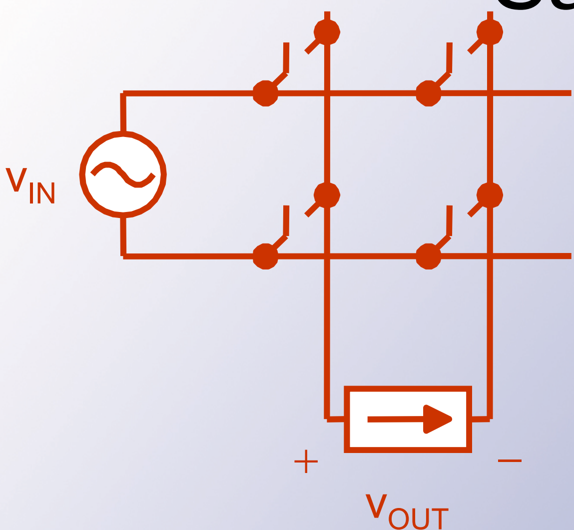
Equivalent Sources

- If switch action is specified, the output waveform becomes fully determined.
- We can treat the waveform as an ideal source (with an unusual shape).

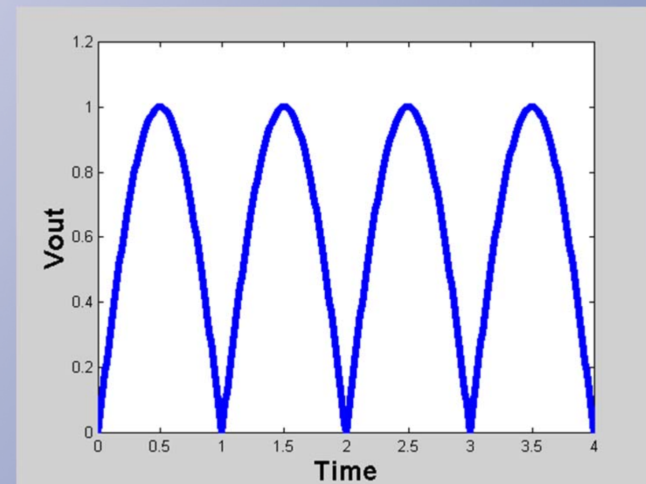
Sample Cases

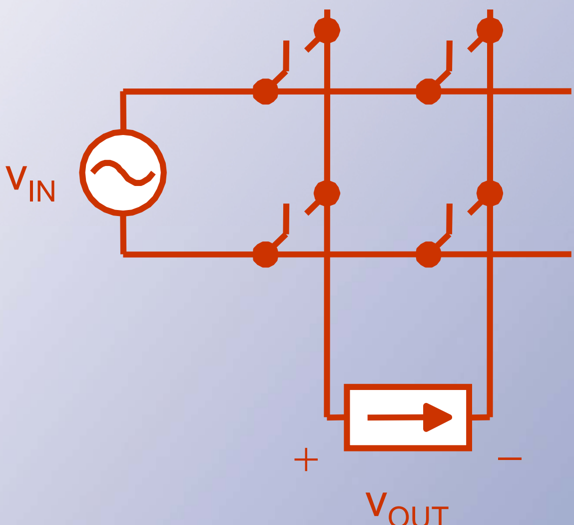
- Full-wave rectifier (Fig. 2.33)
- Phase-delayed rectifier (Fig. 2.17)
- Inverter into an ac current source (Fig. 3.5)
- 60 Hz 3ϕ to 60 Hz 1ϕ conversion
- Fig. 2.19, 60 Hz to 180 Hz

Sample Cases

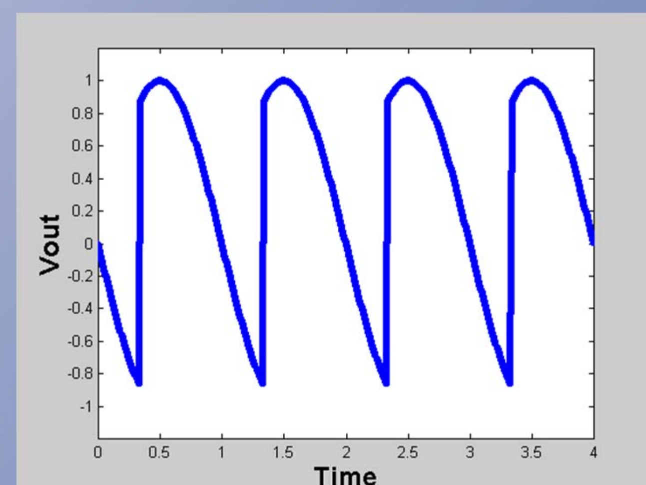


0° delay

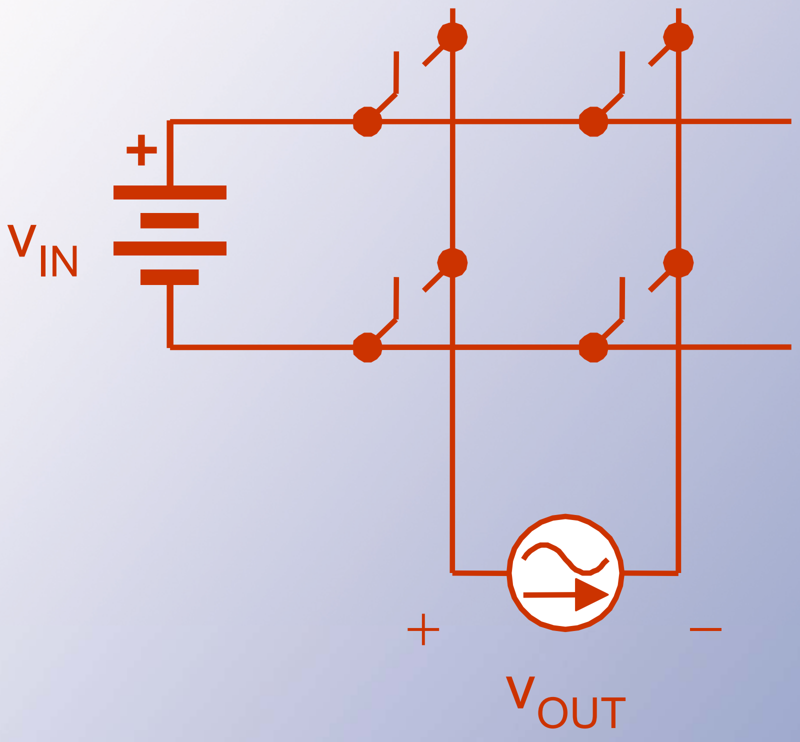


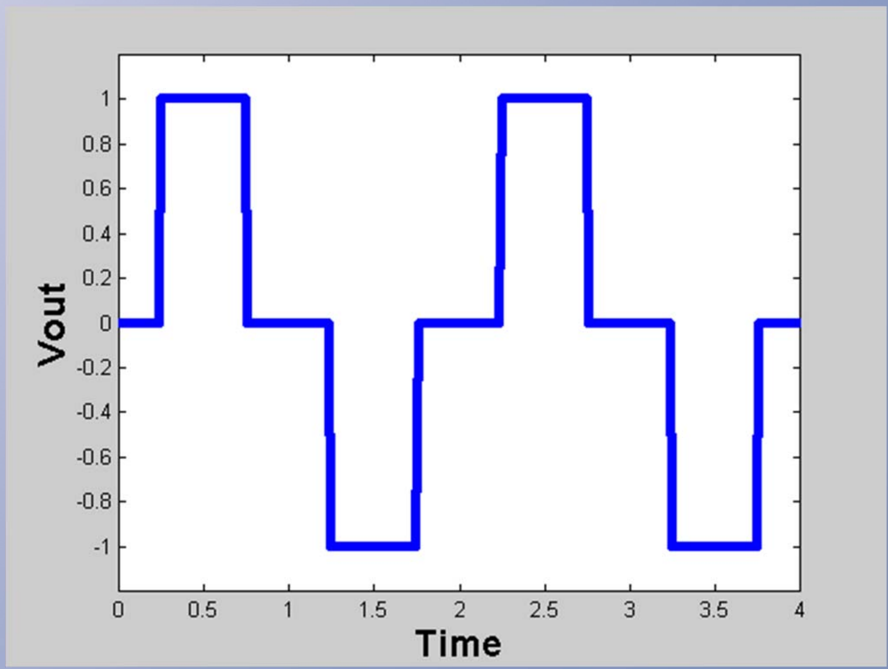


60° delay

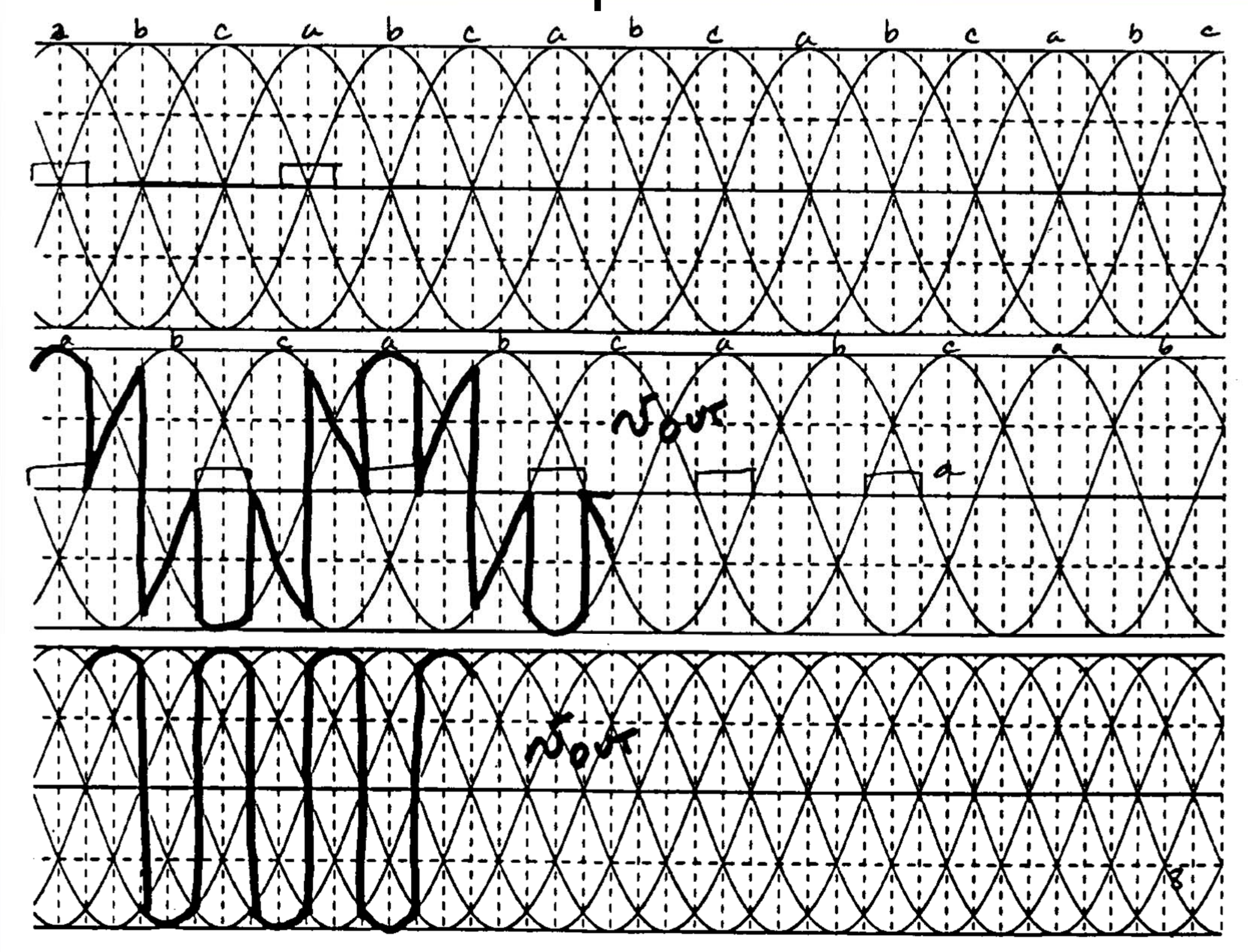


Sample Cases



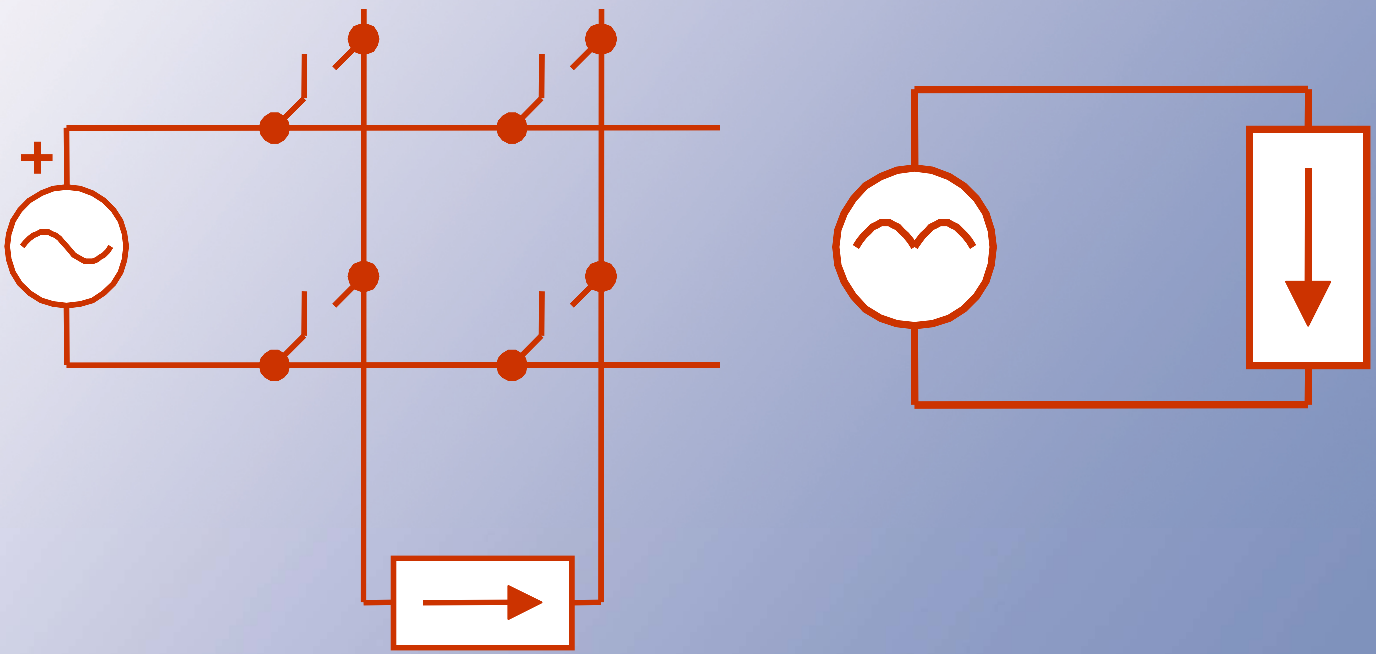


Sample Cases



Equivalent Sources

Any of those waveforms can be a source.



Equivalent Sources

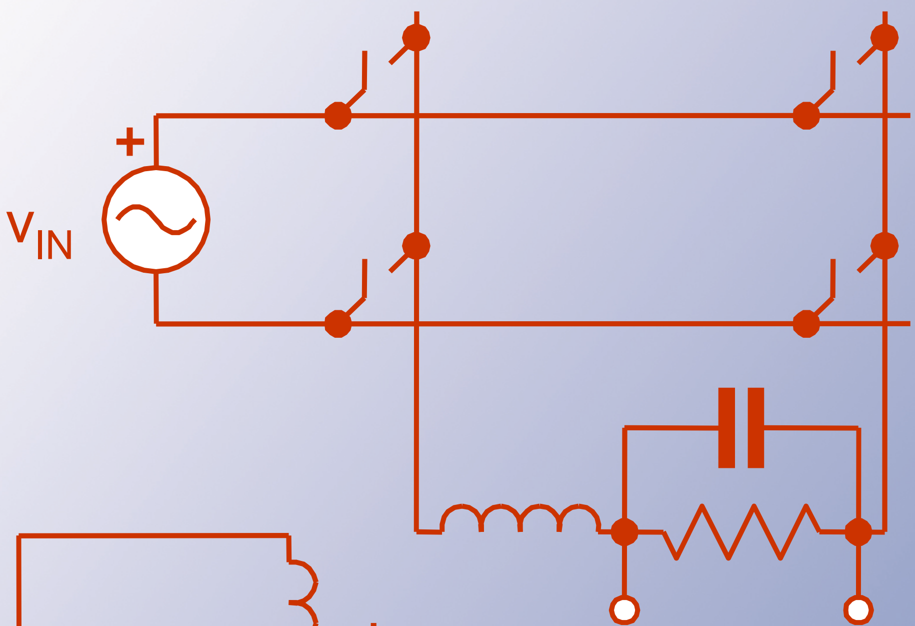
- Equivalent sources can be a powerful tool:
  - Many converters act like an equivalent source in a linear circuit
  - We can represent a source as a combination of Fourier components

Equivalent Sources

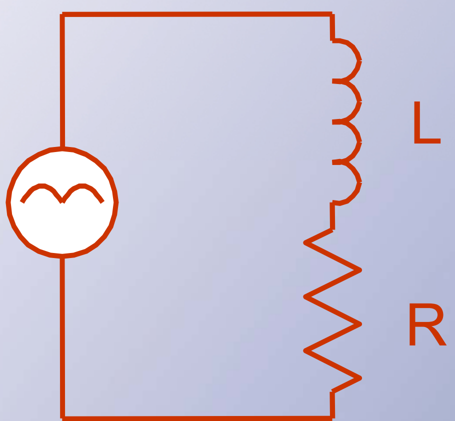
- With a source in a linear circuit, analysis, filter design, etc. can proceed along familiar lines.
- This is a common way to design interfaces for rectifiers and inverters.

Equivalent Sources

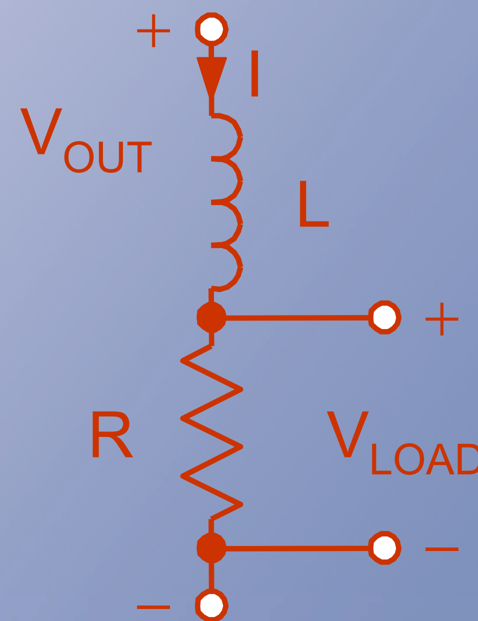
Example:



Output
terminals

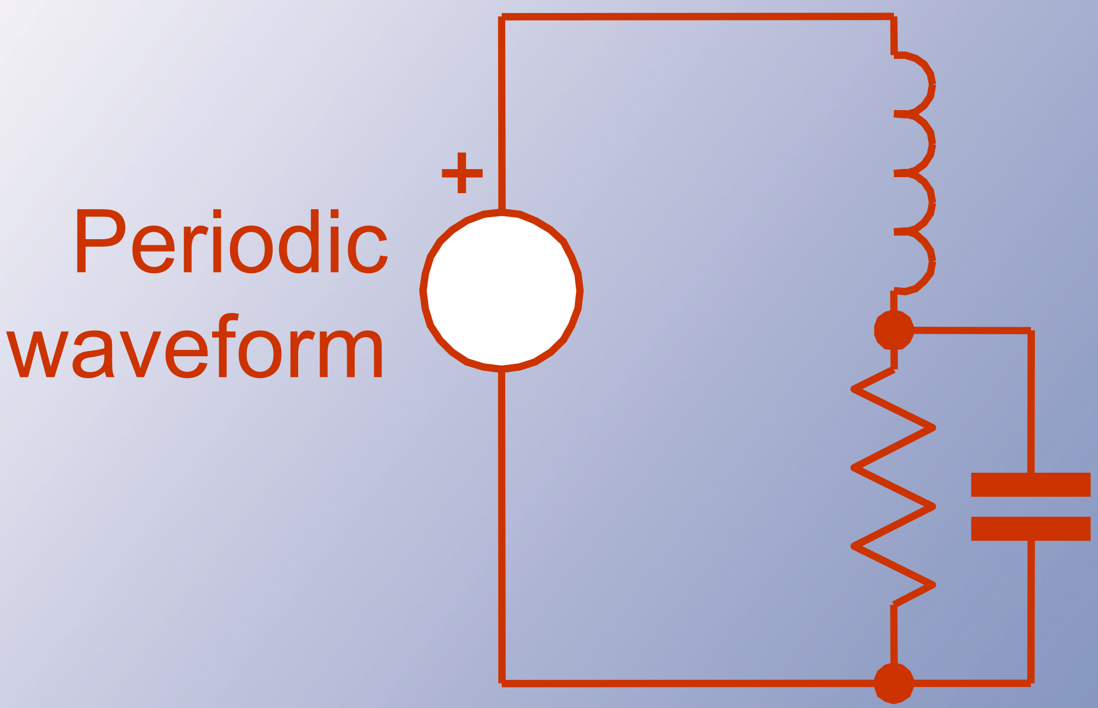


Ignore capacitor for a moment:



