CS440/ECE448 Lecture 9: Minimax Search

Slides by Svetlana Lazebnik 9/2016

Modified by Mark Hasegawa-Johnson 9/2017

Why study games?

- Games are a traditional hallmark of intelligence
- Games are easy to formalize
- Games can be a good model of real-world competitive or cooperative activities
  - Military confrontations, negotiation, auctions, etc.

Game AI: Origins

- Minimax algorithm: Ernst Zermelo, 1912
- Chess playing with evaluation function, quiescence search, selective search:
  Claude Shannon, 1949 (paper)
- Alpha-beta search: John McCarthy, 1956
- Checkers program that learns its own evaluation function by playing against itself: Arthur Samuel, 1956

Types of game environments

	Deterministic	**Stochastic**
Perfect information (fully observable)	Chess, checkers, go	Backgammon, monopoly
Imperfect information (partially observable)	Battleship	Scrabble, poker, bridge

Zero-sum Games

Alternating two-player zero-sum games

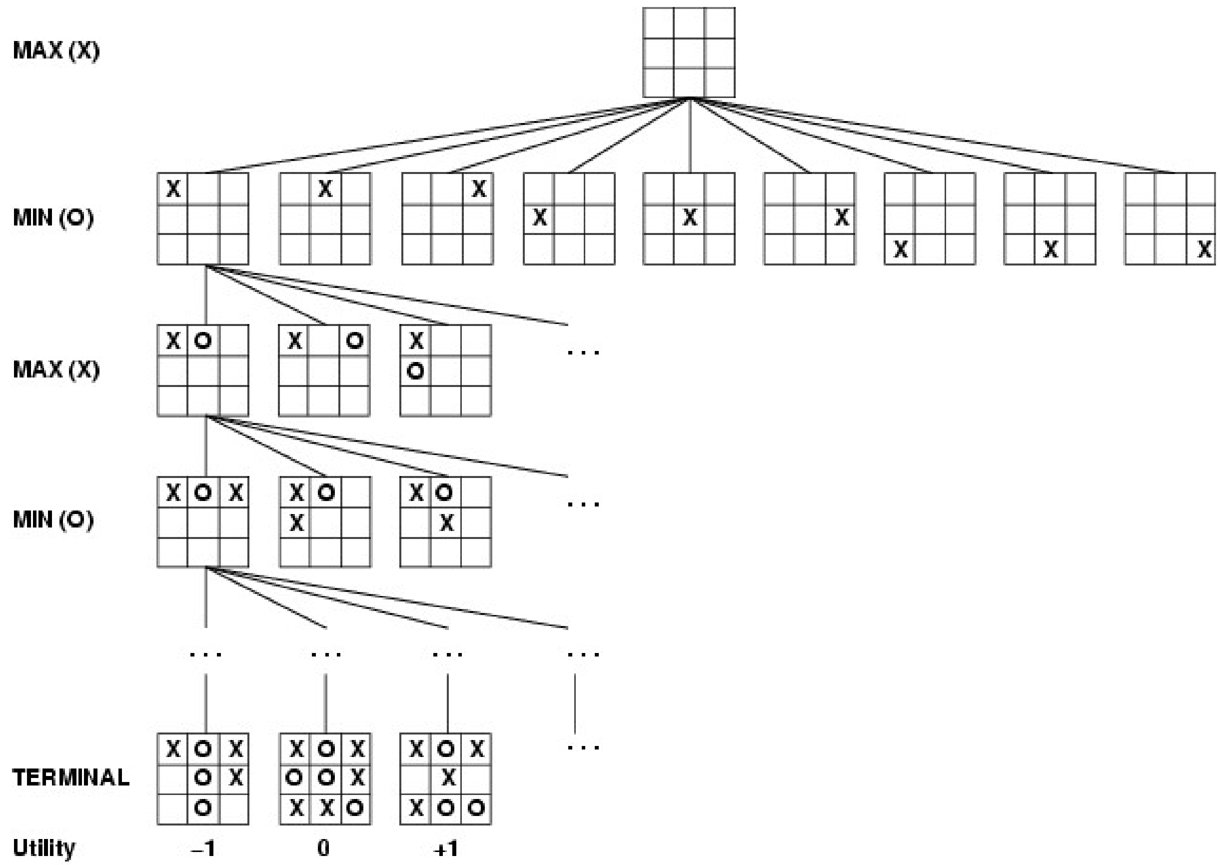
- Players take turns
- Each game outcome or **terminal state** has a **utility** for each player (e.g., 1 for win, 0 for loss)
- The sum of both players' utilities is a constant

