Why study games?

• Games are a traditional hallmark of intelligence
• Games are easy to formalize
• Games can be a good model of real-world competitive or cooperative activities
  • Military confrontations, negotiation, auctions, etc.
Game AI: Origins

• Minimax algorithm: Ernst Zermelo, 1912
• Chess playing with evaluation function, quiescence search, selective search: Claude Shannon, 1949 (paper)
• Alpha-beta search: John McCarthy, 1956
• Checkers program that learns its own evaluation function by playing against itself: Arthur Samuel, 1956
### Types of game environments

<table>
<thead>
<tr>
<th></th>
<th>Deterministic</th>
<th>Stochastic</th>
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<tbody>
<tr>
<td>Perfect information</td>
<td>Chess, checkers, go</td>
<td>Backgammon, monopoly</td>
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<tr>
<td>(fully observable)</td>
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<tr>
<td>Imperfect information</td>
<td>Battleship</td>
<td>Scrabble, poker, bridge</td>
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<td>(partially observable)</td>
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Zero-sum Games
Alternating two-player zero-sum games

• Players take turns
• Each game outcome or terminal state has a utility for each player (e.g., 1 for win, 0 for loss)
• The sum of both players’ utilities is a constant
Games vs. single-agent search

- We don’t know how the opponent will act
  - The solution is not a fixed sequence of actions from start state to goal state, but a \textit{strategy} or \textit{policy} (a mapping from state to best move in that state)
Game tree

- A game of tic-tac-toe between two players, “max” and “min”
COMPLETE MAP OF OPTIMAL TIC-TAC-TOE MOVES

YOUR MOVE IS GIVEN BY THE POSITION OF THE LARGEST RED SYMBOL ON THE GRID. WHEN YOUR OPPONENT PICKS A MOVE, ZOOM IN ON THE REGION OF THE GRID WHERE THEY WENT. REPEAT.

MAP FOR X:

http://xkcd.com/832/
A more abstract game tree

MAX

MIN

Terminal utilities (for MAX)

A two- ply game
Minimax Search
The rules of every game

• Every possible outcome has a value (or “utility”) for me.
• Zero-sum game: if the value to me is $+V$, then the value to my opponent is $-V$.
• Phrased another way:
  • My rational action, on each move, is to choose a move that will maximize the value of the outcome
  • My opponent’s rational action is to choose a move that will minimize the value of the outcome
• Call me “Max”
• Call my opponent “Min”
Game tree search

- Minimax value of a node: the utility (for MAX) of being in the corresponding state, assuming perfect play on both sides
- Minimax strategy: Choose the move that gives the best worst-case payoff
Computing the minimax value of a node

\[ \text{Minimax}(\text{node}) = \]
- Utility(\text{node}) if \text{node} is terminal
- \( \max_{\text{action}} \text{Minimax}(\text{Succ(}\text{node, action})), \) if player = MAX
- \( \min_{\text{action}} \text{Minimax}(\text{Succ(}\text{node, action})), \) if player = MIN
Optimality of minimax

- The minimax strategy is optimal against an optimal opponent

- What if your opponent is suboptimal?
  - Your utility will ALWAYS BE HIGHER than if you were playing an optimal opponent!
  - A different strategy may work better for a sub-optimal opponent, but it will necessarily be worse against an optimal opponent

Example from D. Klein and P. Abbeel
More general games

- More than two players, non-zero-sum
- Utilities are now tuples
- Each player maximizes their own utility at their node
- Utilities get propagated (backed up) from children to parents
Alpha-Beta Pruning
Alpha-beta pruning

• It is possible to compute the exact minimax decision without expanding every node in the game tree
Alpha-beta pruning

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Alpha-beta pruning

• It is possible to compute the exact minimax decision without expanding every node in the game tree

```
MAX

MIN

\[ \geq 3 \]
```

3
12
8
2

\[ \leq 2 \]
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree

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MAX
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Alpha-beta pruning

• It is possible to compute the exact minimax decision without expanding every node in the game tree
Alpha-beta pruning

• It is possible to compute the exact minimax decision without expanding every node in the game tree
Key point that I find most counter-intuitive:

• MIN needs to calculate which move MAX will make.
• MAX would never choose a suboptimal move.
• So if MIN discovers that, at a particular node in the tree, she can make a move that’s REALLY REALLY GOOD for her...
• She can assume that MAX will never let her reach that node.
• ... and she can prune it away from the search, and never consider it again.
Alpha-beta pruning

• \( \alpha \) is the value of the best choice for the MAX player found so far at any choice point above node \( n \)

• More precisely: \( \alpha \) is the highest number that MAX knows how to force MIN to accept

• We want to compute the MIN-value at \( n \)

• As we loop over \( n \)’s children, the MIN-value decreases

• If it drops below \( \alpha \), MAX will never choose \( n \), so we can ignore \( n \)’s remaining children
Alpha-beta pruning

- $\beta$ is the value of the best choice for the MIN player found so far at any choice point above node $n$.
- More precisely: $\beta$ is the lowest number that MIN know how to force MAX to accept.
- We want to compute the MAX-value at $m$.
- As we loop over $m$’s children, the MAX-value increases.
- If it rises above $\beta$, MIN will never choose $m$, so we can ignore $m$’s remaining children.
**Alpha-beta pruning**