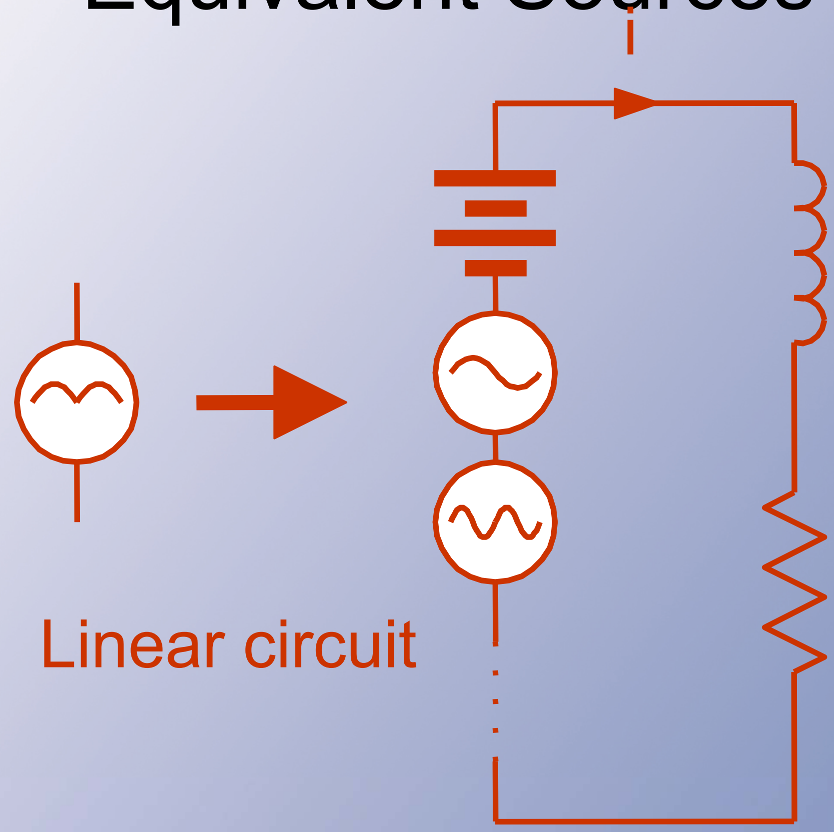
We know V_{OUT}

Equivalent Sources



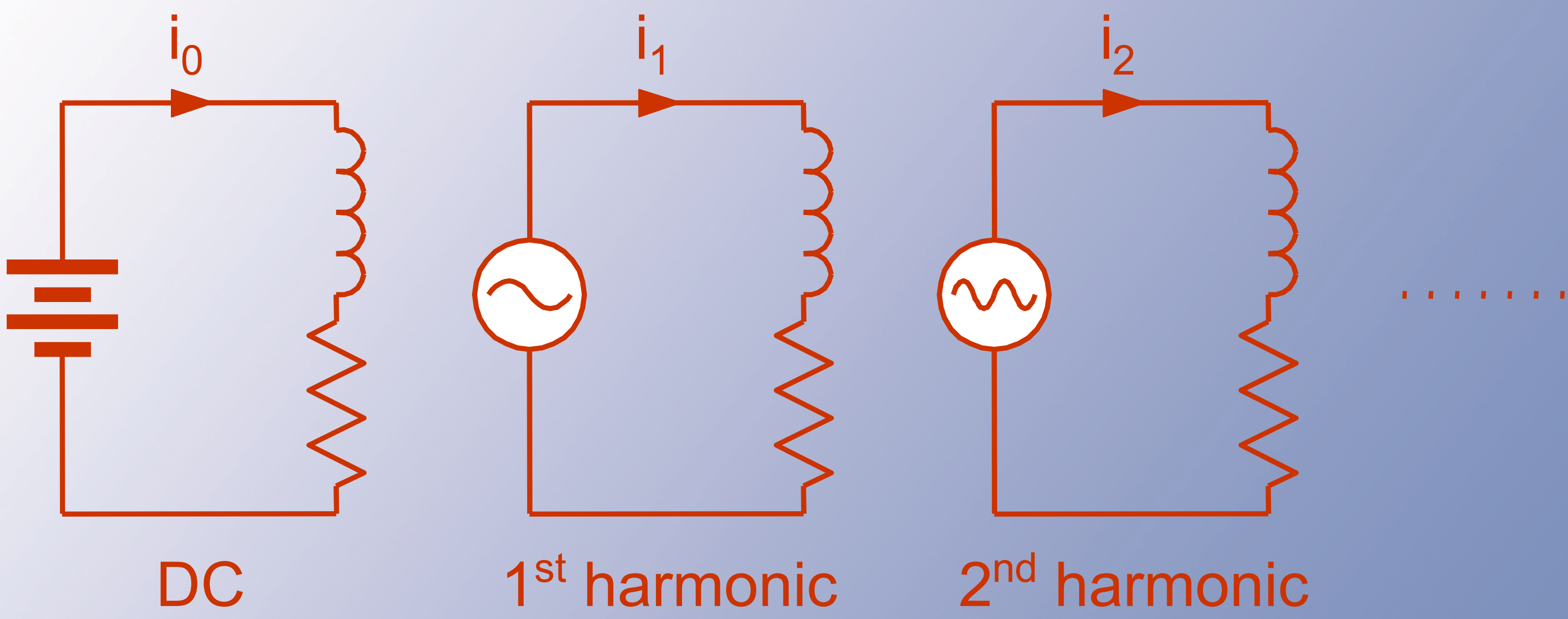
We can represent the periodic waveform with a Fourier series.

Equivalent Sources



i is the sum of the contributions from each of the sources. We can break up the circuit.

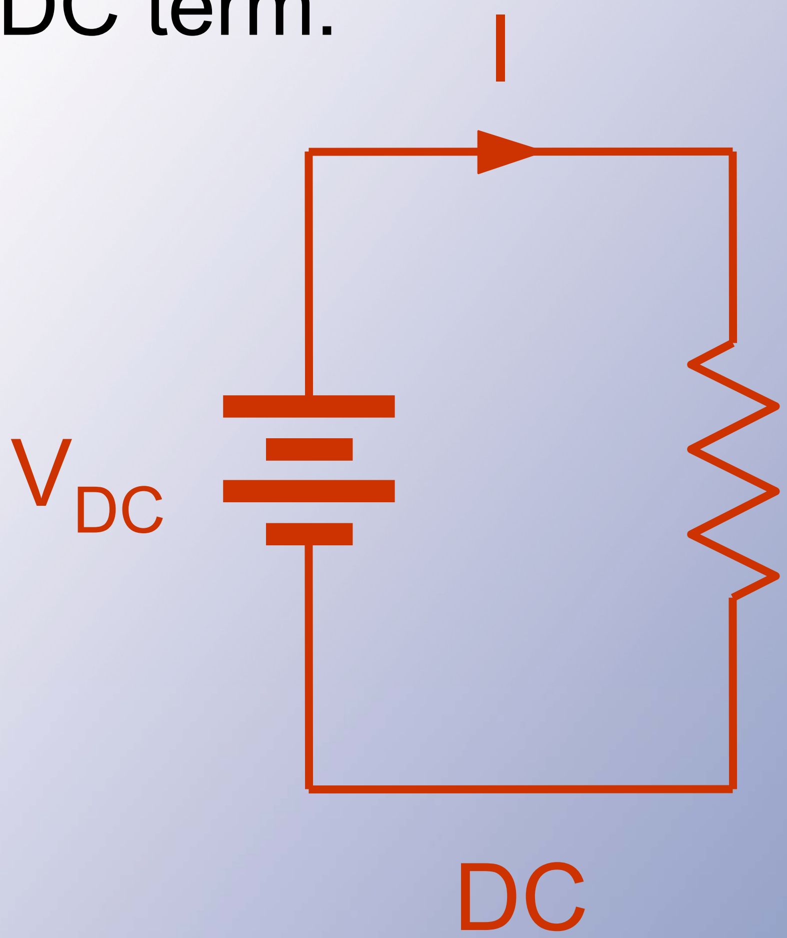
Equivalent Sources



$$i = \sum_{n=0}^{\infty} i_n$$

Equivalent Sources

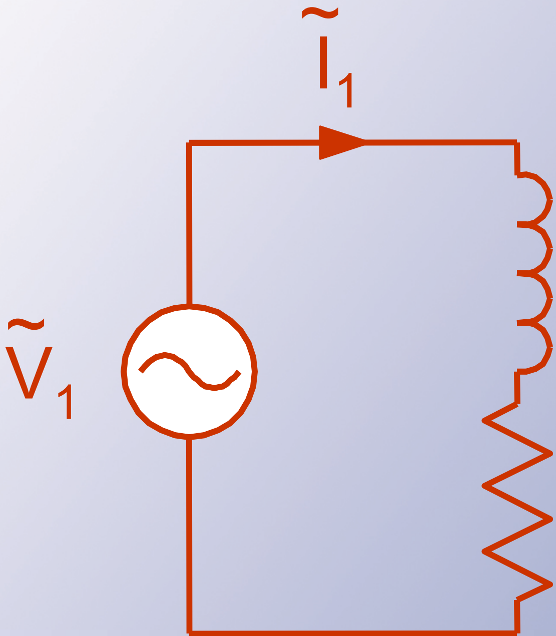
DC term:



$$I_{dc} = \frac{V_{dc}}{R}$$

Equivalent Sources

AC terms, based on phasor analysis.



$$\tilde{I}_1 = \frac{\tilde{V}_1}{R + jw_1 L}$$

Want low ripple

→ e.g., want $\left|\tilde{I}_1\right|$ low

Usually, Fourier terms decrease in amplitude as 1/n. The fundamental is the largest.

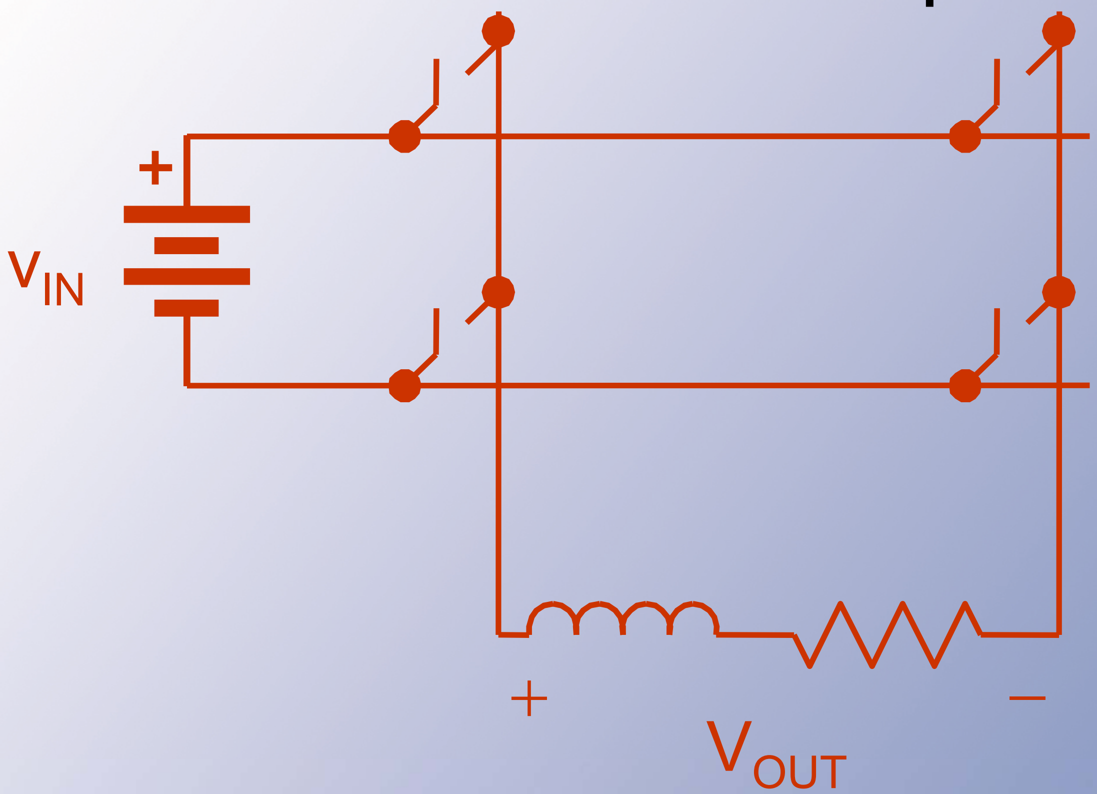
DAY 4 START Power Filtering

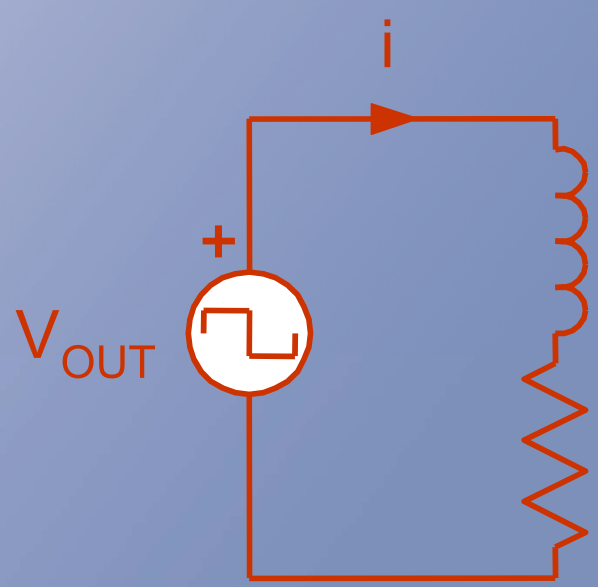
- Filters (or interfaces) for converters have needs distinct from those in signal applications.
- Filters must be lossless, and impedances of sources and loads are unknown.

Power Filtering

- Two common methods of analysis
  - Equivalent sources
  - "Ideal action" assumption

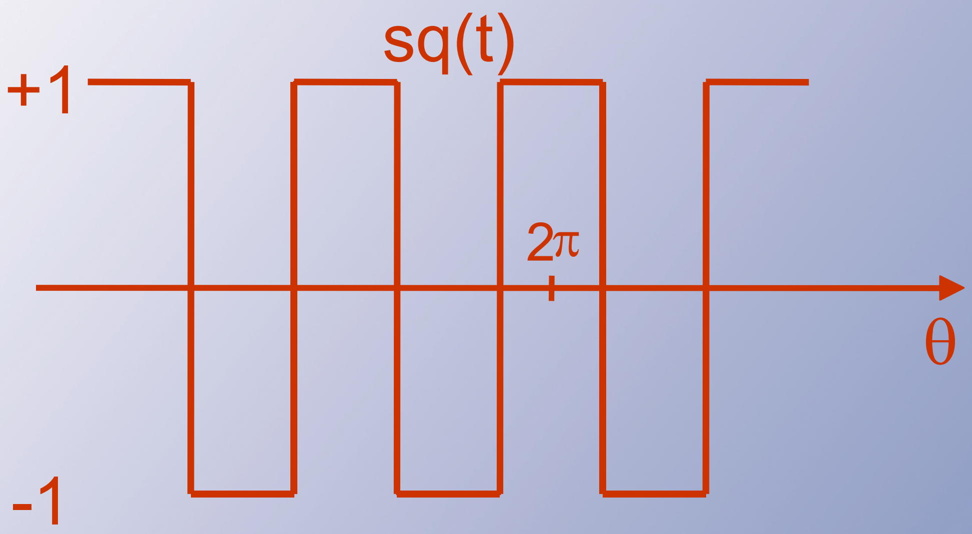
Filter Examples





Filter Examples

$v_{OUT} = V_{in}\, sq(t)$



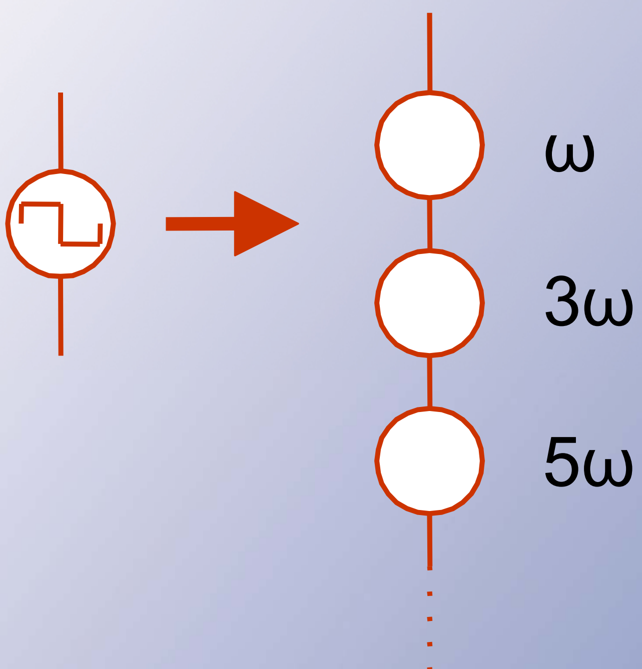
f = 100HZ

$$sq(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos(n\omega t)$$

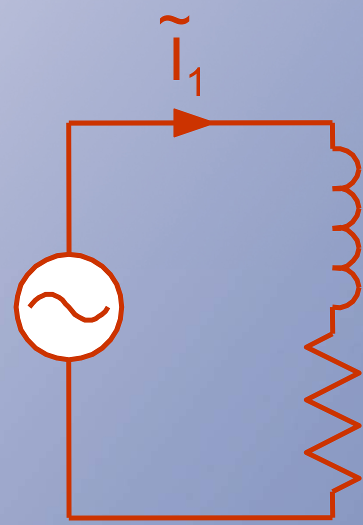
Filter Examples

$$v_{out}(t) = \frac{4V_{in}}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \cos(n\omega t)$$

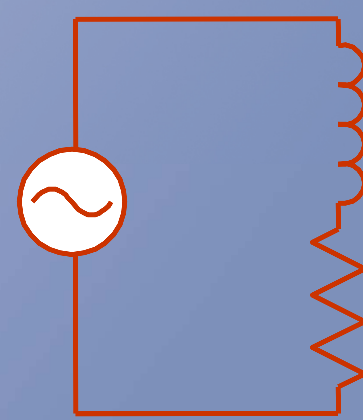
$\omega = 2\pi 100$ rad/s



Fundamental



3rd term



Filter Examples

Look at examples based on the equivalent source method (such as Example 3.6.1).

Ideal Action Assumption

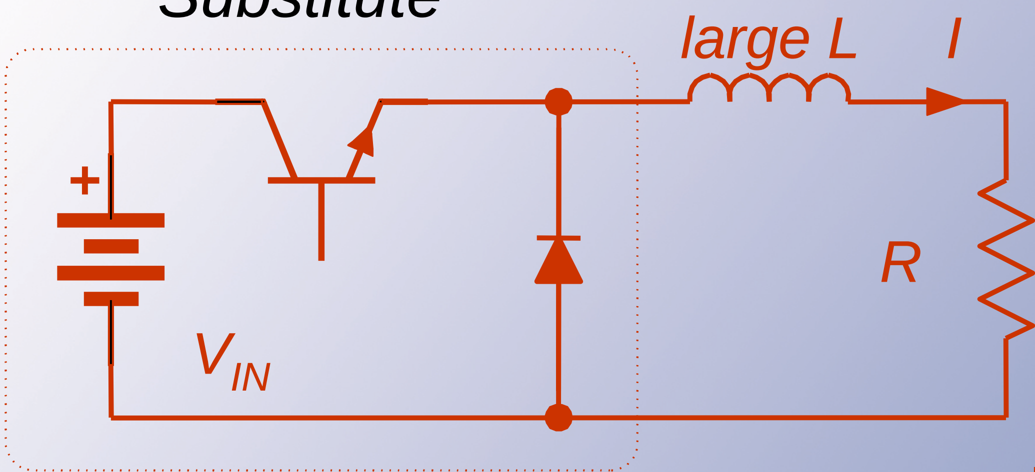
- In a power converter, we know what a filter is trying to achieve.
- Examples: low-ripple dc, ideal ac sine wave, etc.
- In general: give a large *wanted* component and small *unwanted* components.

Ideal Action Assumption

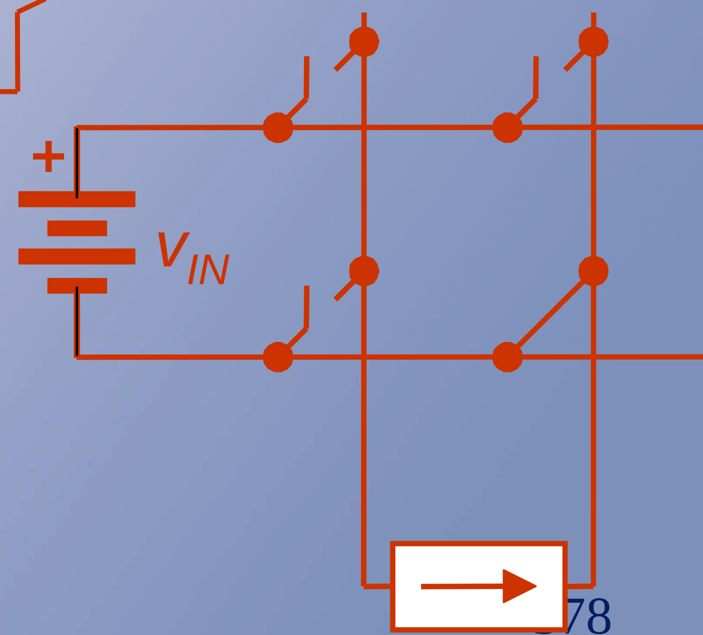
- If the filter is well-designed, it ought to work.
- If it works, we know its output.
- Now, use the “known” output with the known input to compute values.

Filter Examples

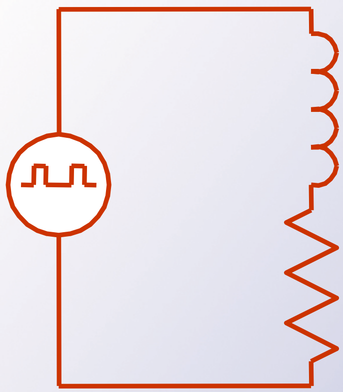
Substitute

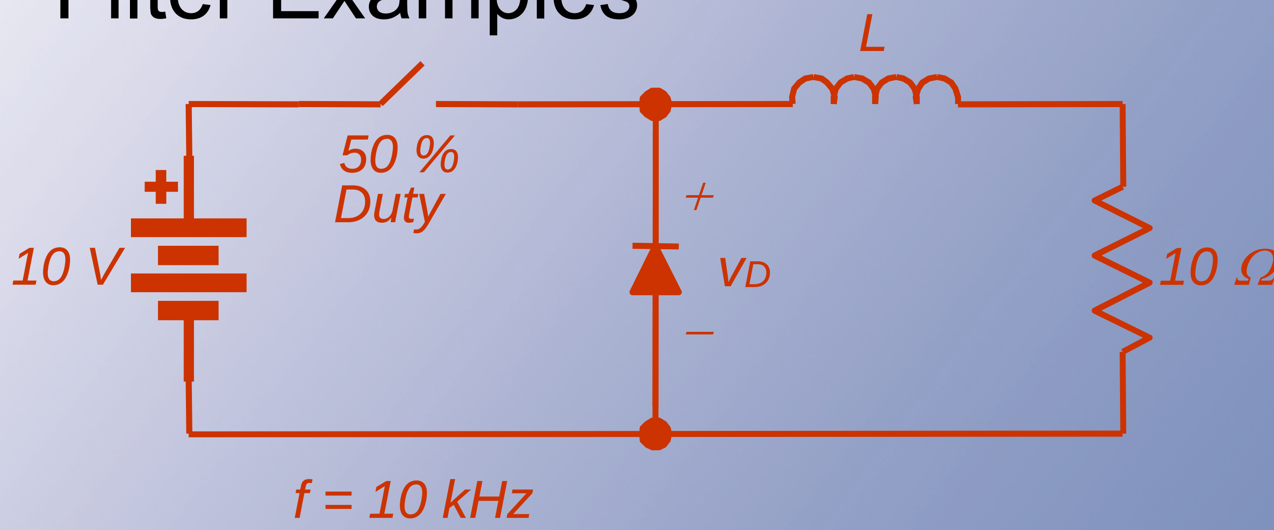


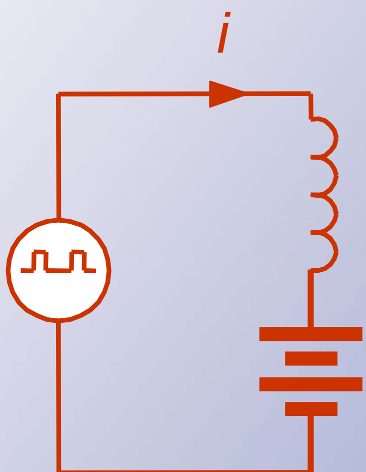
If L is large, then I is just dc.