Games vs. single-agent search

- We don’t know how the opponent will act
  - The solution is not a fixed sequence of actions from start state to goal state, but a ***strategy*** or ***policy*** (a mapping from state to best move in that state)

Game tree

- A game of tic-tac-toe between two players, “max” and “min”

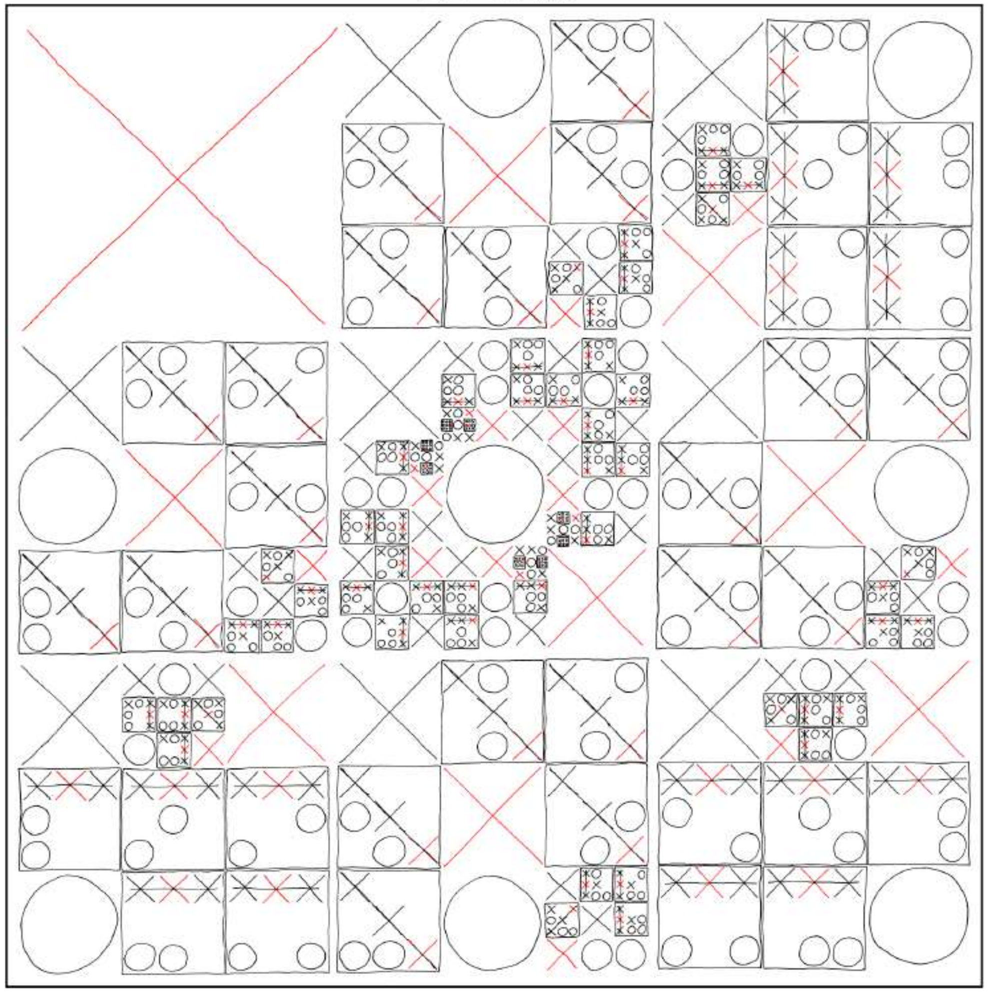