**An unexpected result:**

- $\alpha$ is the highest number that MAX knows how to force MIN to accept
- $\beta$ is the lowest number that MIN knows how to force MAX to accept

So

$$\alpha \leq \beta$$
Alpha-beta pruning

Function \( \text{action} = \text{Alpha-Beta-Search}(node) \)

\[
\begin{align*}
v &= \text{Min-Value}(node, -\infty, \infty) \\
&= \text{Min-Value}(node, \alpha, \beta) \\
\end{align*}
\]

return the action from node with value \( v \)

\( \alpha: \) best alternative available to the Max player

\( \beta: \) best alternative available to the Min player

Function \( v = \text{Min-Value}(node, \alpha, \beta) \)

if Terminal\((node)\) return Utility\((node)\)

\[
\begin{align*}
v &= +\infty \\
\text{for each} & \text{ action from node} \\
v &= \text{Min}(v, \text{Max-Value}(\text{Succ}(node, action), \alpha, \beta)) \\
\end{align*}
\]

if \( v \leq \alpha \) return \( v \)

\( \beta = \text{Min}(\beta, v) \)

end for

return \( v \)
**Alpha-beta pruning**

**Function** \( \text{action} = \text{Alpha-Beta-Search}(\text{node}) \)

\[
\begin{align*}
\alpha & : \text{best alternative available to the Max player} \\
\beta & : \text{best alternative available to the Min player}
\end{align*}
\]

\[
\begin{align*}
\text{Function } v &= \text{Max-Value}(\text{node}, \alpha, \beta) \\
\text{if } \text{Terminal}(\text{node}) & \text{ return } \text{Utility}(\text{node}) \\
v &= -\infty \\
\text{for each } \text{action from node} \\
v &= \text{Max}(v, \text{Min-Value}(\text{Succ}(\text{node}, \text{action}), \alpha, \beta)) \\
\text{if } v & \geq \beta \text{ return } v \\
\alpha &= \text{Max}(\alpha, v) \\
\text{end for} \\
\text{return } v
\end{align*}
\]
Alpha-beta pruning

• Pruning does not affect final result

• Amount of pruning depends on move ordering
  • Should start with the “best” moves (highest-value for MAX or lowest-value for MIN)
  • For chess, can try captures first, then threats, then forward moves, then backward moves
  • Can also try to remember “killer moves” from other branches of the tree

• With perfect ordering, the time to find the best move is reduced to $O(b^{m/2})$ from $O(b^m)$
  • Depth of search is effectively doubled
Limited-Horizon Computation
Games vs. single-agent search

• We don’t know how the opponent will act
  • The solution is not a fixed sequence of actions from start state to goal state, but a strategy or policy (a mapping from state to best move in that state)
Games vs. single-agent search

• We don’t know how the opponent will act
  • The solution is not a fixed sequence of actions from start state to goal state, but a **strategy** or **policy** (a mapping from state to best move in that state)

• Efficiency is critical to playing well
  • The time to make a move is limited
  • The branching factor, search depth, and number of terminal configurations are huge
    • In chess, branching factor ≈ 35 and depth ≈ 100, giving a search tree of $10^{154}$ nodes
      • Number of atoms in the observable universe ≈ $10^{80}$
  • This rules out searching all the way to the end of the game
Evaluation function

• Cut off search at a certain depth and compute the value of an evaluation function for a state instead of its minimax value
  • The evaluation function may be thought of as the probability of winning from a given state or the expected value of that state

• A common evaluation function is a weighted sum of features:

\[
\text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)
\]

• For chess, \(w_k\) may be the material value of a piece (pawn = 1, knight = 3, rook = 5, queen = 9) and \(f_k(s)\) may be the advantage in terms of that piece

• Evaluation functions may be learned from game databases or by having the program play many games against itself
Cutting off search

• **Horizon effect:** you may incorrectly estimate the value of a state by overlooking an event that is just beyond the depth limit
  • For example, a damaging move by the opponent that can be delayed but not avoided

• Possible remedies
  • **Quiescence search:** do not cut off search at positions that are unstable – for example, are you about to lose an important piece?
  • **Singular extension:** a strong move that should be tried when the normal depth limit is reached
Advanced techniques

- **Transposition table** to store previously expanded states
- **Forward pruning** to avoid considering all possible moves
- **Lookup tables** for opening moves and endgames
Chess playing systems

- Baseline system: 200 million node evaluations per move (3 min), minimax with a decent evaluation function and quiescence search
  - 5-ply ≈ human novice
- Add alpha-beta pruning
  - 10-ply ≈ typical PC, experienced player
- Deep Blue: 30 billion evaluations per move, singular extensions, evaluation function with 8000 features, large databases of opening and endgame moves
  - 14-ply ≈ Garry Kasparov
- More recent state of the art (Hydra, ca. 2006): 36 billion evaluations per second, advanced pruning techniques
  - 18-ply ≈ better than any human alive?
Summary

• A zero-sum game can be expressed as a minimax tree
• Alpha-beta pruning finds the correct solution. In the best case, it has half the exponent of minimax (can search twice as deeply with a given computational complexity).
• Limited-horizon search is always necessary (you can’t search to the end of the game), and always suboptimal.
  • Estimate your utility, at the end of your horizon, using some type of learned utility function
  • Quiescence search: don’t cut off the search in an unstable position (need some way to measure “stability”)
  • Singular extension: have one or two “super-moves” that you can test at the end of your horizon