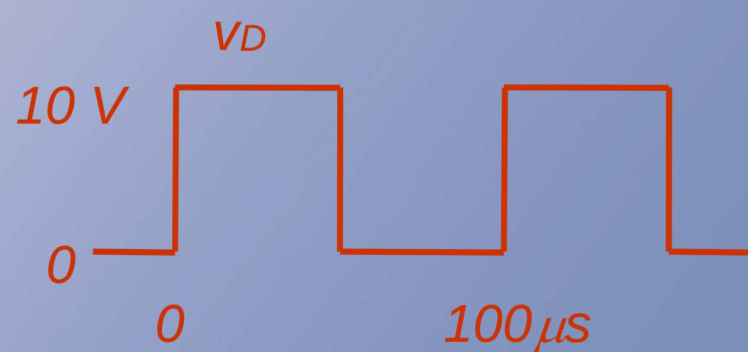
Filter Examples



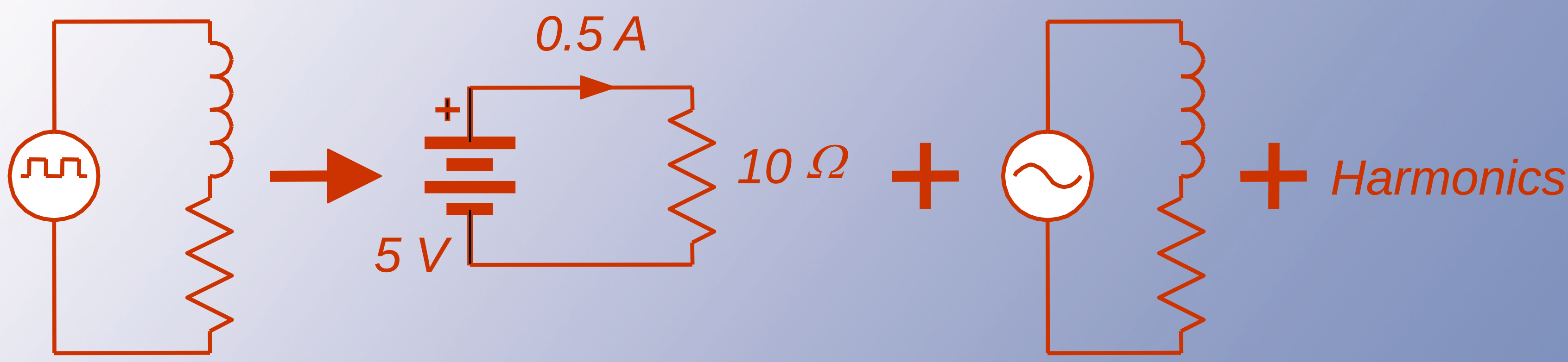




Ideal action assumption



Filter Examples



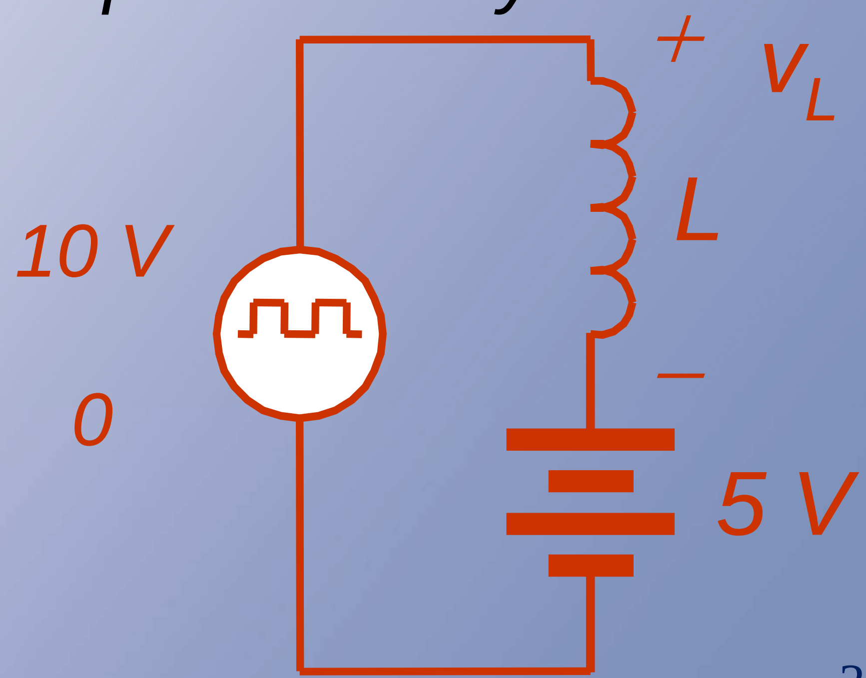
Choose L to make $\left|\tilde{I}_1\right| <$ *Limit.*

Too much work!

Filter Examples

If L is large and the circuit works, the inductor current is almost constant and so is the voltage across the load resistor.

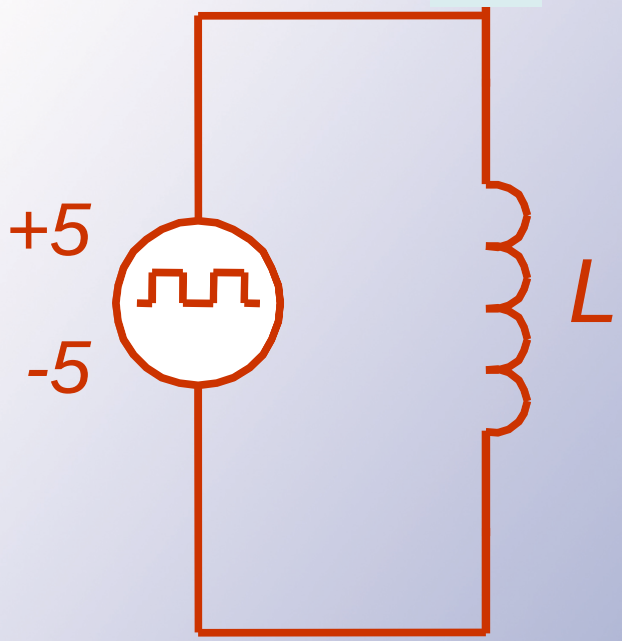
This voltage can be represented by a constant voltage source.



Switch on: $V_L = 5\ V$

Switch off: $V_L = -5\ V$

Filter Examples



$$V_L = L\,di/dt$$

$$If\ \ V_L = 5V = L\,di/dt$$

$$\frac{5V}{L} = di/dt$$

$$= \frac{\Delta i}{\Delta t}$$

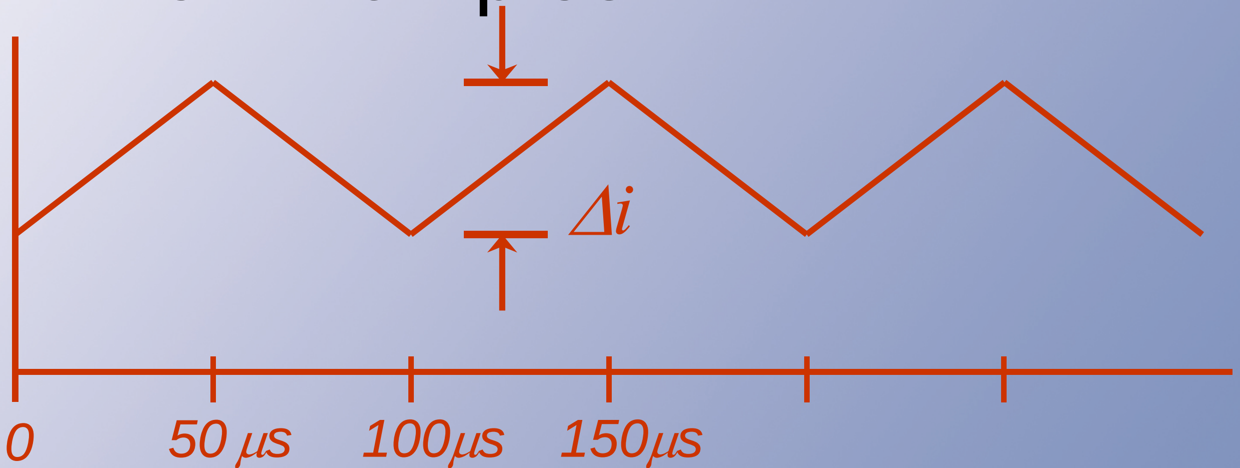
$$If\ \ V_L = -5V = L\,di/dt$$

$$\frac{-5\,V}{L} = di/dt$$

$$= \frac{-\Delta\, i}{\Delta t}$$

Filter Examples

$<i_L> = 0.5\ A$



$$\frac{5V}{L} = \frac{\Delta i}{\Delta t}$$

$$= \frac{\Delta i}{50\mu s}$$

$$\Delta i = \frac{5V}{L} \times 50\mu s$$

Choose L to make $\Delta i = 0.005\ A$

Filter Examples

$$0.005\ A = \frac{5V}{L} \times 50 \times 10^{-6}$$

$$L = \frac{250 \times 10^{-6}}{5 \times 10^{-3}}$$

$$L = 0.005\ H$$

$L \geq$ 5 mH makes $\Delta i \leq 0.005\ A$

Results and Comments

- Since we know the objective of our filters, it is reasonable to design them based on the assumption that the objective is met!
- This simple expedient is a very effective simplifying step.

Results and Comments

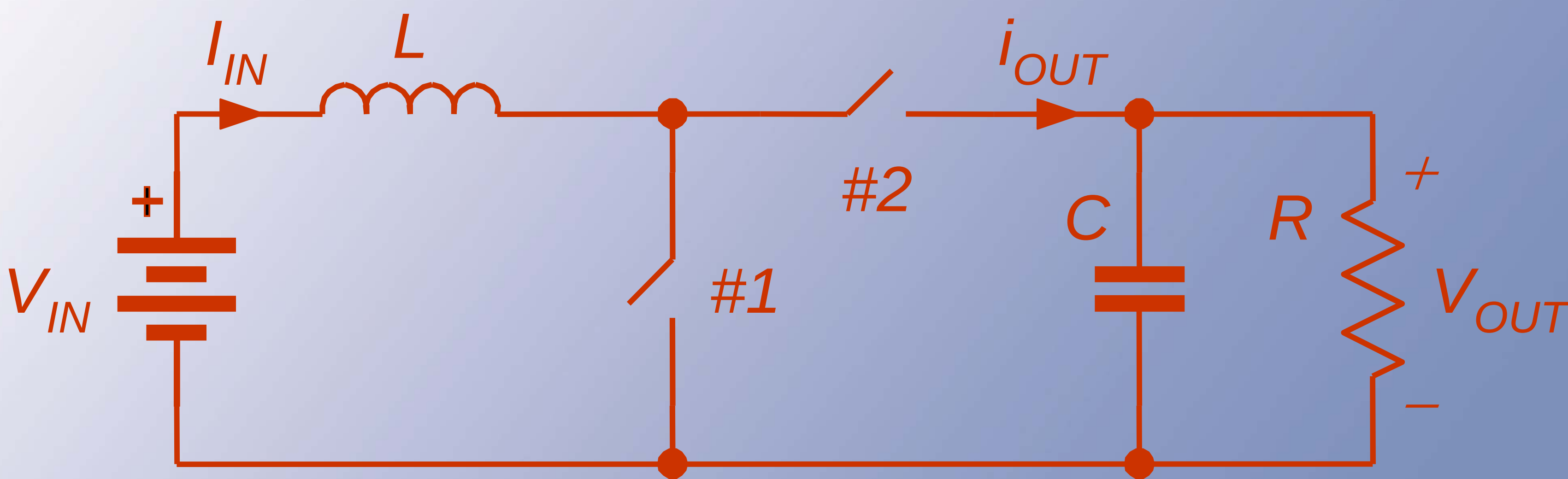
- The *ideal action assumption* works better than one might expect.
- We will analyze this as we build up converter designs.

Summary So Far

- We can analyze the quality of a converter output.
- Equivalent sources give us a way to deal with the interface problem.
- The ideal action assumption helps considerably with design.

Filter Example

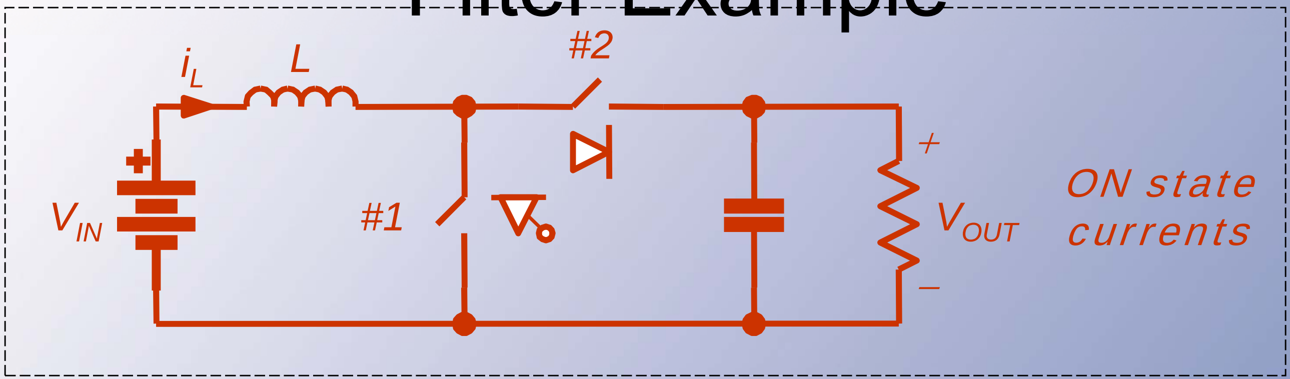
- Consider a converter, shown, with switch #1 duty ratio at 3/4.

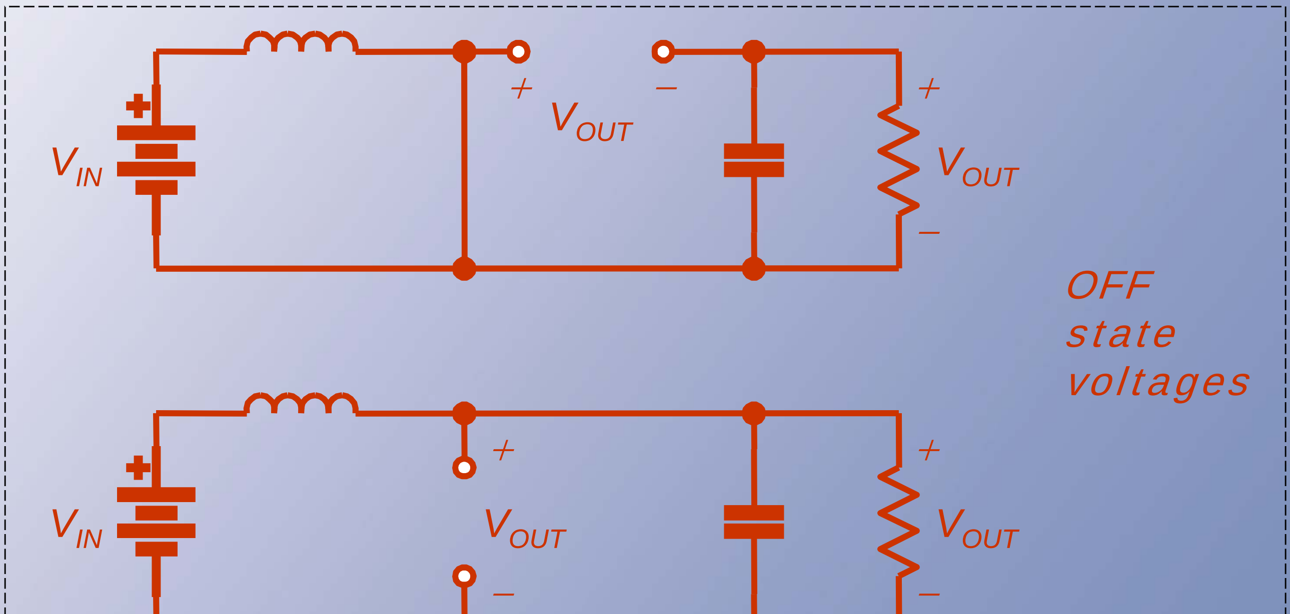


Filter Example

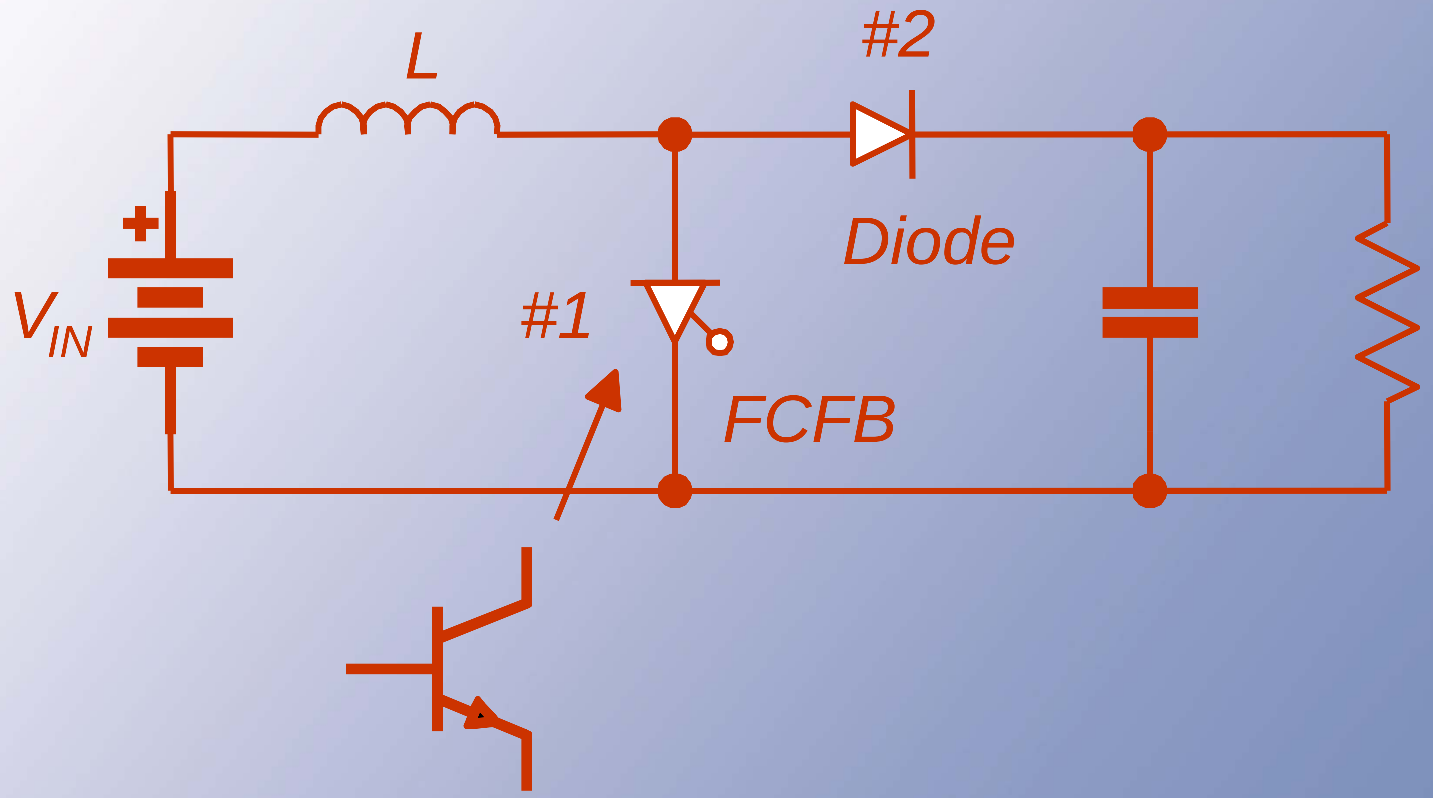
- Let the switching frequency be 200 kHz, L = 1 mH, C = 10 μF, R = 10 Ω, V_{in} = 5 V.
- By KVL and KCL, the switches need to alternate.
- We can determine the device types.

Filter Example

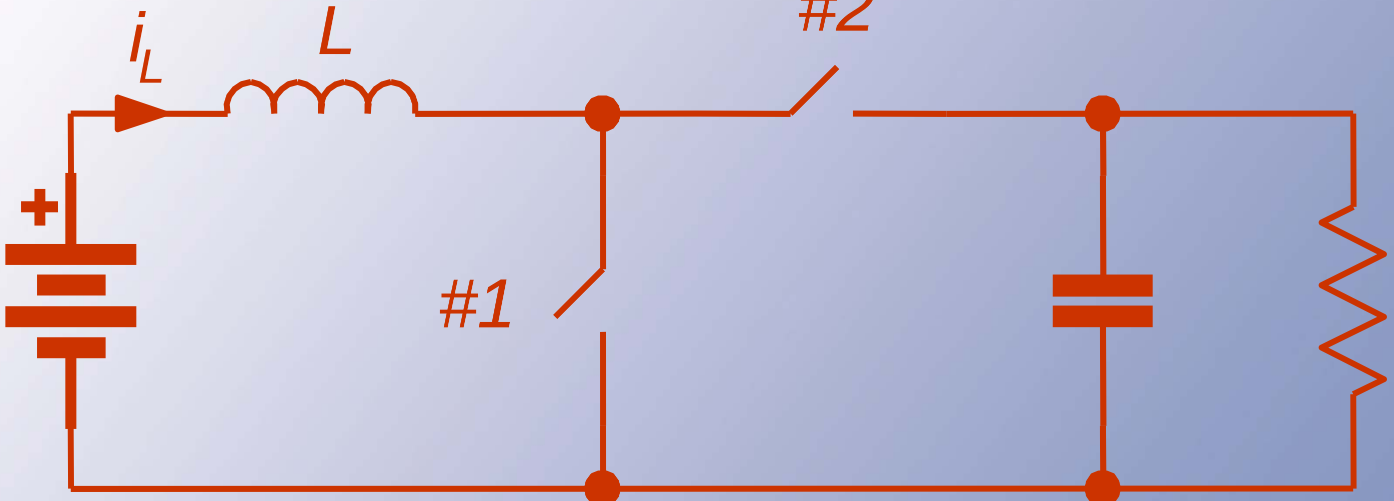


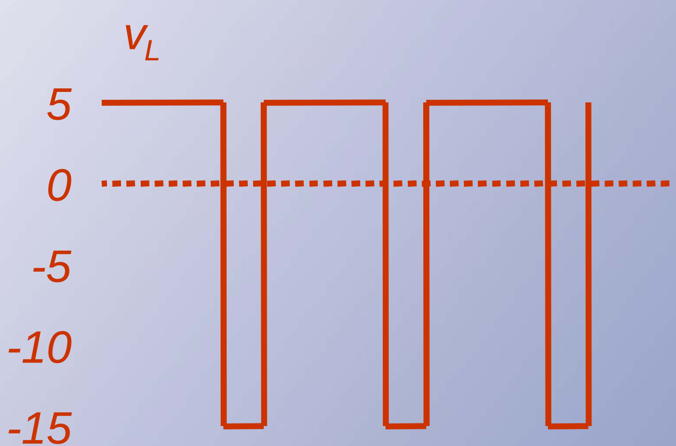


Filter Example



Load Current





$$\langle v_L \rangle = 0$$

Energy Balance

- With switch #1 on, the input energy to the inductor is $(V_{in})(i_L)(3T/4)$. With switch #2 on, the input is $(V_{in} - V_{out})(i_L)(T/4)$.
- The total must be zero. This requires $V_{out} = 4\ V_{in} = 20$ V.