COMPLETE MAP OF OPTIMAL TIC-TAC-TOE MOVES

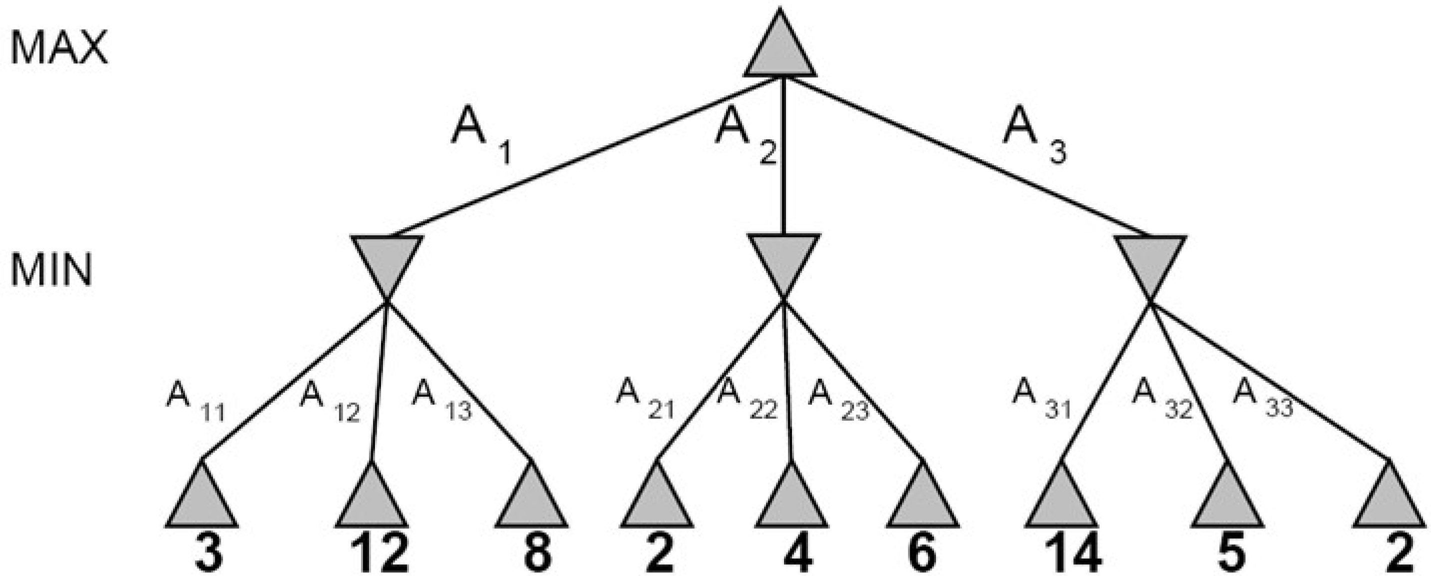
YOUR MOVE IS GIVEN BY THE POSITION OF THE LARGEST RED SYMBOL ON THE GRID. WHEN YOUR OPPONENT PICKS A MOVE, ZOOM IN ON THE REGION OF THE GRID WHERE THEY WENT. REPEAT.

http://xkcd.com/832/

MAP FOR X:



A more abstract game tree



Terminal utilities (for MAX)

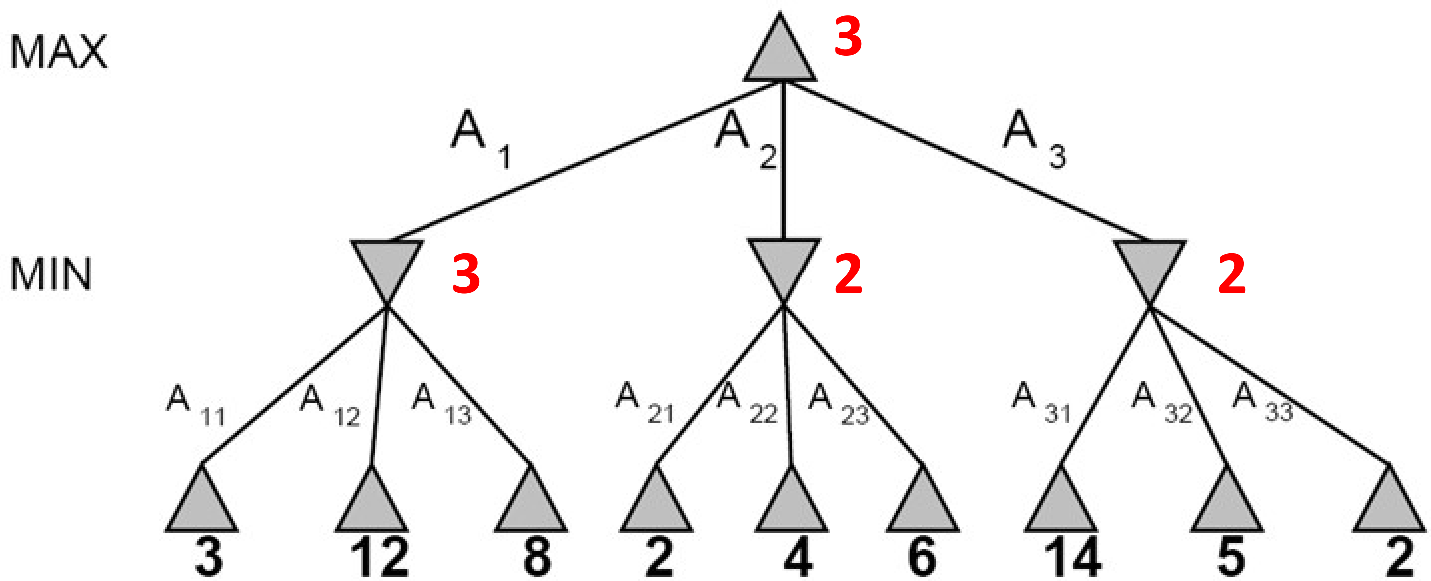
A *two-ply* game

Minimax Search

The rules of every game

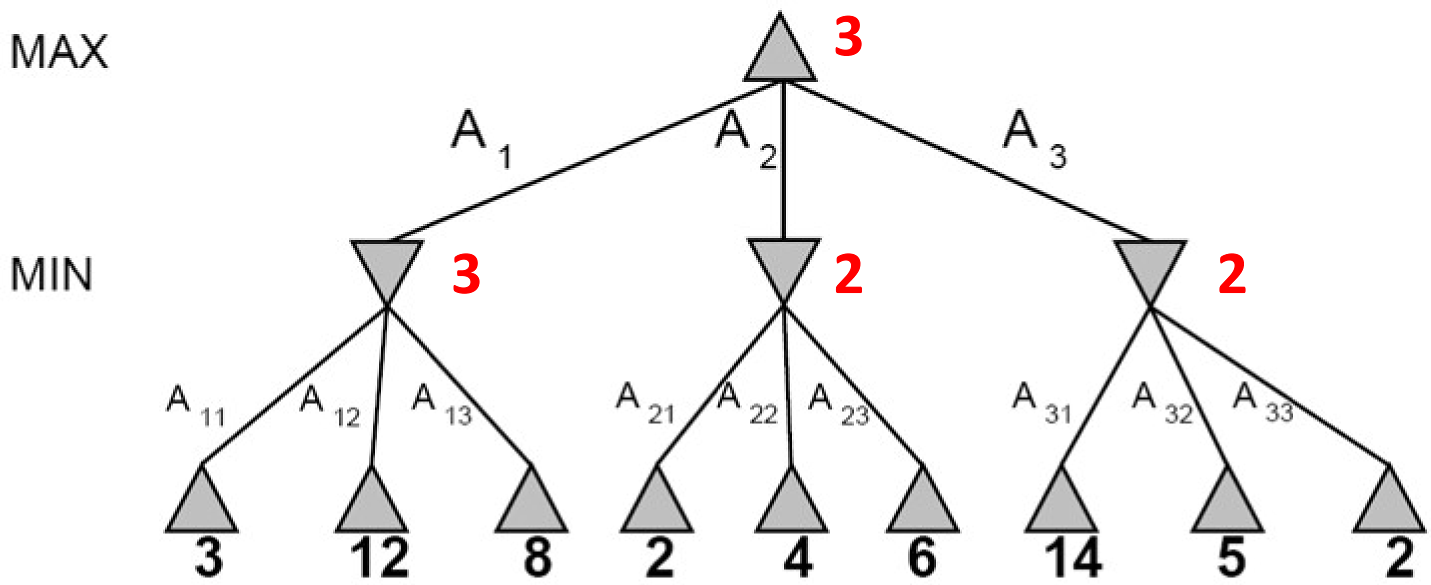
- Every possible outcome has a value (or “utility”) for me.
- Zero-sum game: if the value to me is +V, then the value to my opponent is –V.
- Phrased another way:
  - My rational action, on each move, is to choose a move that will maximize the value of the outcome
  - My opponent’s rational action is to choose a move that will minimize the value of the outcome
- Call me “Max”
- Call my opponent “Min”