Load Current

- The load current is 2 A, and the load power is 40 W.
- The average input current must be (40 W)/(5 V) = 8 A. This is i_L.

Current Ripple

- If the inductor and capacitor are large (we will check this), then i_L and V_{out} are nearly constant.
- The inductor sees 5 V when #1 is on, so its current increases for 3.75 us.

Current Ripple

- The inductor sees 5 V - 20 V = -15 V when switch #1 is off, and the current falls for 1.25 us.
- During the rise, v_L = 5 V = L di/dt, but the rise is linear over 3.75 us, so (5 V)/L = $\Delta i/\Delta t$, Δt = 3.75 us.

Current ripple

With a 1 mH inductor, this means

Δi = (5 V)(3.75 us)/(1 mH),

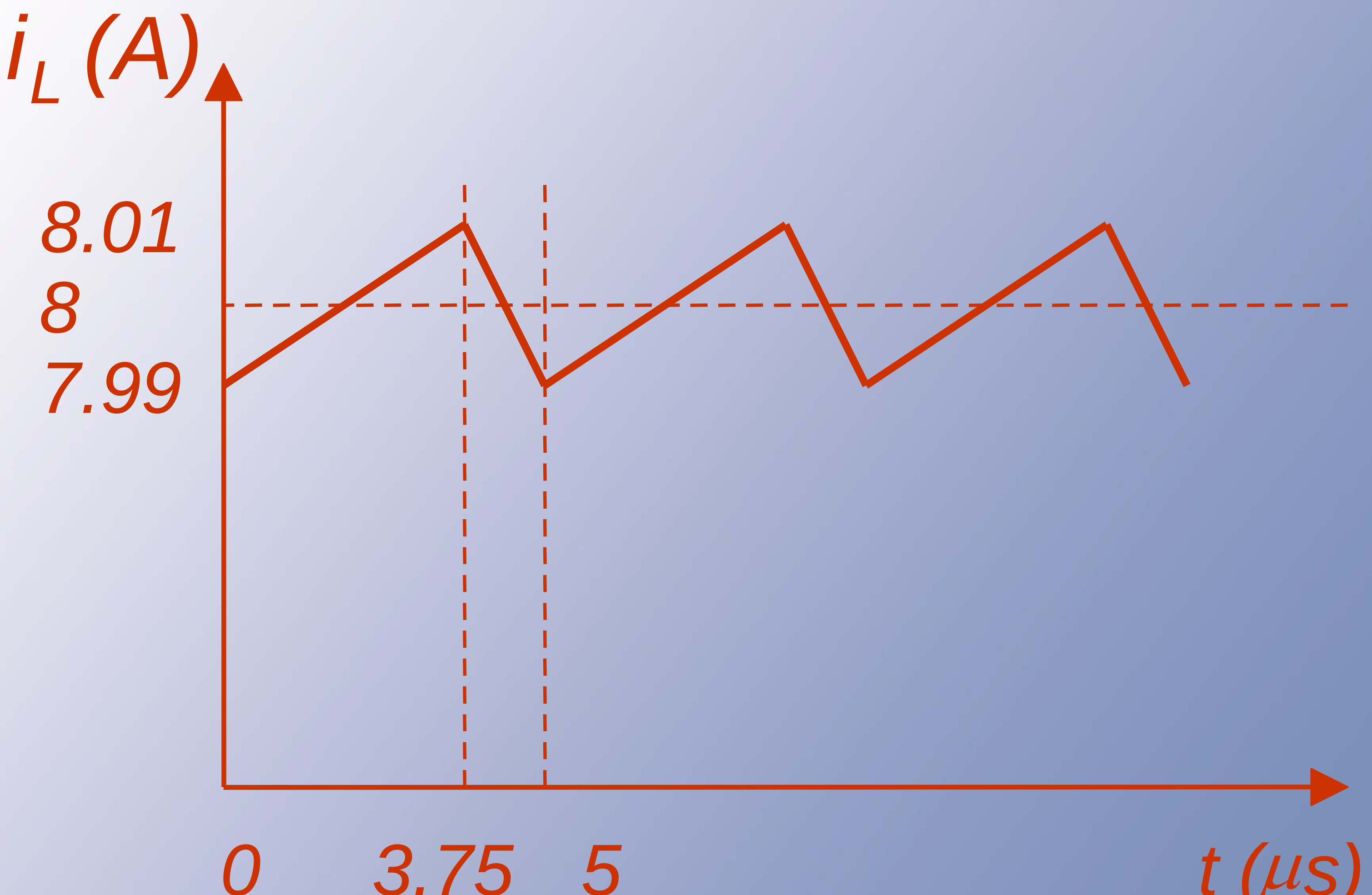
Δi = 0.0188 A.

This is less than 0.25% of i_L.

Check the current fall. Does it match?

Why?

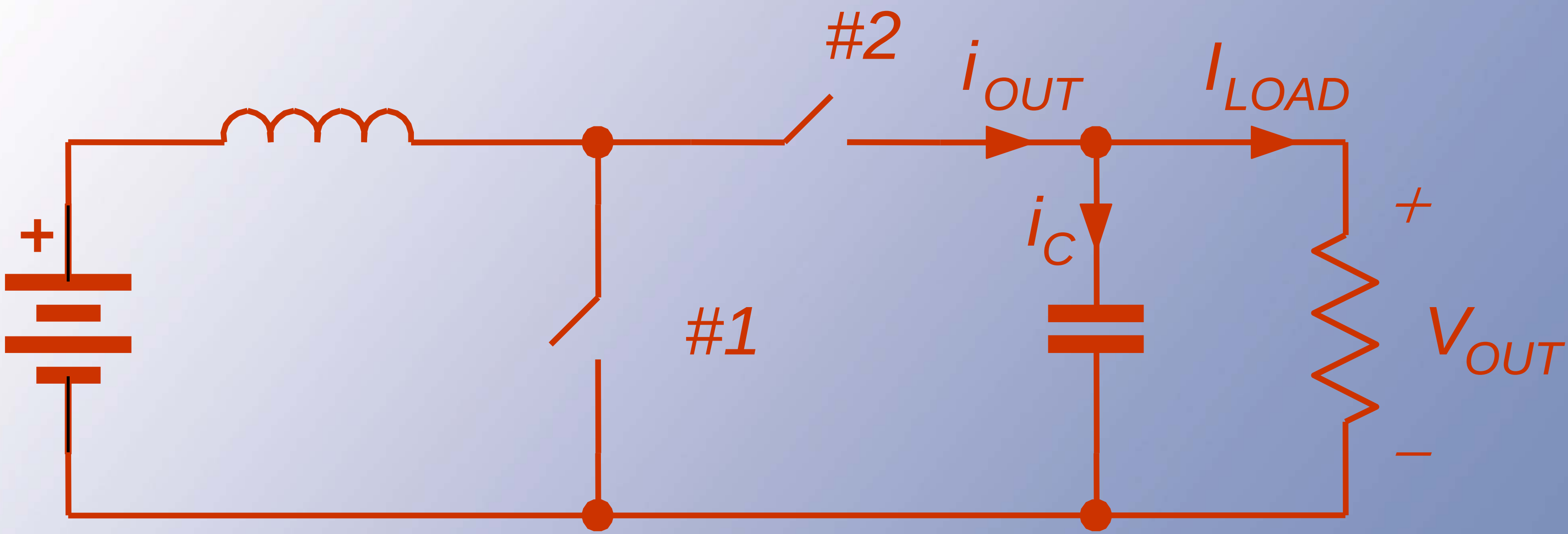
Current ripple



Voltage Ripple

- We can do the same thing to find ripple on the output capacitor.
- The capacitor current is known: With switch #2 off, the resistor draws out 2 A. With switch #2 on, the current is 8 A - 2 A = 6 A.

Voltage Ripple



- i_C is fully determined.
- #2 off : i_C = -2 A v_C decreases
- #2 on : $i_C = i_L - 2 = 8 - 2 = 6$ A v_C increases

Voltage Ripple

- Thus i_C = 6 A for 1.25 us, and -2A for 3.75 us.
- Since $i_C = C\, dv/dt$ gives linear voltage ramps, the voltage rises when i_C = 6 A: $(6\text{ A})/C = \Delta v/\Delta t$.
- The time involved is 1.25 us.

Voltage Ripple

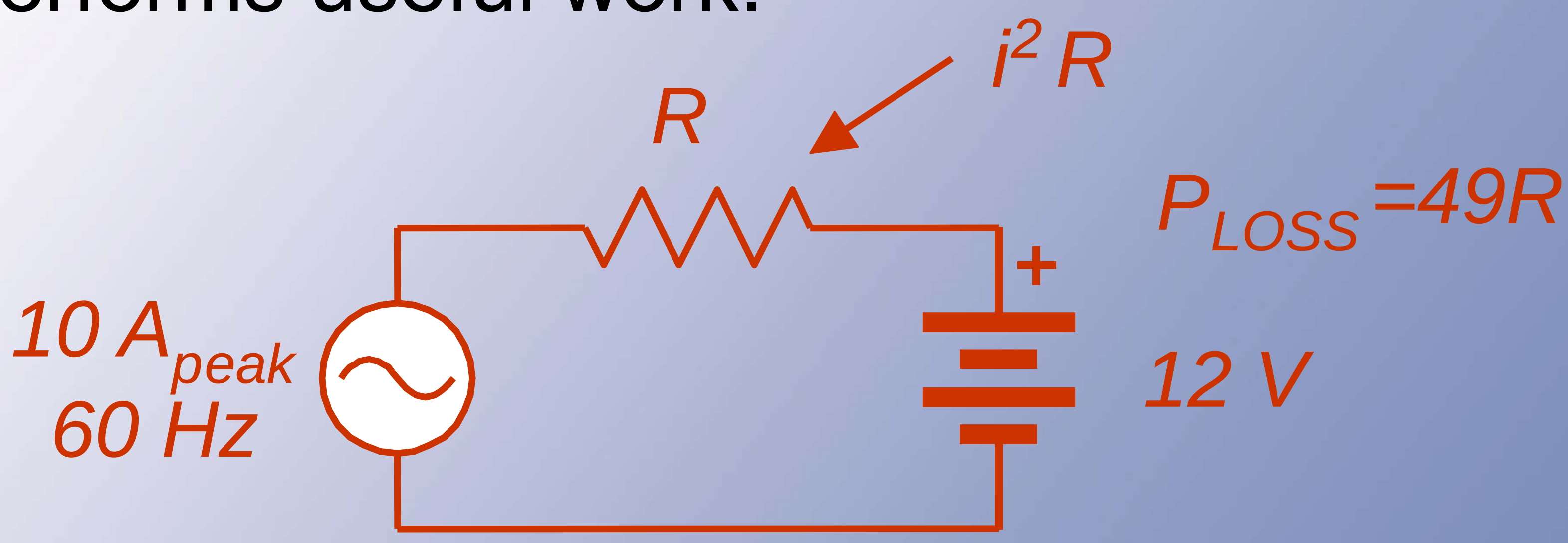
- (6 A)(1.25 us)/(10 uF) = Δv = 0.75 V.
- This is 3.75% of the 20 V dc level.
- Not perfect, but still very nearly constant.
- Thus with switching frequency of 200 kHz, L = 1 mH, C = 10 µF, R = 10 Ω, V_{in} = 5 V, we get 20 V out and 3.75% peak-to-peak output ripple.

Power Factor

- A conventional measure in utility systems is *power factor* -- the fraction of energy flow that does useful work.
- Recall that cross-frequency terms do not contribute <P>.
- But, the cross terms *do* require current and voltage.
- The extra current means extra I^2R loss, and should be avoided is possible.

Power Factor

Capture fraction of energy flow that erforms useful work.



$\langle P_{OUT} \rangle = 0 \Rightarrow pf = 0$

Power Factor

- Power factor is defined by

$$pf = \frac{\langle P \rangle}{V_{RMS} I_{RMS}} \leq 1$$

- Ideally, this is 1. When harmonics or phase shifts are present, it is less than 1.
- *pf* can be less than 1 even in a linear circuit, but it is never greater than 1.

Power Factor Example

$$\langle P \rangle = 0 \Rightarrow pf = 0$$

Two contributions to the pf : “Distortion power” and “Displacement power.” The “*displacement factor*:”

$$df = \frac{\langle P \rangle}{V_{RMS1} I_{RMS1}} = \cos(\theta_1)$$

Power Factor Issues

- *pf* is often divided into a phase effect at the wanted frequency (*displacement power*, with a *displacement factor*), and a distortion effect at unwanted frequencies.
- *pf* < 1 causes extra loss, and limits flow capabilities.

Power Factor Issues

Why do we want pf = 1 ?

1) Minimizes system loss. Maximizes "device utilization."

2) Gives more available power.

120 V, 12 A

pf = 1 → 1440 W

pf = 0.5 → 720 W

3) Examples

Rectifiers can have pf ~ <u>0.3</u>

Dc-Dc Converters

- We would like to have a dc transformer -- a device with $P_{in}=P_{out}$ and $V_{out}/V_{in} = a$.
- Magnetic transformers cannot handle dc, but the dc transformer is still a valid concept.
- Our objective in dc-dc converter design is to approach a dc transformer as best we can.

Dc Transformers

- We would like to have a box like this, for DC.

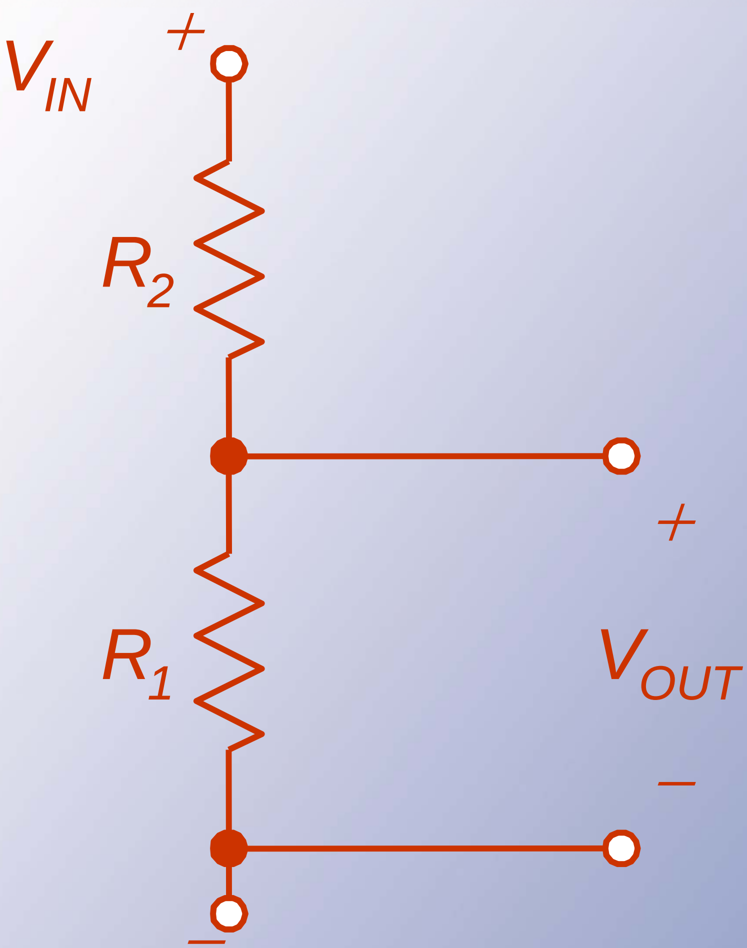
$$P_{in} = V_{in} I_{in} = P_{out} = V_{out} I_{out}$$

$$\frac{V_{out}}{V_{in}} = a \qquad \frac{I_{in}}{I_{out}} = a$$

Dividers?

- We might try a voltage divider.
- Two problems:
  - No regulation
  - Losses within the "converter"

Dividers?

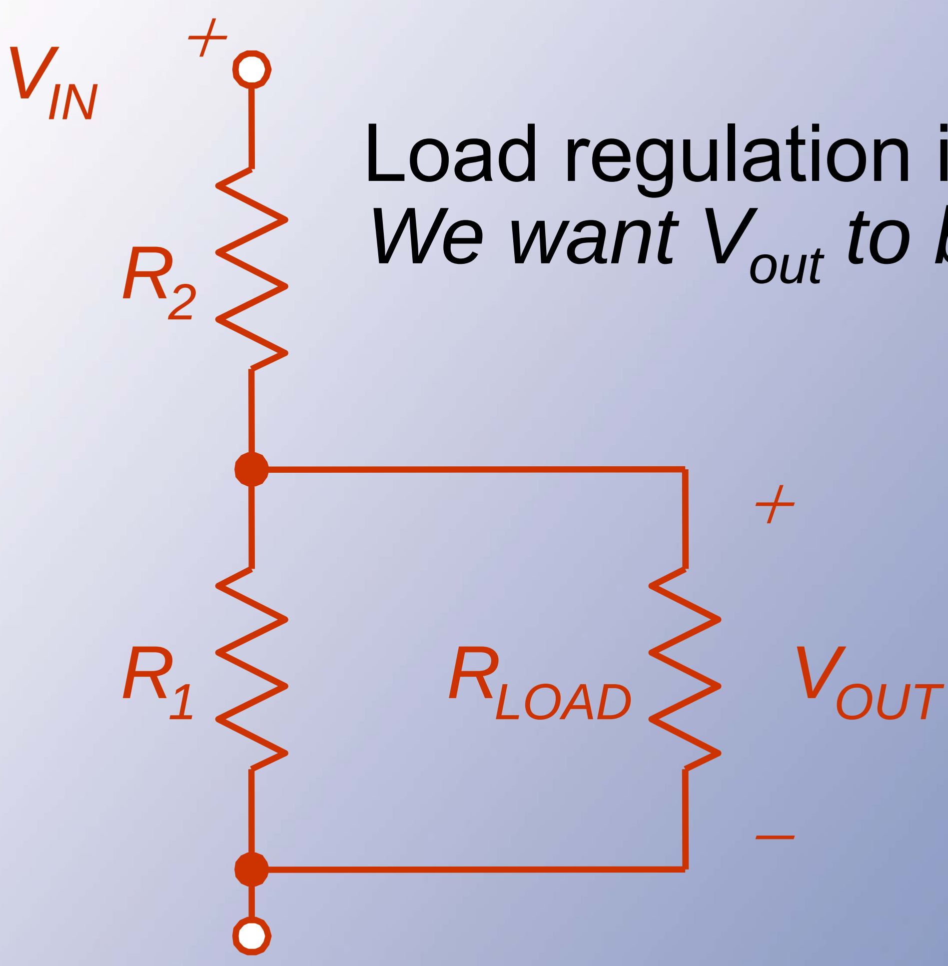


$$\eta = \frac{P_{out}}{P_{in}}$$

$$No\ load:\ V_{out} = \frac{R_1}{R_1 + R_2} V_{in}$$

$$If\ P_{out} = 0,\ then\ \eta = 0$$

Dividers?



Load regulation issue.

We want V_{out} to be almost constant.

Dividers?

- The load regulation problem can be addressed through excess loading:
- Make the divider input draw so much power that the load power causes no change.

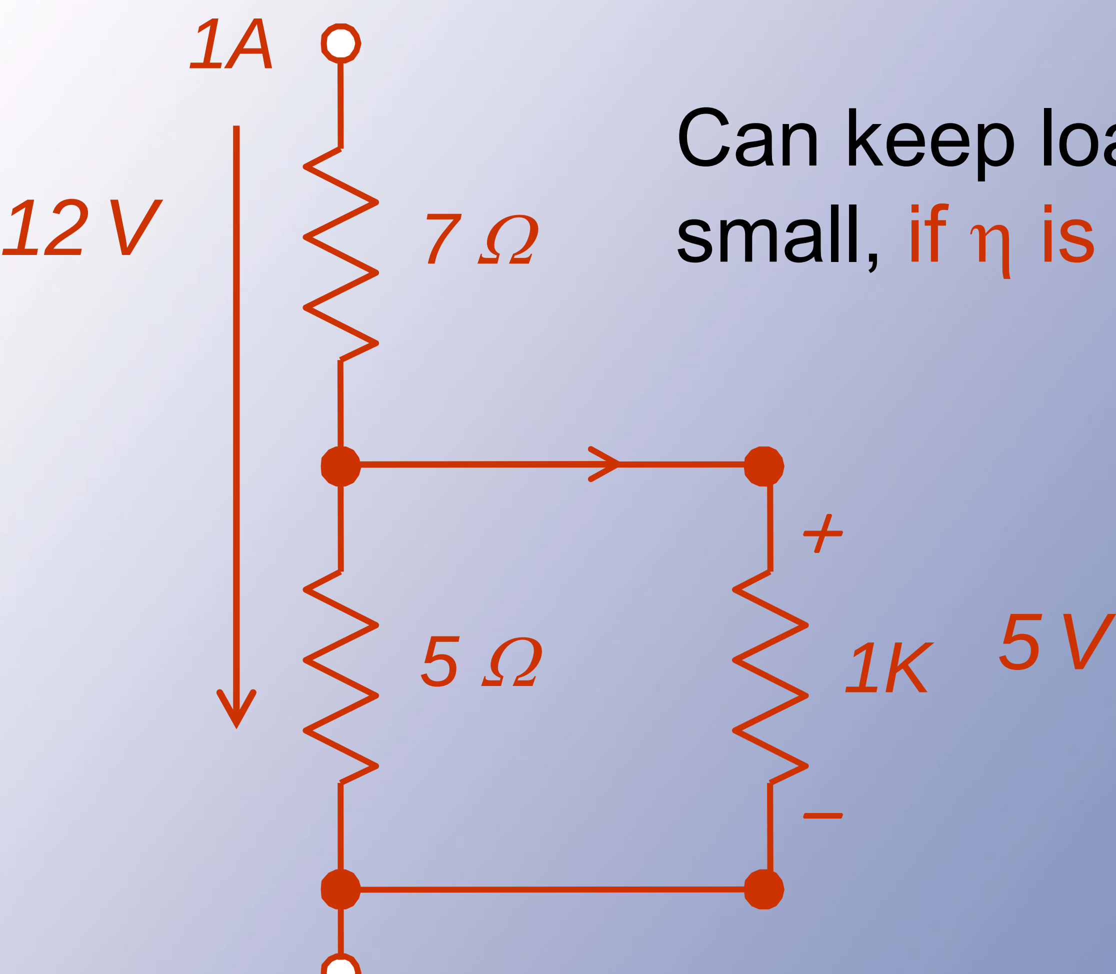
Divider Efficiency

- Instead, if somehow all output power is delivered to the load (best possible case), the efficiency is V_{out}/V_{in}.
- This occurs only at a single load value, if designed in advance. The design has no load regulation.
- Reality is always worse.

Dividers -- Conclusion

- Voltage dividers are useful for sensing applications when the load power is intended to be zero.
- A voltage divider is *not* useful for dc-dc conversion.
- It is not a power electronic circuit, since the efficiency cannot be 100%.

Sensing application



Can keep load change small, if η is low

Sensing application

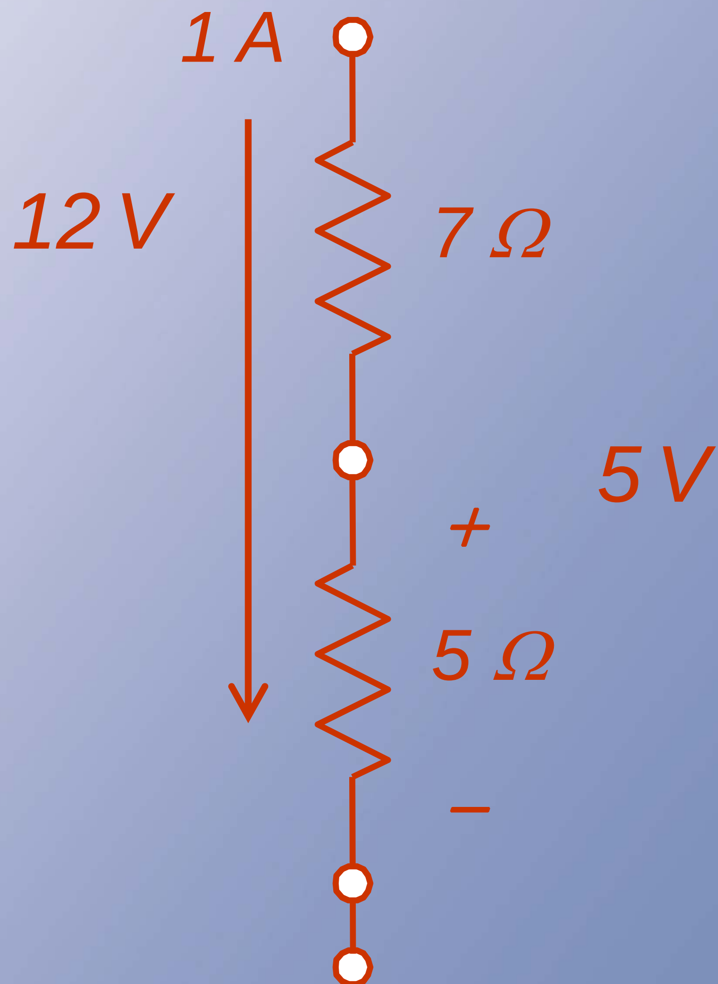
$$P_{in} = V_{in} I_{in}$$

$$P_{out} = V_{out} I_{out}$$

$$I_{in} = I_{out}$$

$$\frac{P_{out}}{P} = \frac{V_{out} I_{in}}{V_{in} I_{out}}$$

$$= \frac{V_{out}}{V_{in}}$$



$\eta = 5/12$

Dc Regulators

- Since a divider has no regulation, it motivates new types of circuits.
- In these types of "converters," the output is independent (within limits) of the input and of the load.
- They perform a regulation function rather than energy conversion.
- We call them "dc regulators."

Amplifiers

- It is also possible to use amplifier methods for dc-dc conversion.
- These are common, because they have excellent regulation properties.
- In general, efficiency is poor.

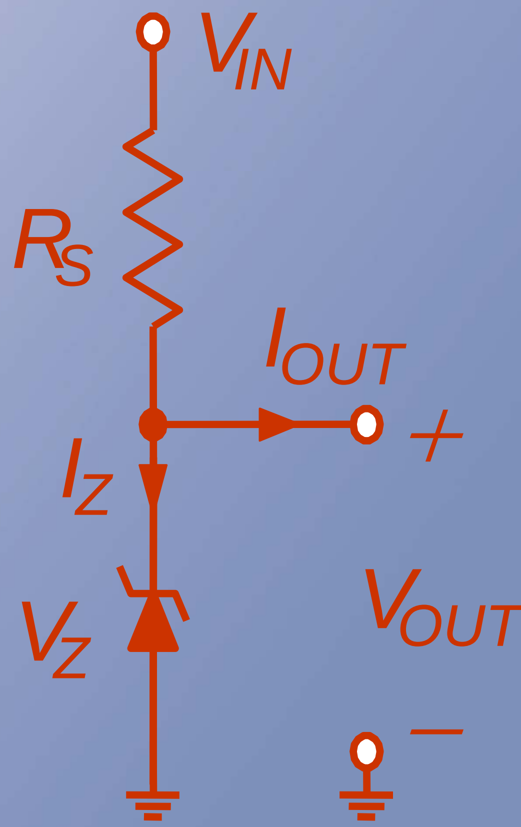
Shunt Regulator

Voltage divider, 12 V to 5 V, 1 W.

- With exact values, best efficiency is 5/12.
- To provide regulation, the divider current path must carry much more than the load current.
- Problems: line regulation, load regulation, loss even if $P_{out} = 0$, low η.

Shunt regulator.

- Zener diode in place of low-side resistor.
- Requires $I_Z > 0$.
- For 12 V to 5 V, 1 W, $R_1 < 35\ \Omega$.
- Solves the line and load regulation challenges, but not the others.



Example

12 V to 5 V regulation at up to 0.2 A.

At 0.2 A load, the input current must be at least 0.2 A to ensure $I_Z > 0$.

This current flows through a drop of 7 V, so $R_s < 35\ \Omega$.

Try it . . .

Example

- Test a load of 0.1 A. The input current, if the regulator works, is (12 V - 5 V)/(35 Ω) = 0.2 A. The load current is 0.1 A, so the zener current must be 0.1 A.
- This is wasteful, but it works.
- Useful for generating low-power reference voltages.

Example

$$P_{OUT} = (0.1\ A)(5\ V) = 0.5\ W$$

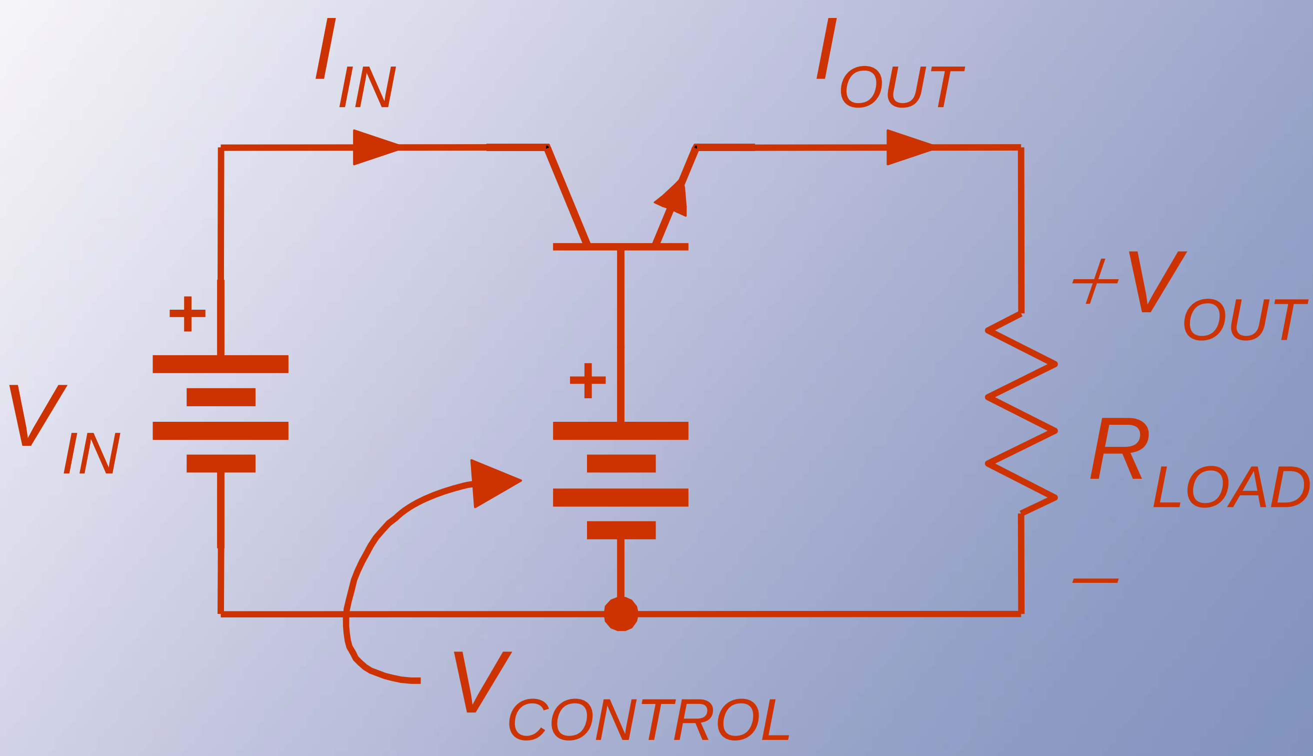
$$P_{IN} = (12\ V)(0.2\ A) = 2.4\ W$$

$$\eta = \frac{P_{out}}{P_{in}} = 20.8\%$$

Series Regulator

- Instead find a series device that can provide an output that is approximately independent of the input.
- A bipolar transistor can do the job – in its linear operating region.
- With proper bias, the output depends on the base voltage.
- Not a switching method.

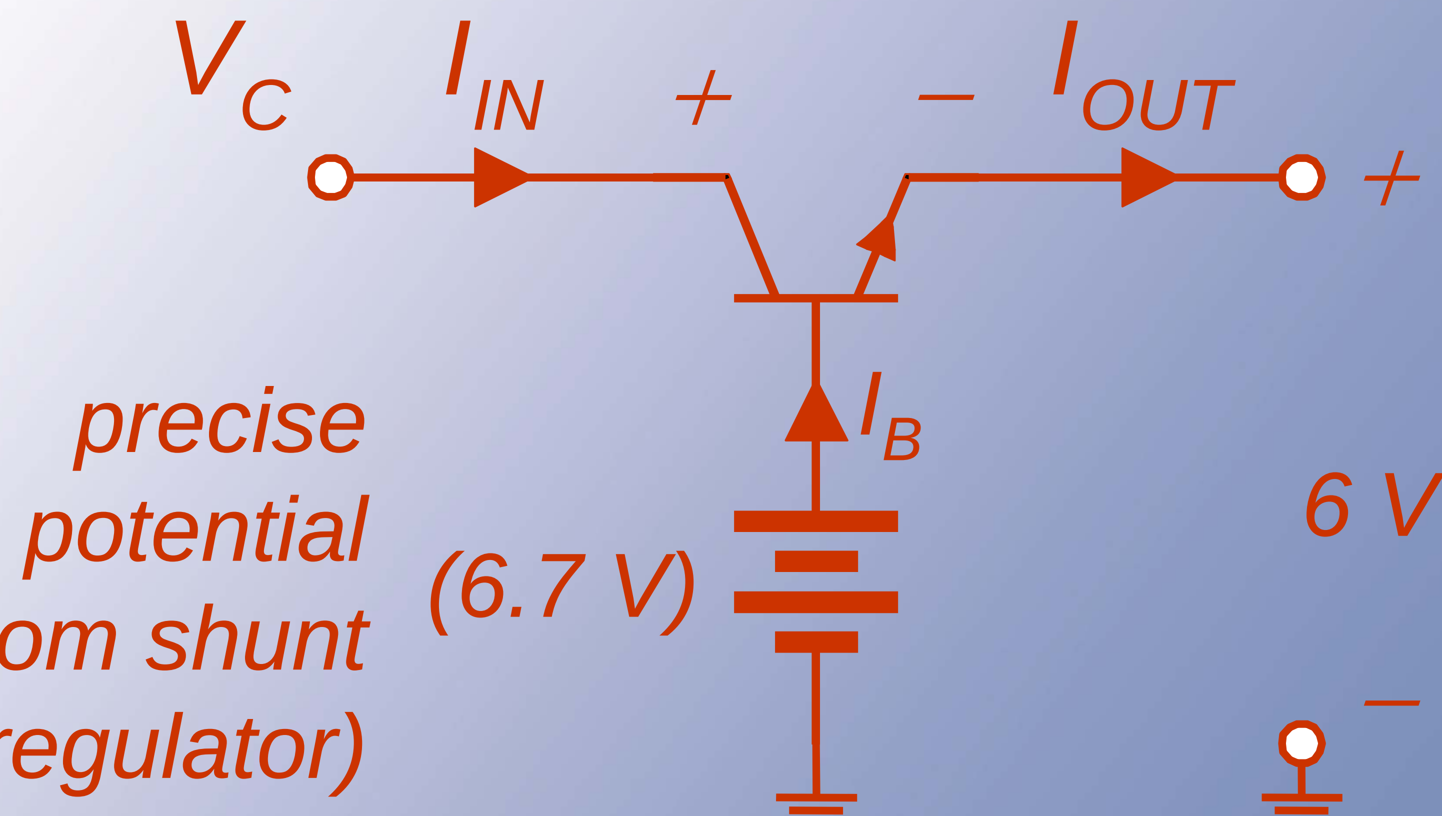
Series Pass Arrangement



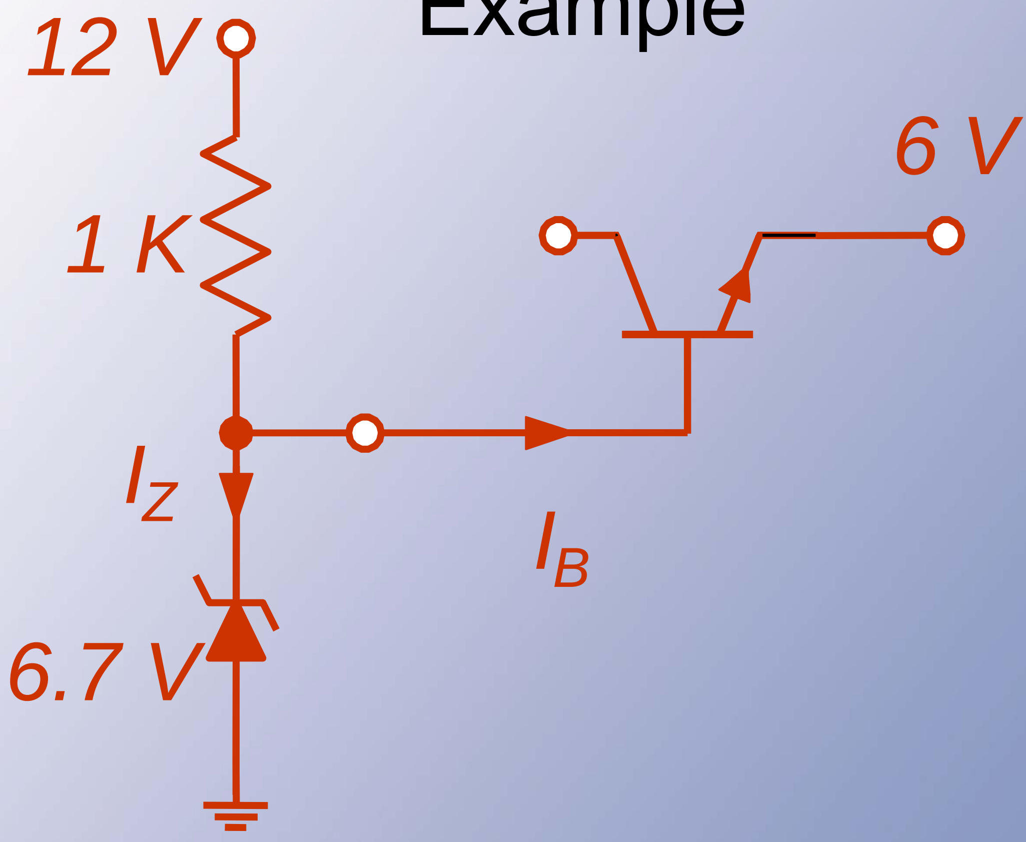
The emitter voltage follows the (low-power) base voltage.

Series Pass Arrangement

Suppose a 6 V output is needed.



Example



Here, a shunt regulator provides the reference voltage for a series regulator.

Series Pass Arrangement

- In the bipolar case, if there is high gain, the base current is very low.
- The emitter voltage will be roughly 0.7 V below the base voltage.
- This works provided the collector input is high enough.

Series Pass Arrangement

$I_{IN}=I_C$ If I_B is small (high gain), then

$I_{OUT}=I_E$ $I_C = I_E$

$=I_B+I_C$ $I_{IN}= I_{OUT}$.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_{out} I_{out}}{V_{in} I_{in}} = \frac{V_{out}}{V_{in}}$$

Series Pass Comments

- Common for local dc power, e.g., 12 V in, 5 V out, but extremely inefficient unless voltages are nearly the same.
- Notice that $I_{in} \approx I_{out}$.
- Best-case efficiency is V_{out}/V_{in} since current is conserved.
- Requires $V_{in} > V_{out} + {\sim}2$ V

More Comments

- Although this is common, it is only acceptable when voltages are close.
- Useful example: 14 V to 12 V regulator for automotive application. Efficiency could be 86%.
- Poor example: 48 V to 5 V regulator for telephone application. Efficiency is only 10%.

Key Advantage

- $V_{out} = V_{control} - V_{be}$ --- entirely independent of input, load, etc.
- This is a “linear regulator,” since V_{out} is a linear function of a control potential.

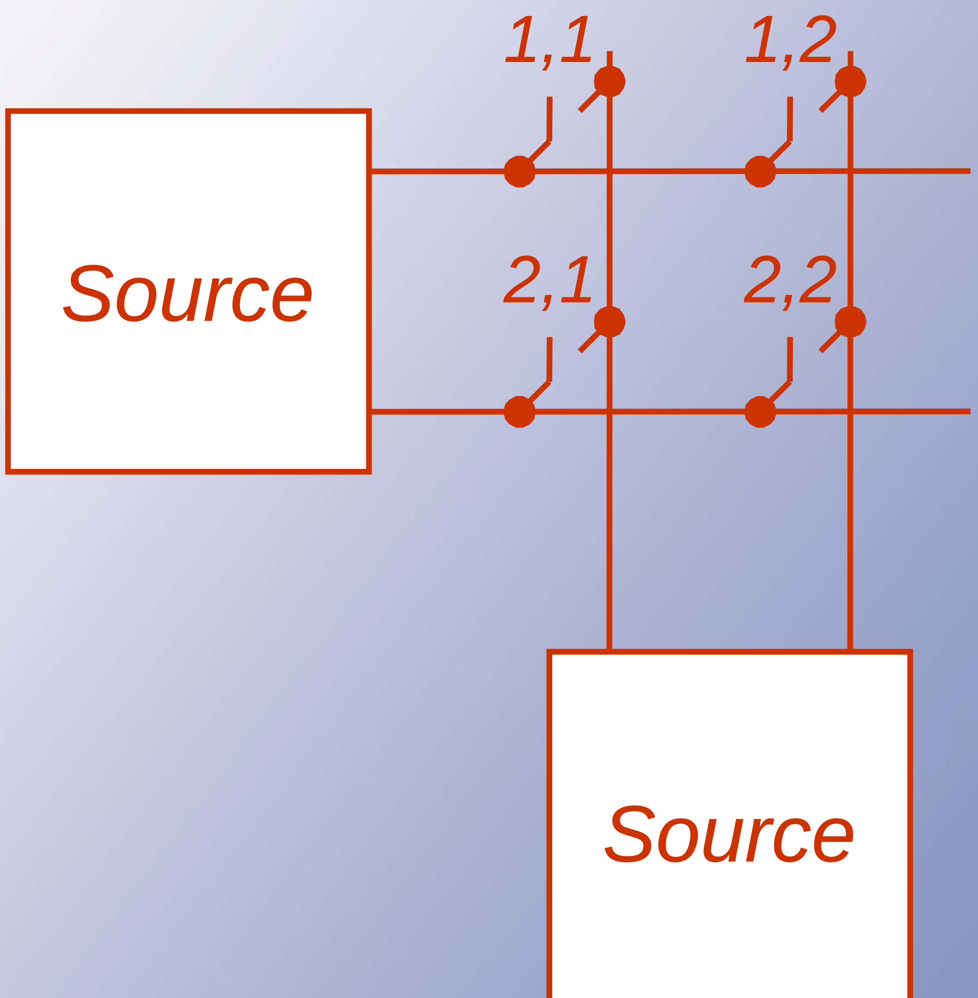
Parting Comments

- Series linear regulators make good filters -- if we can keep the input and output close together.
- Shunt regulators provide fine fixed reference voltages but are not so useful for power.

Now, Switching

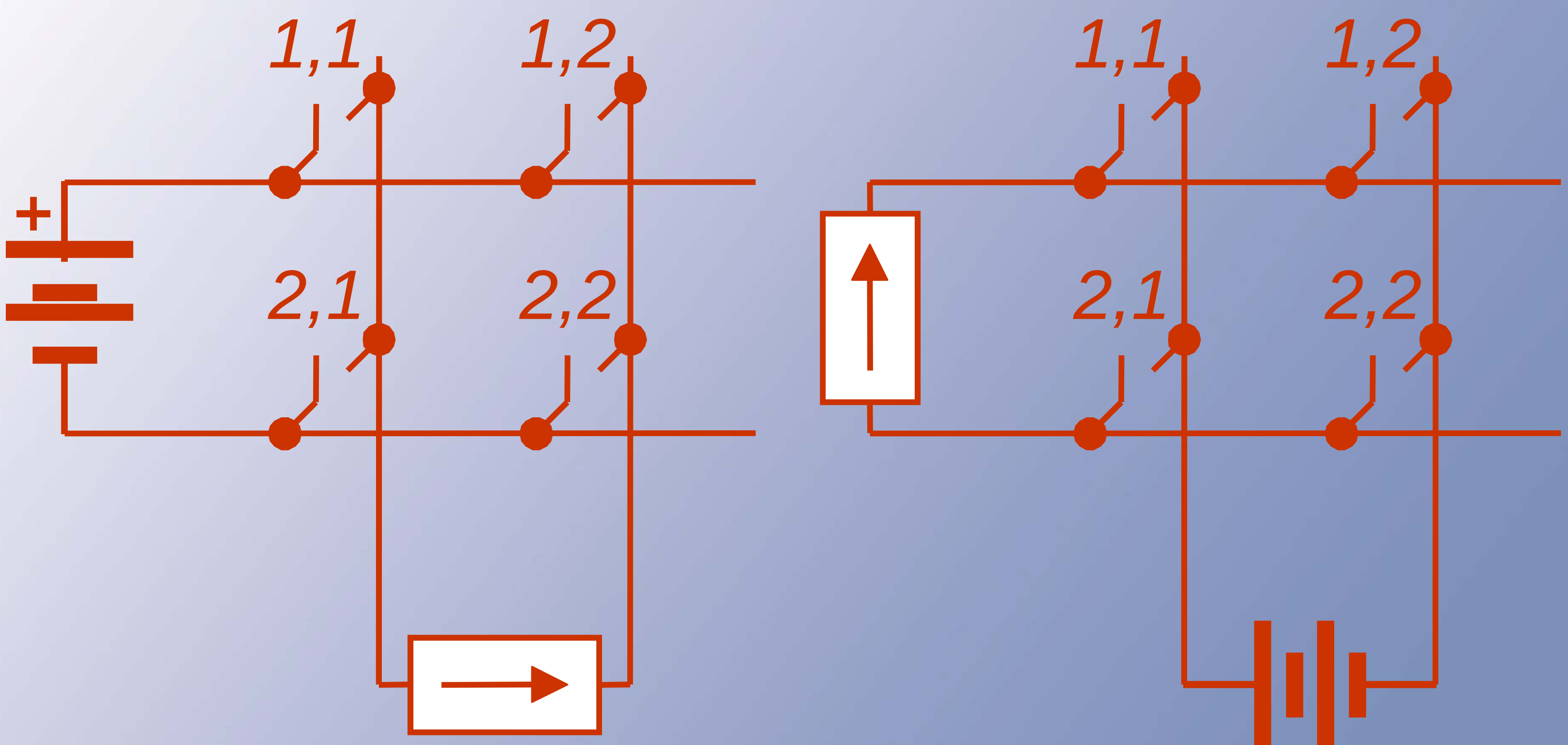
- The circuits so far cannot provide 100% efficiency. *We need switching.*
- Two possibilities of general dc-dc conversion:
  - 2 x 2 matrix, voltage in, current out
  - 2 x 2 matrix, current in, voltage out.
- These are the direct dc-dc converters.