Game tree search



- **Minimax value of a node**: the utility (for MAX) of being in the corresponding state, assuming perfect play on both sides
- **Minimax strategy:** Choose the move that gives the best worst-case payoff

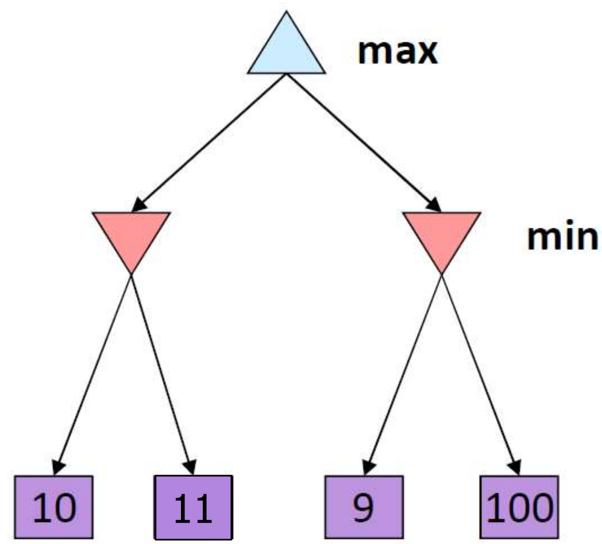
Computing the minimax value of a node



- **Minimax**(*node*) =
  - Utility(*node*) if *node* is terminal
  - $\max_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MAX
  - $\min_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MIN