Direct DC-DC Converters

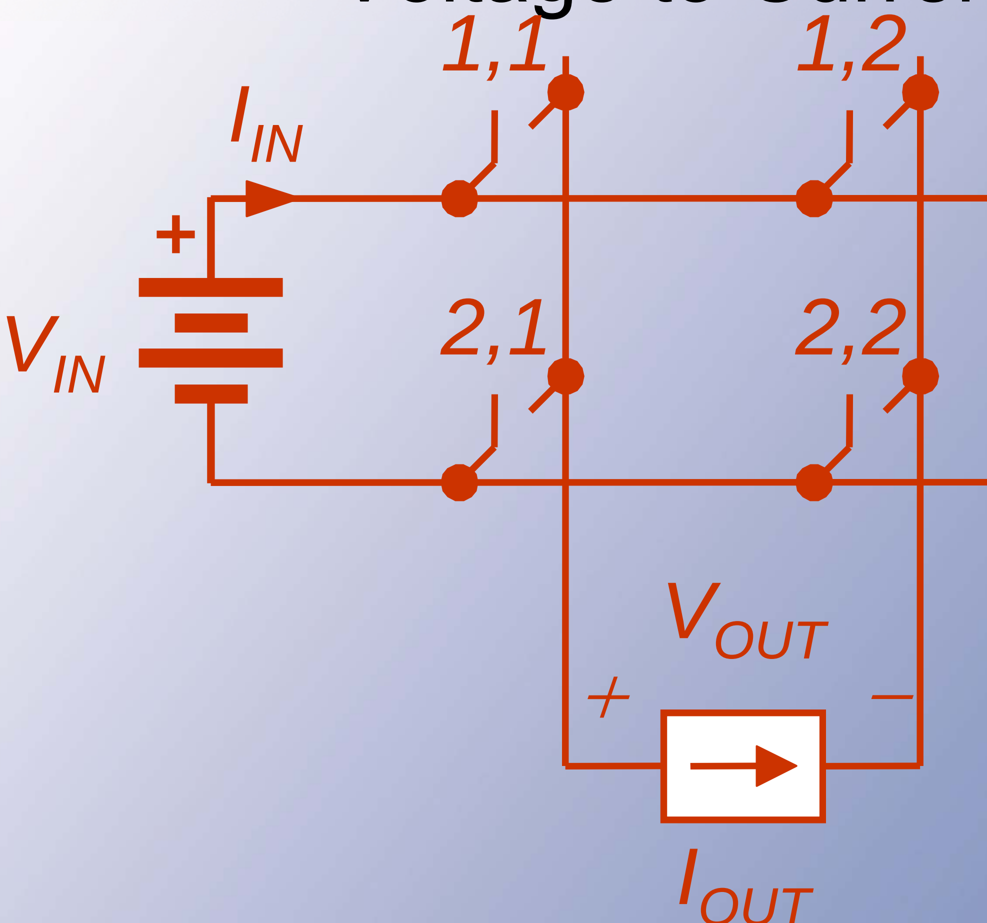


Direct DC-DC Converters

Two direct converters for DC-DC:



Voltage to Current



Output voltage is +V , 0, or –V

Switch Relations

- Output is $+V_{in}$ if 1,1 and 2,2 are on together, etc.
- A switching function representation is $v_{out}(t) = q_{11}\, q_{22}\, V_{in} - q_{12}\, q_{21}\, V_{in}$
- But KVL, KCL require $q_{11}+q_{21}=1$, $q_{12}+q_{22}=1$.

Switch Relations

In switching function form:

$$v_{out}(t) = q_{11}q_{22}V_{in} - q_{21}q_{12}V_{in}$$

$$i_{in}(t) = q_{11}q_{22}I_{out} - q_{21}q_{12}I_{out}$$

KVL+KCL: $q_{11} + q_{21} = 1$

$$q_{12} + q_{22} = 1$$

$$v_{out}(t) = q_{11}q_{22}V_{in} - (1 - q_{11})(1 - q_{22})V_{in}$$

Switch Relations

$$v_{out}(t) = (q_{11} + q_{22} - 1)V_{in}$$

In this dc application, we are interested in <v_{out}(t)>. The switching function averages are the duty ratios, and

$$\langle v_{out}(t) \rangle = (D_{11} + D_{22} - 1)V_{in}$$

We can choose duty ratios D_{11} and D_{22} to provide a desired <v_{OUT}>.

Switch Relations

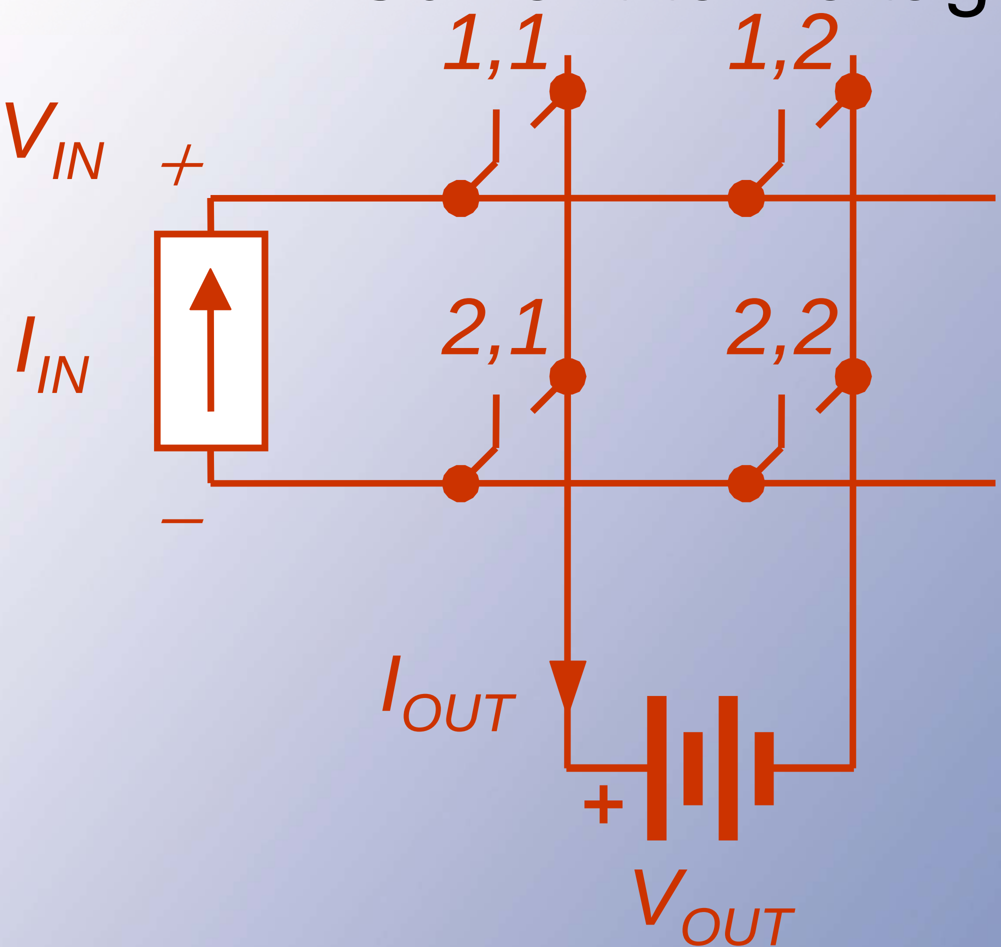
$$0 \le D_{ii} \le 1 \quad \Rightarrow \quad 0 \le D_{11} + D_{22} \le 2$$

$$\Rightarrow \quad -V_{in} \le \langle v_{out} \rangle \le V_{in} \quad \Rightarrow \quad \left| \langle v_{out} \rangle \right| \le V_{in}$$

"Buck Converter" or
"Step-Down Converter"

$$\langle i_{in} \rangle = (D_{11} + D_{22} - 1) I_{out}$$

Current to Voltage



Output current is +I, 0, or -I

Switch Relations

$$\langle i_{out} \rangle = (D_{11} + D_{22} - 1) I_{in}$$

$$\langle v_{in} \rangle = (D_{11} + D_{22} - 1) V_{out}$$

$$V_{out} = \frac{\langle v_{in} \rangle}{(D_{11} + D_{22} - 1)}$$

$$0 \le D_{ii} \le 1 \quad \Rightarrow \quad 0 \le D_{11} + D_{22} \le 2$$

$$\Rightarrow \quad |\langle v_{out} \rangle| \ge V_{in}$$

Boost Converter

Summary

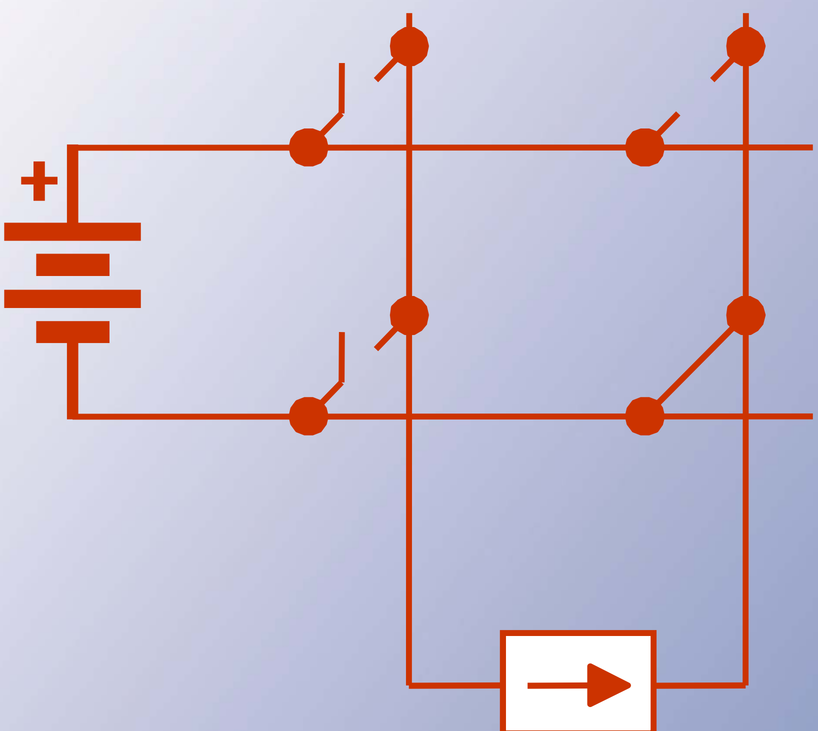
- The dc transformer is an important practical function.
- Non-switching methods, such as voltage dividers and dc regulators, are not really suitable for power conversion.
- We considered two switching circuits that accomplish buck and boost dc-dc conversion functions – types of dc transformers.

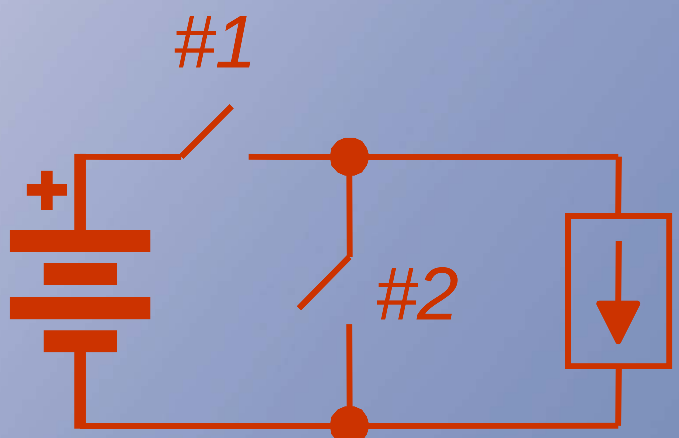
Simplifications

- In many applications, it is desirable to share a common input-output node (ground reference).
- This requires one switch always on and one always off.

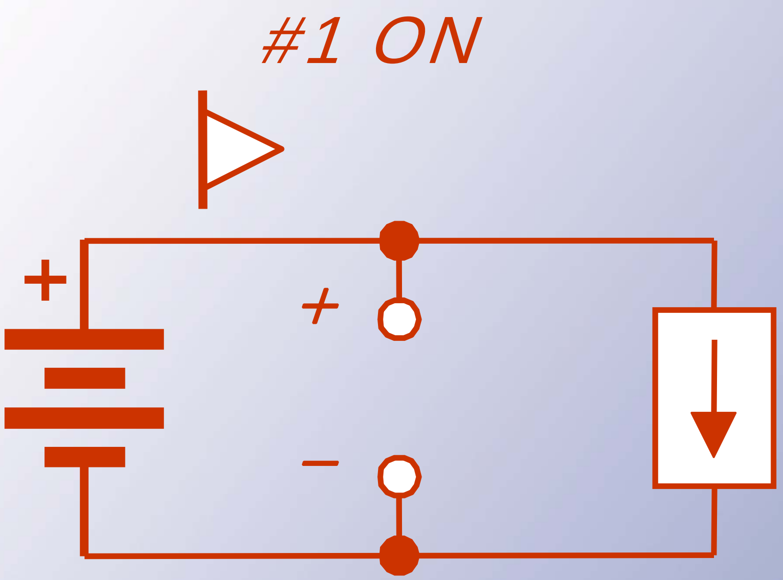
Common-Ground Dc-Dc

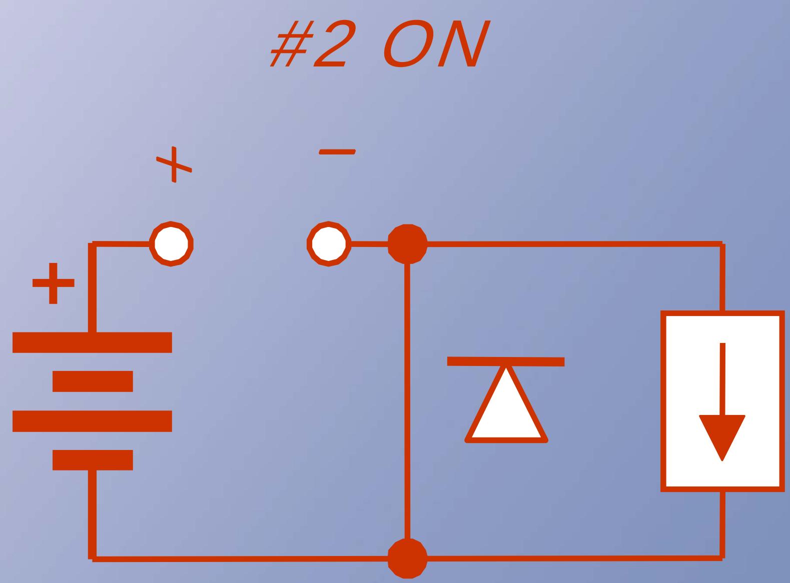
Example: 2x2 switch matrix, with common input-output ground

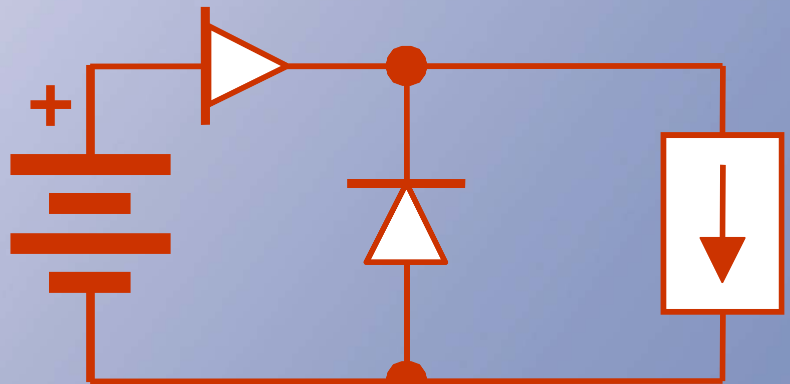




Common-Ground Dc-Dc

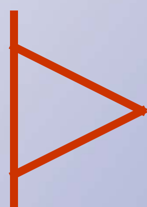
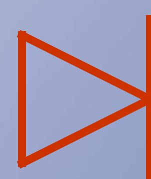






Common-Ground Dc-Dc

With two switches left, label them #1 and #2.

One becomes  and one 

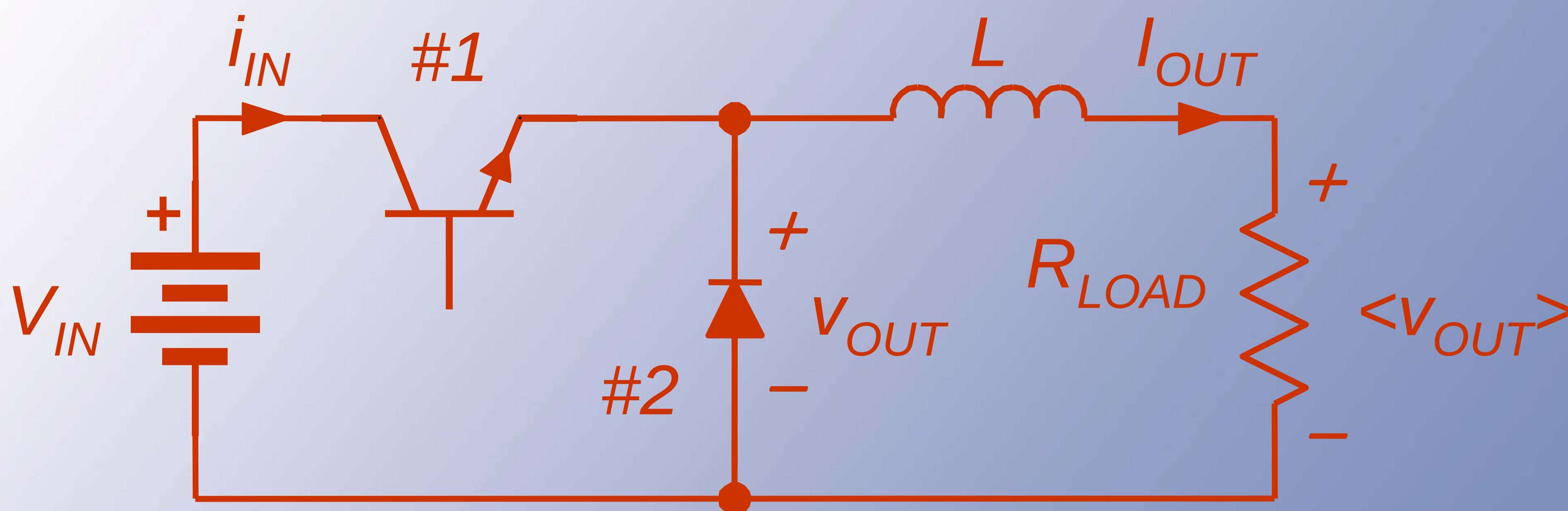
This can be checked by testing current (on) polarity and voltage (off) polarity.

Switching Functions

With ideal, or near-ideal, current and voltage sources, KVL and KCL require $q_1 + q_2 = 1$.

The buck converter:

Buck Converter



- The voltage v_{out} is the “switch matrix output.”
- The load voltage is $\langle v_{out} \rangle$ since $\langle v_L \rangle = 0$.

Relationships

$$v_{out} = q_1 V_{in} \qquad \langle v_{out} \rangle = D_1 V_{in}$$

$$i_{in} = q_1 I_{out} \qquad \langle i_{in} \rangle = D_1 I_{out}$$

There is no loss.

Instantaneous power: $p_{in}(t) = q_1 V_{in} I_{out}$

$$= p_{out}(t)$$

Average power: $\langle p_{out} \rangle = \langle p_{in} \rangle$

$$= D_1 V_{in} I_{out}$$

Relationships

v_{out} is the switching matrix output.

$$v_{out} = q_1 V_{in} \qquad \langle v_{out} \rangle = \langle q_1 V_{in} \rangle$$

$$= V_{in} \langle q_1 \rangle \rightarrow$$

load voltage

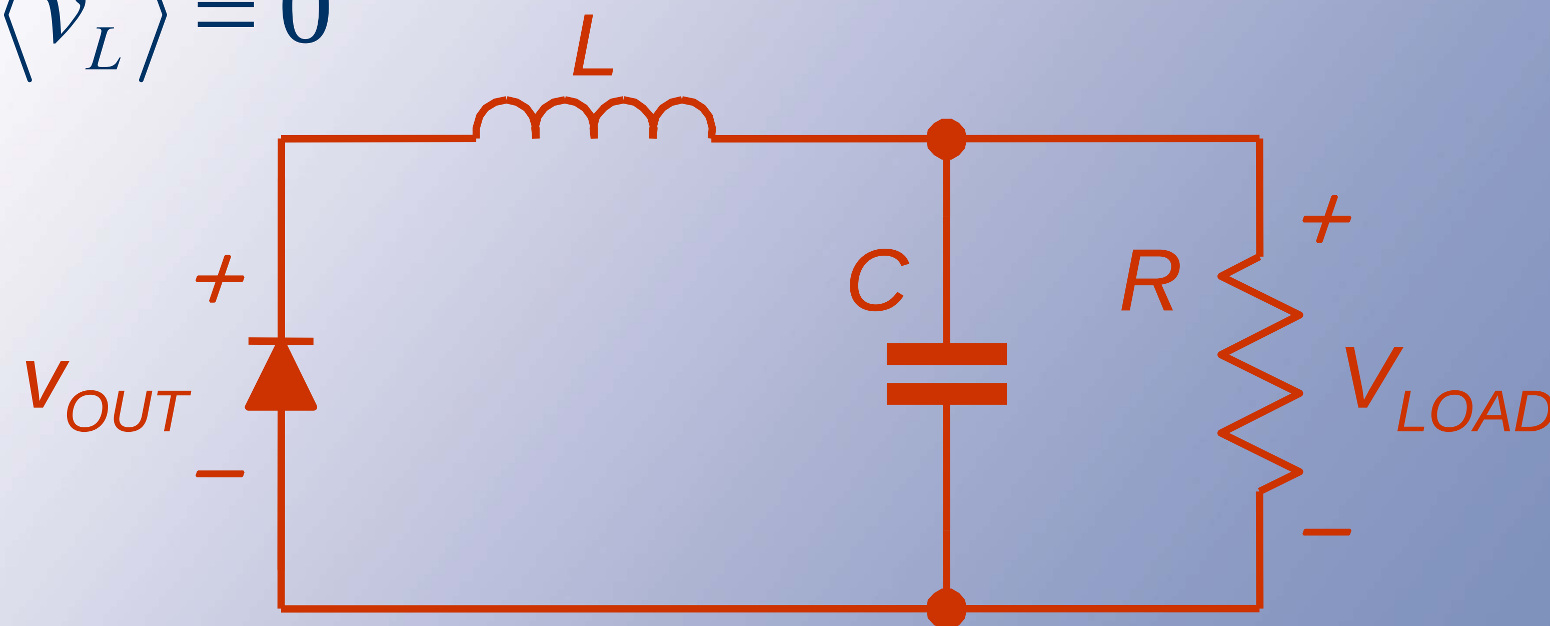
$$\rightarrow V_{out} = D_1 V_{in} \qquad \langle i_{in} \rangle = \langle q_1 I_{out} \rangle$$

$$= D_1 I_{out}$$

$$\langle v_{out} \rangle = V_{out} \rightarrow \textit{load voltage}$$

Relationships

$$\langle v_L \rangle = 0$$



$$V_{out} = \langle v_{out} \rangle = \langle v_{load} \rangle$$

L big $\rightarrow V_{load} \approx$ constant

Relationships

$$p_{in}(t) = V_{in} i_{in}(t)$$

$$= V_{in} q_1 I_{out}$$

$$p_{in}(t) = p_{out}(t)$$

$$p_{out}(t) = v_{out} I_{out}$$

$$= q_1 V_{in} I_{out}$$

$$\langle p_{in} \rangle = \langle p_{out} \rangle$$

$$= D_1 V_{in} I_{out}$$

The RMS "output"

The voltage v_{out} has an RMS value of

$$\sqrt{\frac{1}{T}\int_0^T q_1(t)^2 V_{in}^2 dt} = V_{in}\sqrt{D_1}$$

Is this relevant?

Notice that $q^2(t) = q(t)$

$$q_{RMS} = \sqrt{D}$$

A Design

- A 24 V to 5 V converter, switching at 100 kHz. The nominal load is 25 W, and the ripple is to be less than 1% peak-to-peak.
- This could be met with a buck converter, since $V_{out} < V_{in}$.

A Design

- The duty ratio will need to be V_{out}/V_{in} = (5 V)/(24 V) = 0.208
- The output current is (25 W)/(5 V) = 5 A.
- When switch #1 is on, the inductor sees 24 V - 5 V = 19 V.

A Design

- With #1 off, the inductor sees -5V
- So, since v_L = L di/dt, with #1 on,

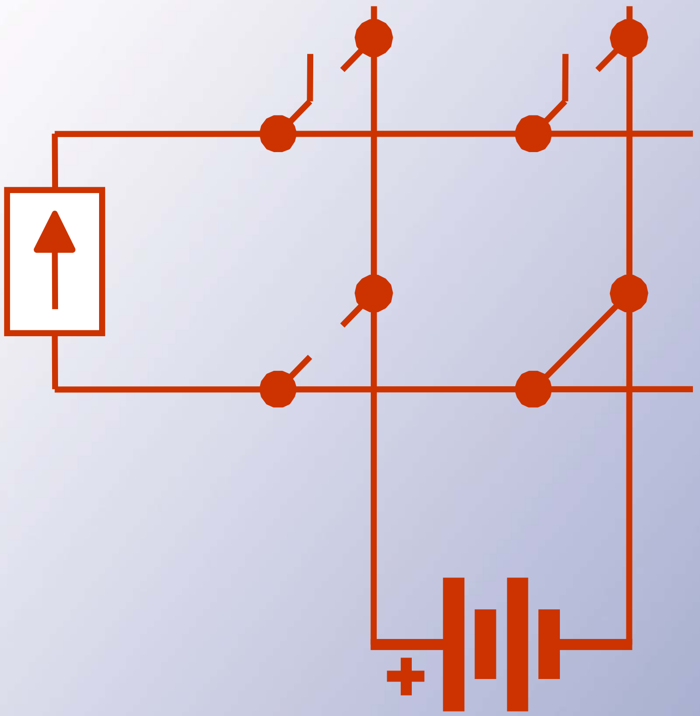
$$19\text{ V} = L\,di/dt$$
$$= L\,\Delta i/\Delta t$$

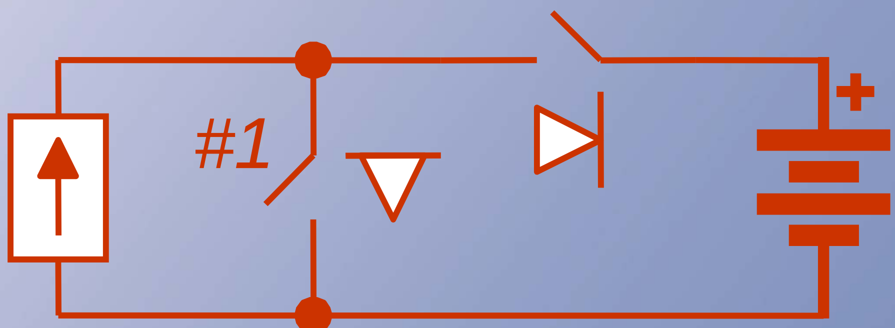
- The time involved is 0.208 T, or 2.08 us. We want $\Delta i < 0.01(5\text{ A})$.
- Thus (19 V)(2.08 us)/L < 0.05 A, and L > 0.792 mH

A Design

- We expect that $D_1 = 0.208$, $f_{switch} = 100$ kHz, L = 0.8 mH, and R = 1 Ω will meet the need.
- Practice: What is the peak-to-peak ripple if L = 8 uH? → it will be 100x as big

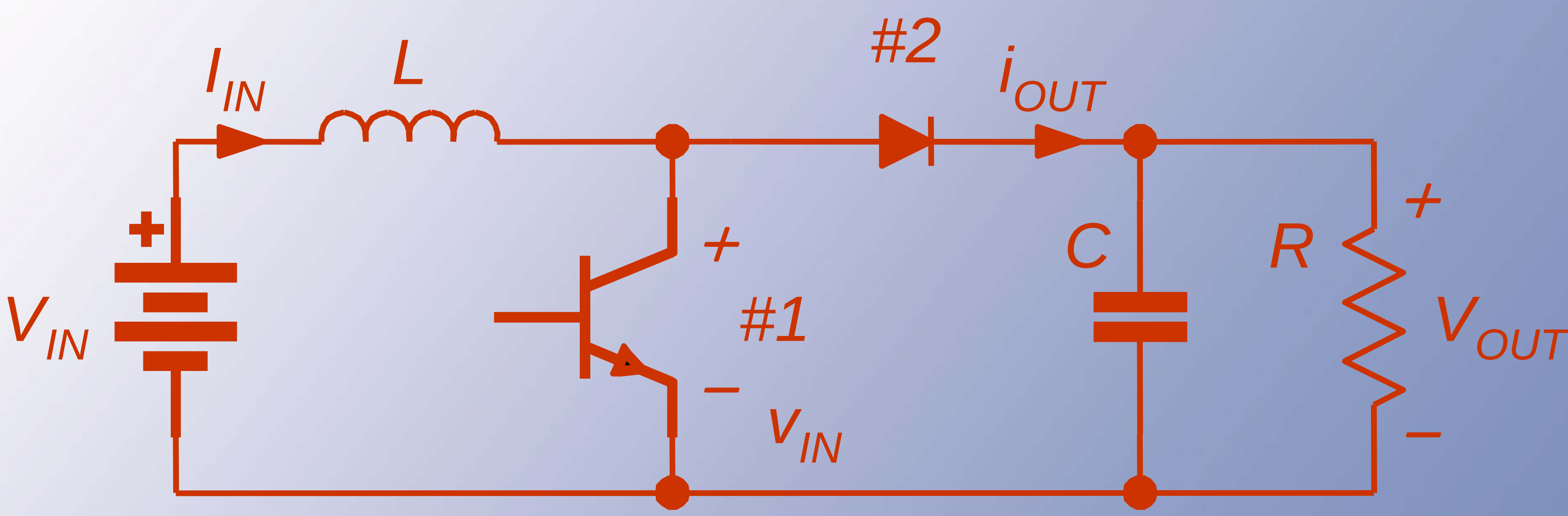
Boost Converter





A boost converter is a buck converter flipped horizontally.

Boost Converter



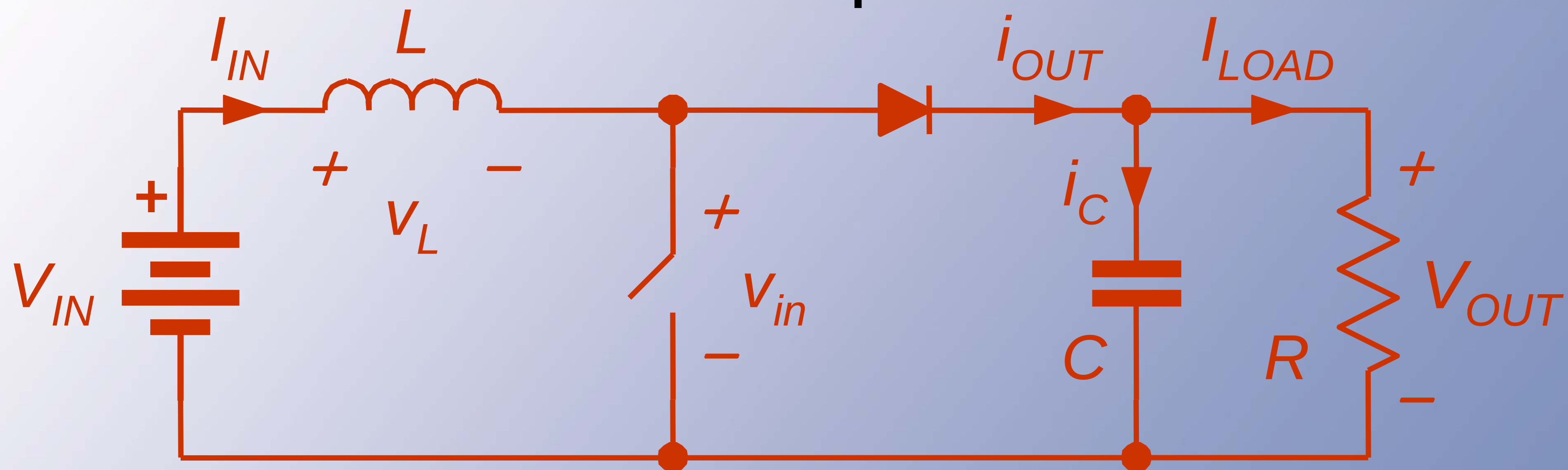
With common ground, the matrix reduces to two switches.

I_{in} is formed as a voltage in series with L.

Relationships

- The input voltage to the switch matrix is v_{in}, the voltage across the transistor.
- Since $\langle v_L \rangle = 0$, the average transistor voltage matches V_{in}.

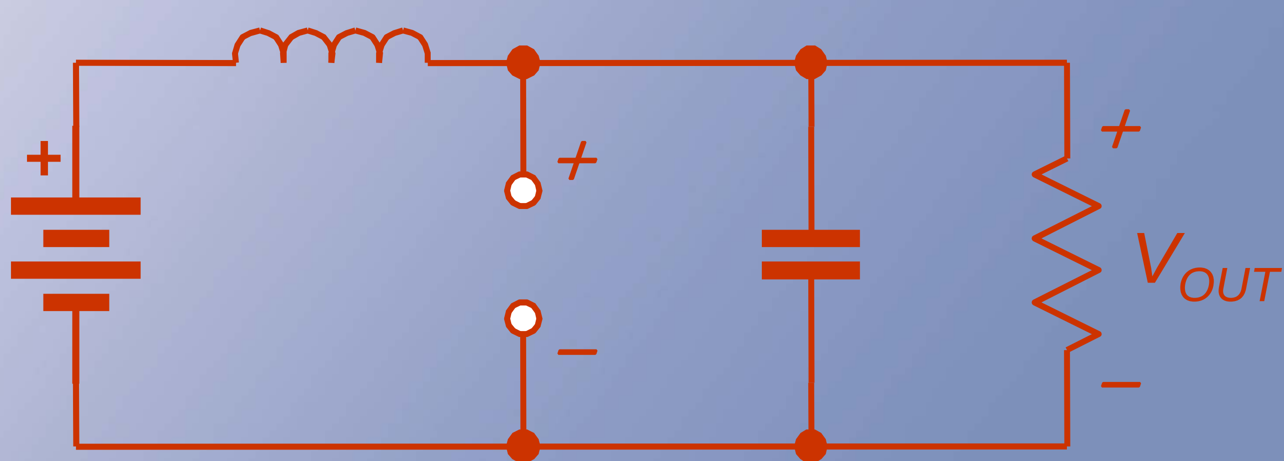
Relationships



$$\langle v_L \rangle = 0$$

$$V_{in} = \langle v_{in} \rangle$$

$$\langle i_C \rangle = 0$$



Relationships

- By KVL and KCL, sources require
  $q_1 + q_2 = 1$.
- Then $v_{in} = q_2 V_{out}$
  $= (1 - q_1) V_{out}$,
  $i_{out} = q_2 I_{in}$
  $= (1 - q_1) I_{in}$.
- The averages require $\langle v_{in} \rangle = V_{in}$, and
  $V_{out} = V_{in}/(1 - D_1)$

Relationships

$$i_{out} = q_2 I_{in}$$

$$v_{in} = (1 - q_1) V_{out} \qquad = (1 - q_1) I_{in}$$

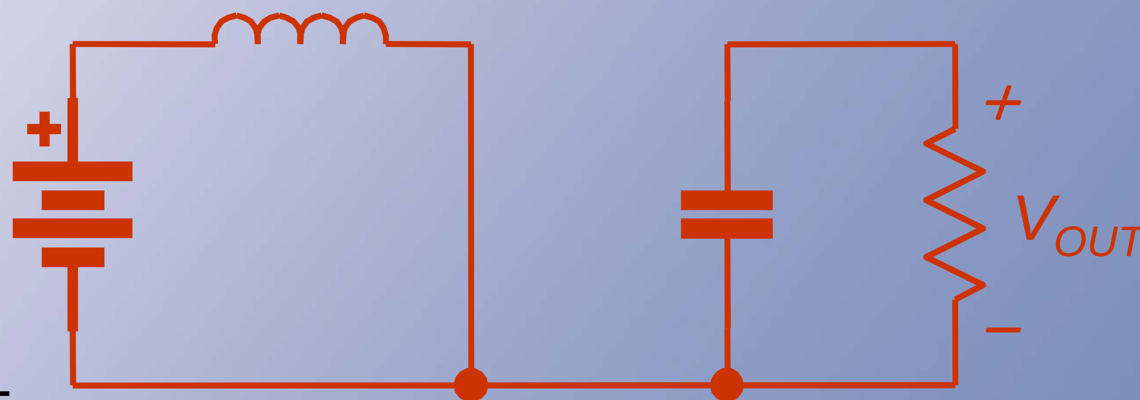
$$\langle v_{in} \rangle = V_{in} \qquad \langle i_{out} \rangle = I_{in}(1 - D_1)$$

$$= \langle (1 - q_1) V_{out} \rangle \qquad = I_{load}$$

$$V_{in} = V_{out}(1 - D_1) \qquad = I_{out}$$

Relationships

$$If\ D_1 = 1:$$



$$If\ D_1 \neq 1$$

$$V_{out} = \frac{V_{in}}{1 - D_1}$$

$$= \frac{V_{in}}{D} > V_{in}$$

$$V_{out} = 0$$

Example

2 V to 5 V boost (input might be one Li-ion cell, for instance, with 2 V as its lowest value).

Switching: 80 kHz. Load: 5 W. Input ripple: ± 10 mA. Output ripple: ± 1%.

This gives a period of 12.5 us.

Boost Example

- With 2 V input and 5 V output, the load current at 5 W is 1 A, but the input current must be (5 W)/(2 V) = 2.5 A.
- With $\pm$ 10 mA input ripple, the peak-to-peak value is 20 mA.

Boost Example

- When switch #1 is on, the inductor voltage is 2 V, and current increases.
- The duty ratios: $D_2 = V_{in}/V_{out} = 0.40$, and $D_1 = 1 - D_2 = 0.60$
- Switch #1 is on 0.60 T = 7.5 us.

Boost Example

v_L = L di/dt = 2 V with #1 on.

Thus (2 V)/L = $\Delta i/\Delta t$,

Δt = 7.5 us.

To get Δi < 0.02 A, we need

L > (2 V)(7.5 us)/(0.02 A), or

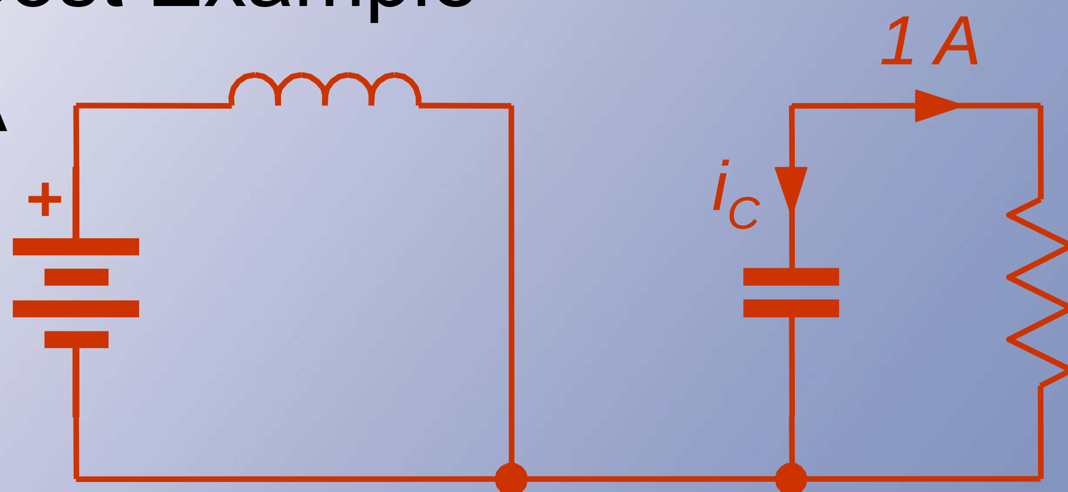
L > 0.75 mH.

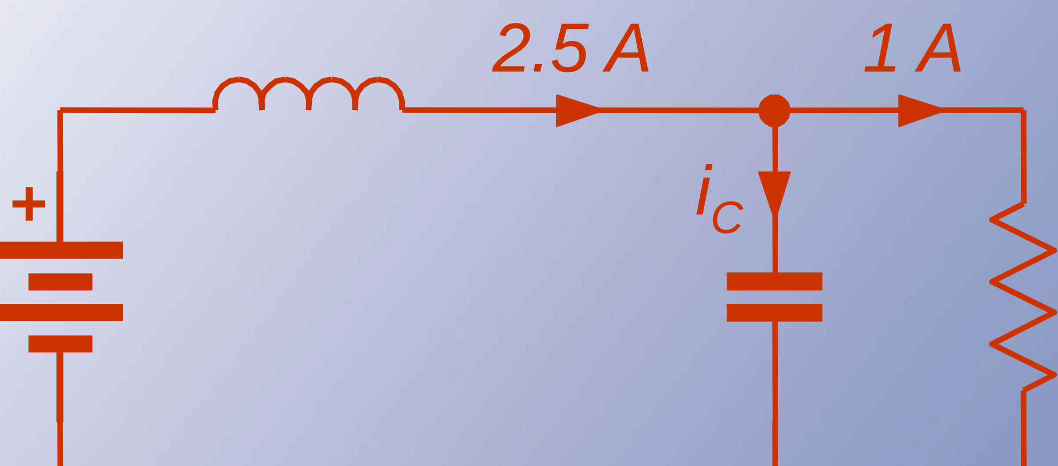
Boost Example

- What about V_{out}?
- The capacitor current is $I_{in} - I_{load} = 2.5\ A - 1\ A$ when switch #2 is on, and -1 A when switch #1 is on.
- We want $\pm$ 1% of 5 V, or a peak-to-peak change below 0.1 V.

Boost Example

#1 ON: i_C= -1 A





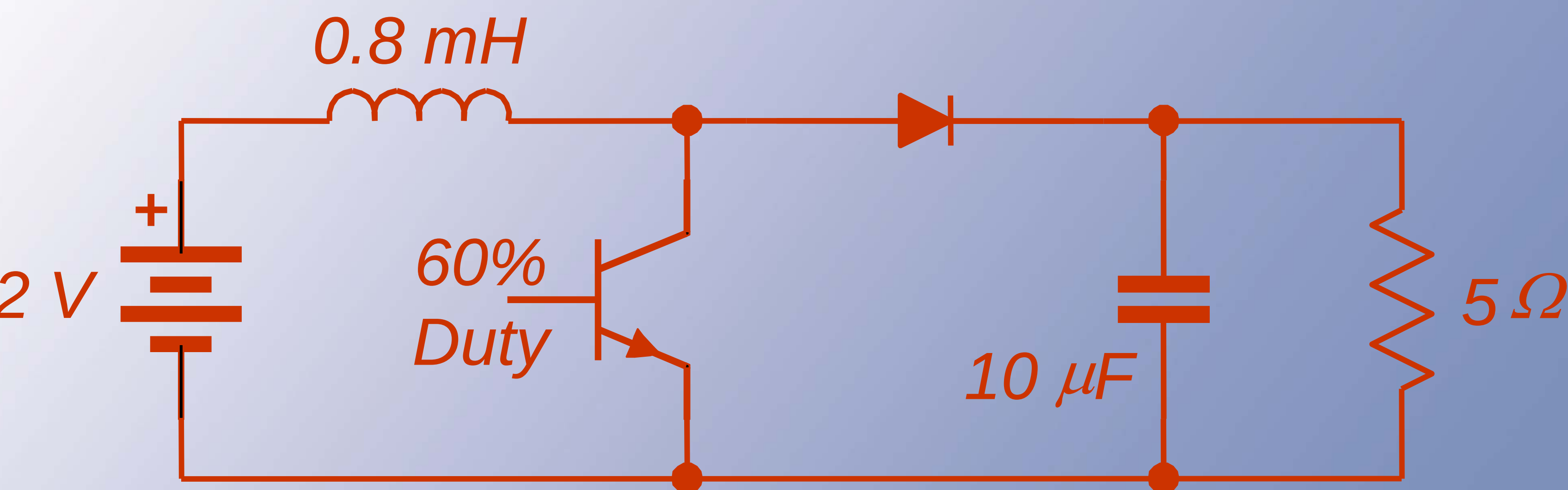
#2 ON: i_C= 1.5 A

Boost Example

- With switch #2 on (duty ratio was found to be 0.4, so time is 5 us),
  $i_C = 1.5$ A
  $= C\, dv/dt$
  $= C\, \Delta v/\Delta t$.
- (1.5 A)(5 us)/C = $\Delta v < 0.1$ V.
- This requires C > 75 uF.