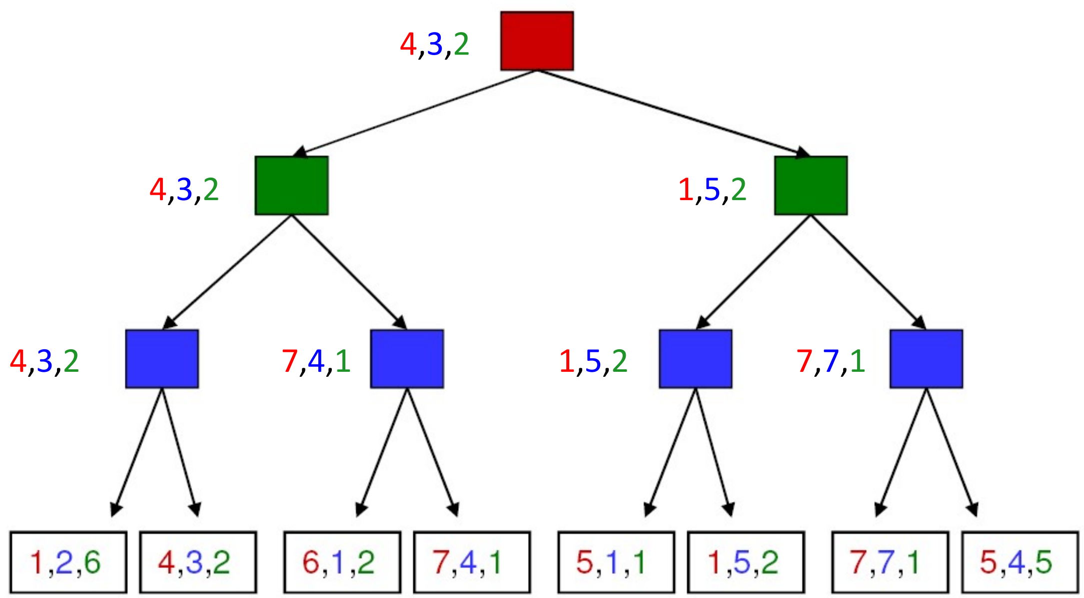
Optimality of minimax

- The minimax strategy is optimal against an optimal opponent
- What if your opponent is suboptimal?
  - Your utility will ALWAYS BE HIGHER than if you were playing an optimal opponent!
  - A different strategy may work better for a sub-optimal opponent, but it will necessarily be worse against an optimal opponent



Example from D. Klein and P. Abbeel

More general games

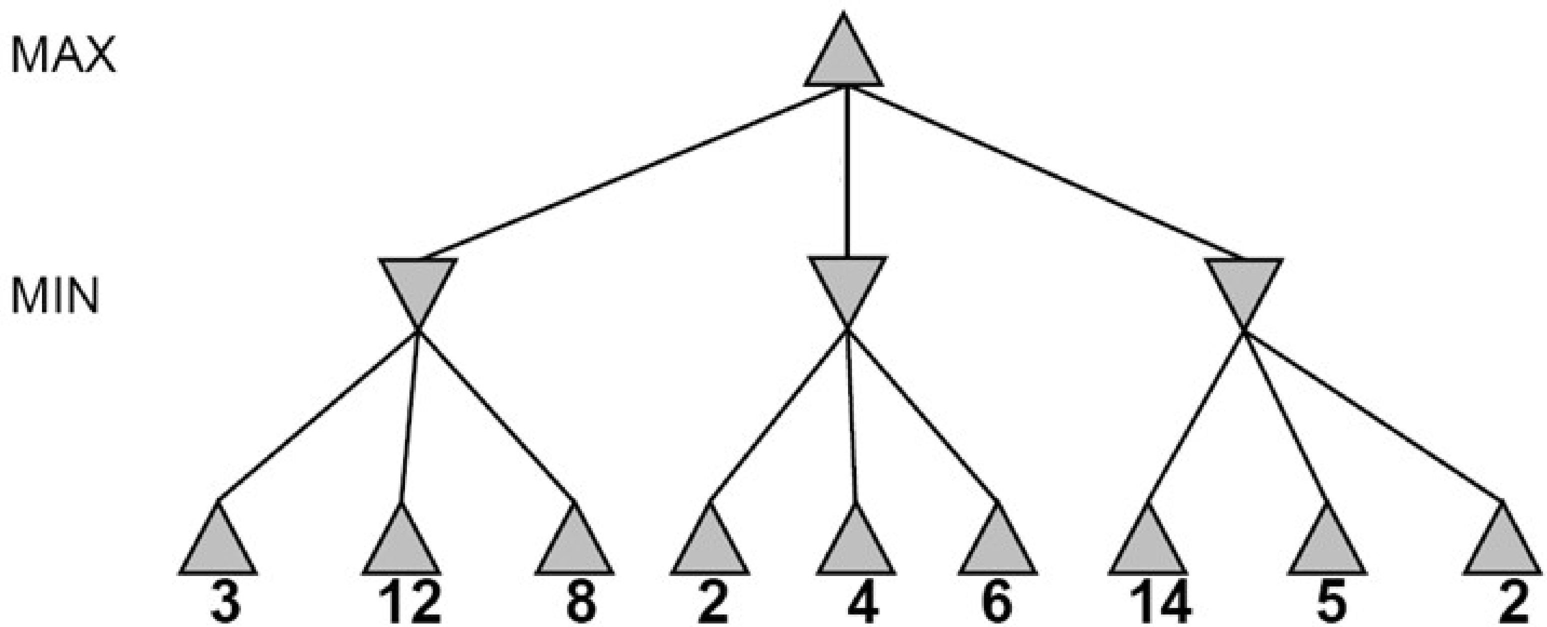


- More than two players, non-zero-sum
- Utilities are now tuples
- Each player maximizes their own utility at their node
- Utilities get propagated (*backed up*) from children to parents