Boost Example

2 to 5 V, 80 kHz boost converter:

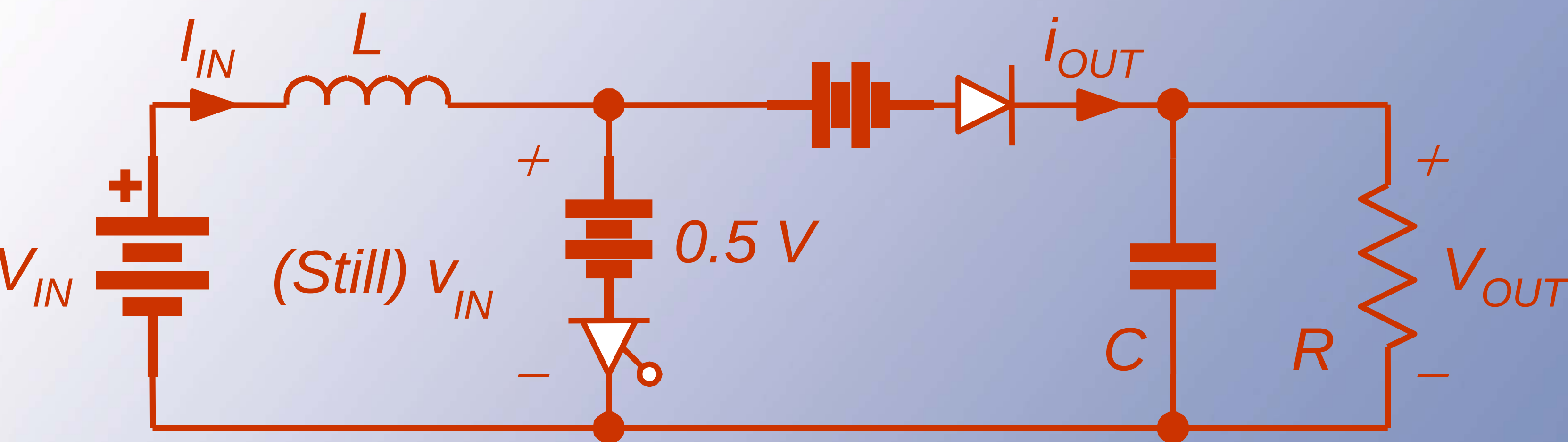


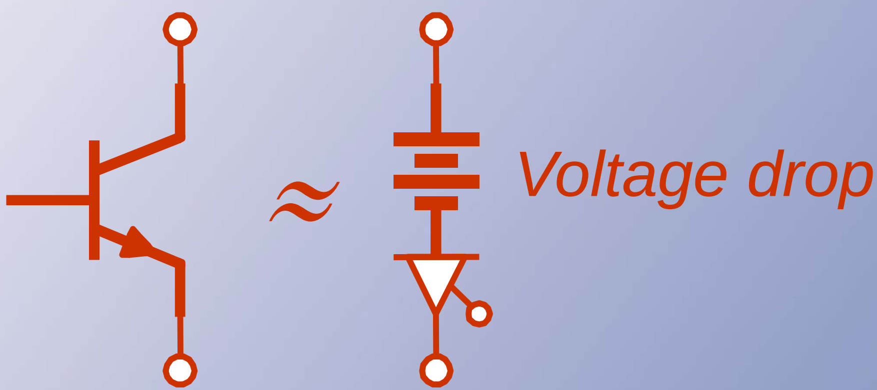
Practice: What if f_s is changed to 40 kHz? → average values are the same, ripple 2x

Comments

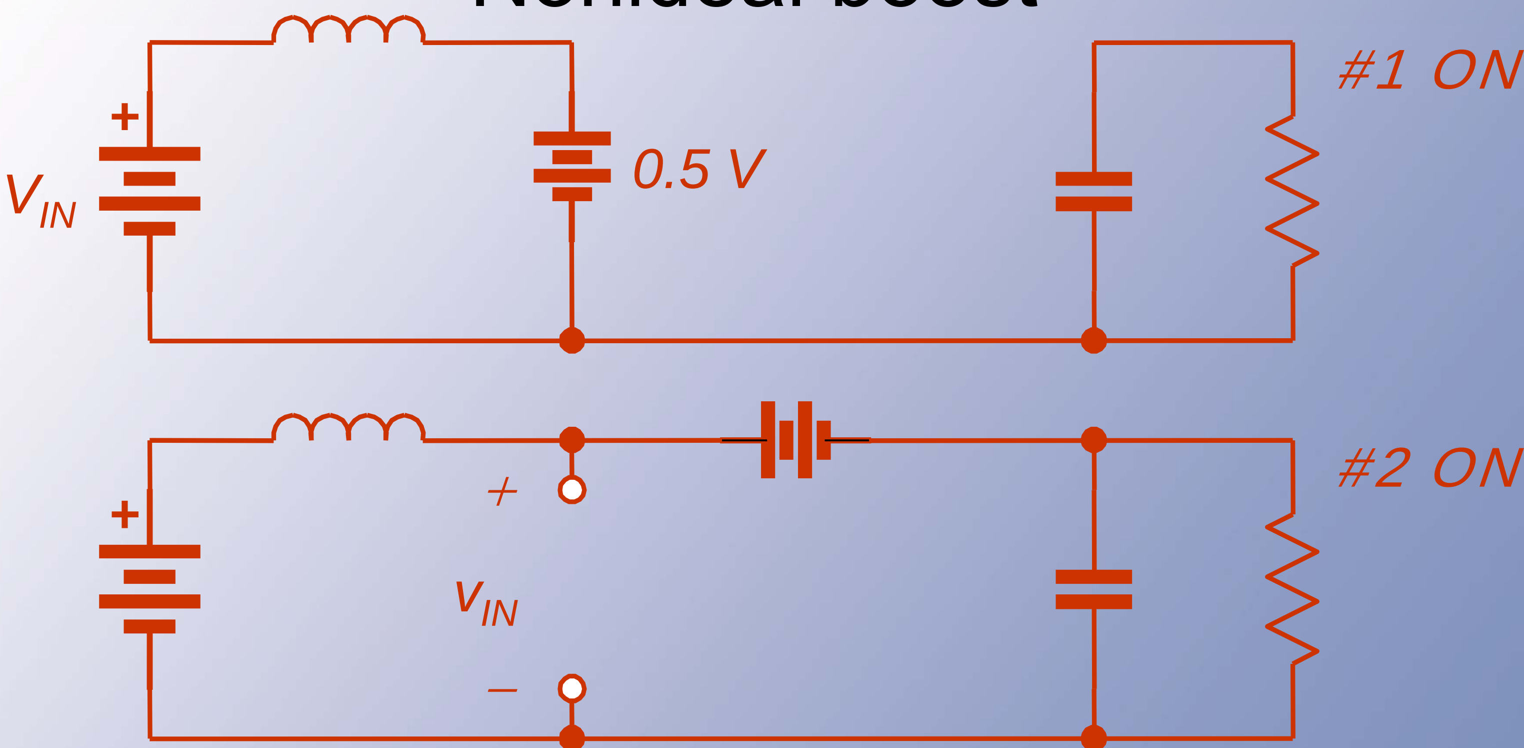
- With a few practice examples, you should be able to design a common-ground buck or boost converter.
- Challenge: Think about effects of nonideal switching.
- It is not so difficult to include some basic nonideal effects, such as switching device voltage drops and resistances.
- Consider an example with switch and diode voltage drop.

Nonideal boost





Nonideal boost



$$v_{in} = q_1(0.5\,V) + q_2(V_{out} + 1)$$

$$V_{in} = \langle v_{in} \rangle = D_1(0.5\,V) + (1 - D_1)(V_{out} + 1)$$

Nonideal boost

- Switching function expressions still apply.
- Boost: $v_{in} = q_1(0.5\text{ V}) + q_2(V_{out} + 1\text{ V})$.
- On average,

$$\langle v_{in} \rangle = V_{in}$$
$$= D_1(0.5\text{V}) + (1 - D_1)(V_{out} + 1\text{ V}), \text{ and}$$
$$V_{out} = (V_{in} + 0.5D_1 - 1)/(1 - D_1)$$

- For current, $i_{out} = q_2 I_L$, $\langle i_{out} \rangle = D_2 I_L$.
- Since $\langle i_{out} \rangle$ is the load current I_{load}, we have $I_L = I_{load}/D_2 = I_{load}/(1 - D_1)$.

Nonideal boost

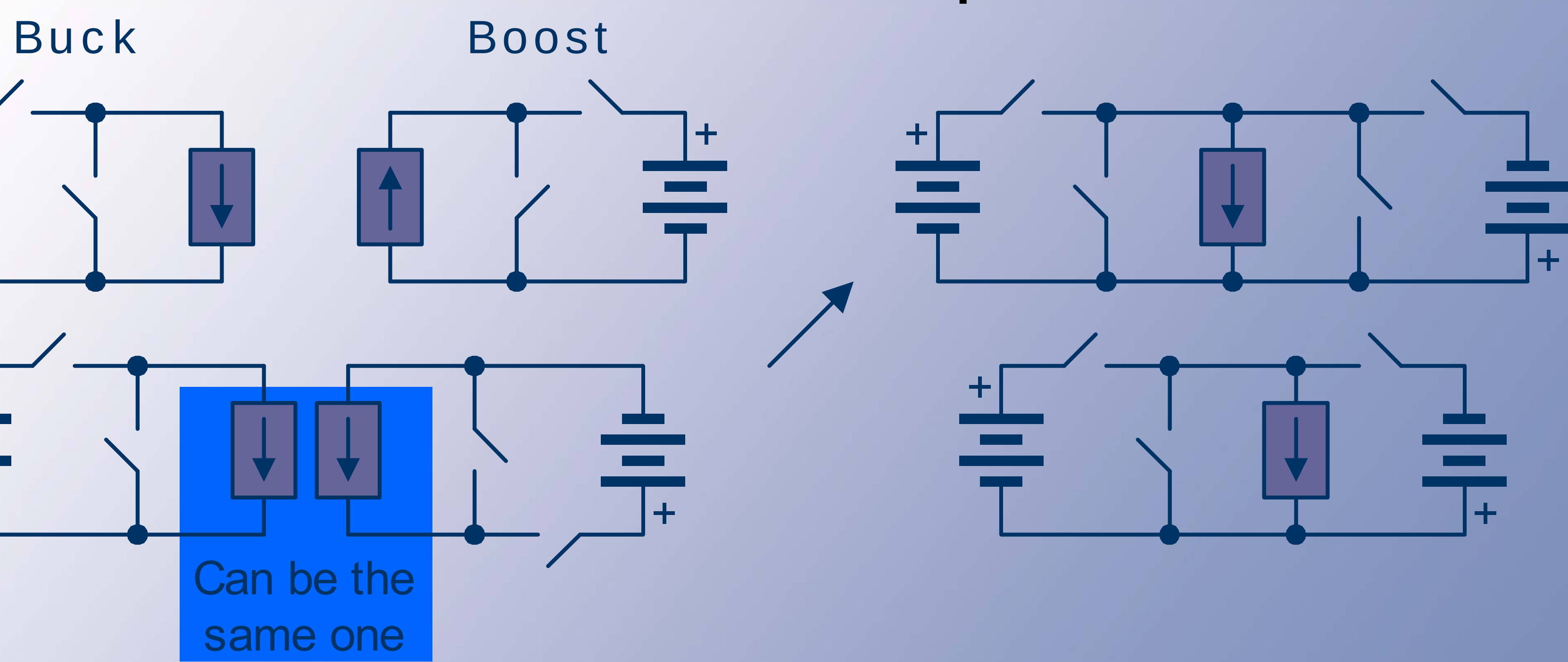
- The efficiency: $P_{in} = V_{in} I_L$, $P_{out} = V_{out} I_{load}$.
- So $P_{in} = V_{in} I_{load}/(1 – D_1)$ and

  $$P_{out} = (V_{in} + 0.5D_1 - 1) I_{load}/(1 - D_1)$$
- The efficiency ratio $\eta = (V_{in} + D_1/2 - 1)/V_{in}$, and $\eta = 1 – (1 – D_1/2)/V_{in}$.
- This is less than 100%, reflecting the losses in the switch forward drops.
- Switching functions support analysis of converters even with these extra parts.

Indirect Dc-Dc Converters

- The buck is a dc transformer with $V_{out} < V_{in}$.
- The boost gives $V_{out} > V_{in}$.
- How can we give full range? *Use a buck as the input for a boost.*
- That is, use the current source output of a buck to provide the input source for a boost.
- Remove redundant or unnecessary switches. Result is the polarity reverser: buck-boost.

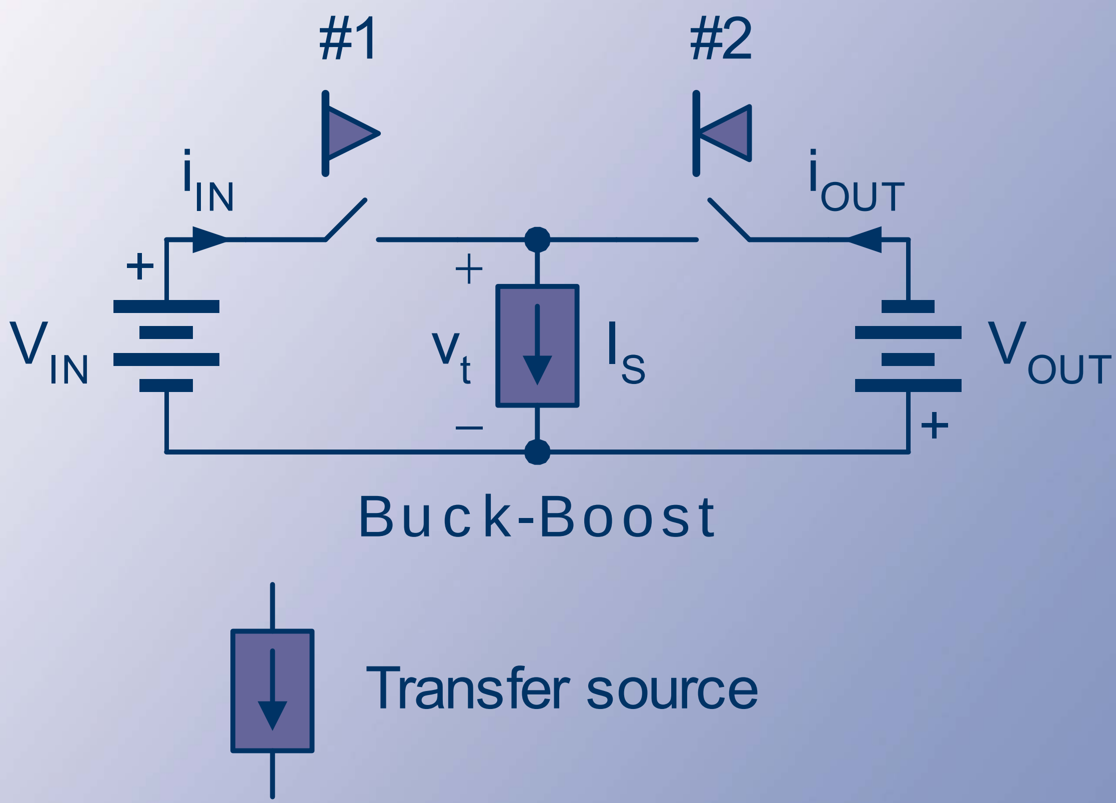
Buck-Boost Development



Final Simplification

- The switch across the current source is not necessary for KCL.
- Try removing it.
- The current source is a *transfer source.*

Buck-Boost Converter



Left switch is FCFB. Right switch is FCRB.

Relationships

- To meet KVL and KCL, $q_1+q_2 = 1$.
- There are really two matrices now. Let us consider the transfer source, which is manipulated by both matrices.
- Transfer voltage is subject to control.
- Transfer voltage $v_t = q_1 V_{in} - q_2 V_{out}$.
- Transfer source power is $v_t\, I_s = q_1\, V_{in}\, I_s - q_2\, V_{out}\, I_s$.
- We want the average power in the transfer source to be zero -- no loss.

Relationships

$$KVL + KCL:$$

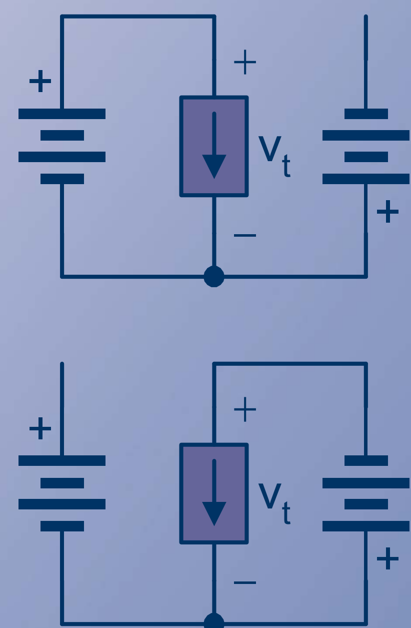
$$q_1 + q_2 = 1$$

$$v_t = q_1 V_{in} - q_2 V_{out}$$

$$v_t I_s = q_1 V_{in} I_s - q_2 V_{out} I_s$$

$$\langle v_t \rangle = D_1 V_{in} - D_2 V_{out}$$

$$\langle v_t I_s \rangle = I_s \langle v_t \rangle = I_s \left(D_1 V_{in} - D_2 V_{out} \right)$$



<v_tI_s> must be zero, not to have losses in the transfer source.

Relationships

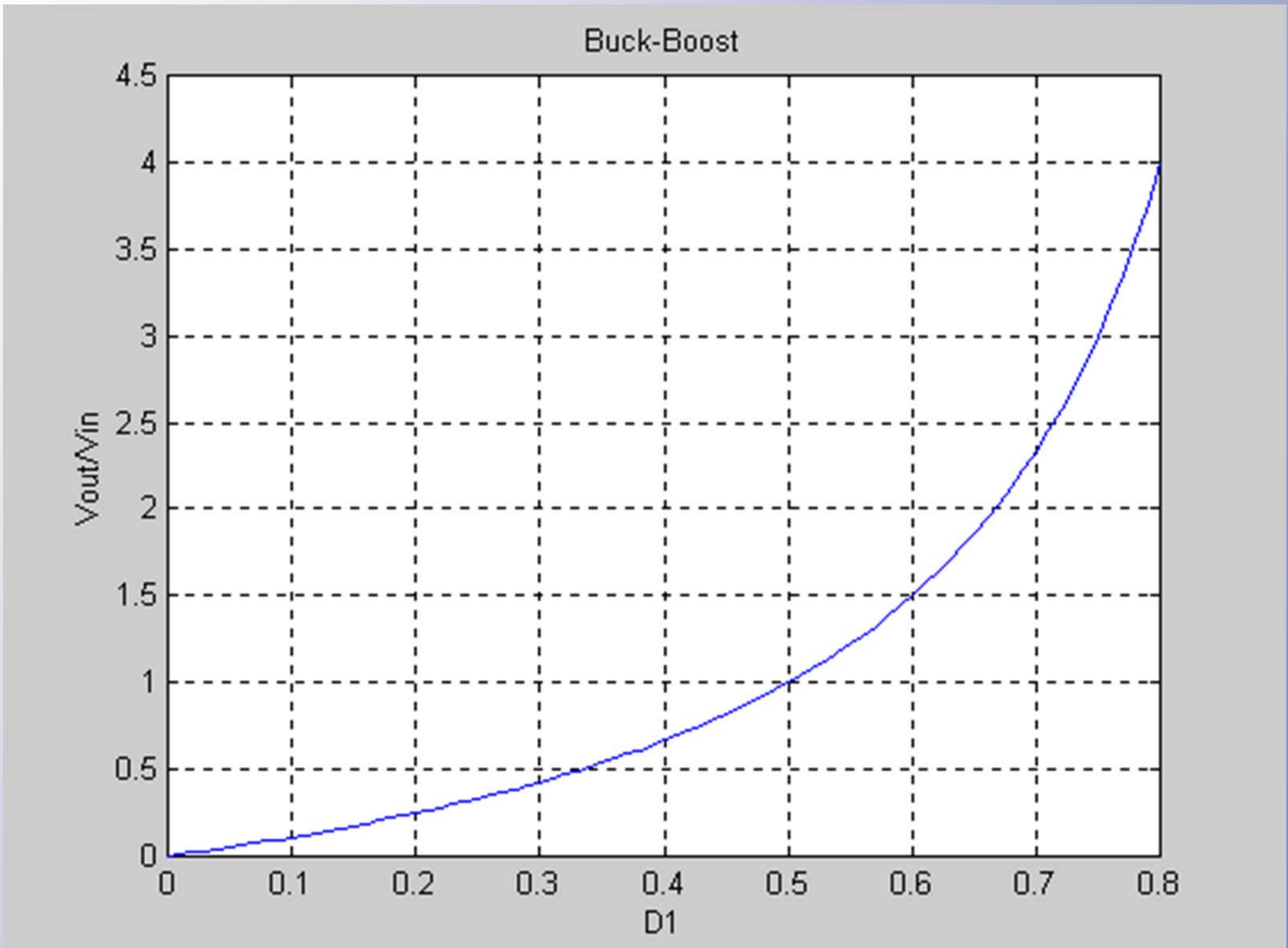
This can be done if $D_1V_{in} = D_2V_{out}$.

Since $D_1 + D_2 = 1$, we have $D_1V_{in} = (1 - D_1)V_{out}$.

This becomes $V_{out} = D_1V_{in}/(1-D_1)$.

The polarity reversal comes from the cascade process.

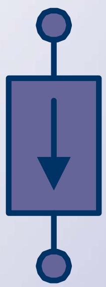
Buck-Boost



Relationships

- The buck-boost allows outputs both higher and lower than the input, but a polarity shift is present.
- The transfer source can be an inductor alone to avoid loss.

Relationships



Consumes no average power.
Maintains fixed I.

Can be approximated by an inductor.

This will be our transfer current source.

What About Currents?

The input current: $i_{in} = q_1 I_s$,
The output current: $i_{out} = q_2 I_s$,
Average input: $I_{in} = D_1 I_s$,
Average output: $I_{out} = D_2 I_s$.
We do not really know I_s. Add the above:
$I_{in} + I_{out} = (D_1 + D_2)I_s = I_s$.

Currents and Stresses

- The transfer source sees a current equal to the sum of input and output average currents.
- Each switch must carry I_s, and each must block $V_{in} + V_{out}$.
- All device ratings are higher than either the input or output needs.