Alpha-Beta Pruning

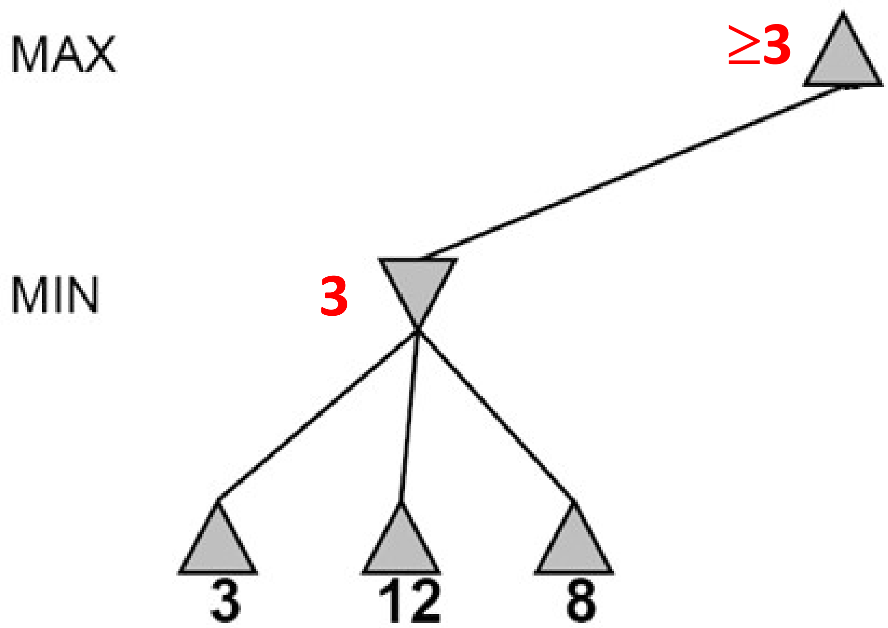
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



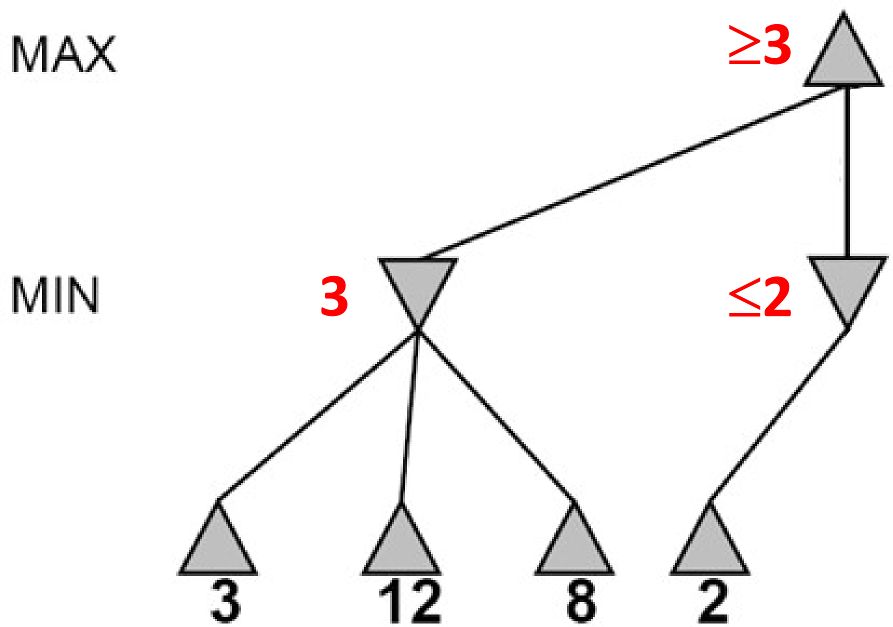
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



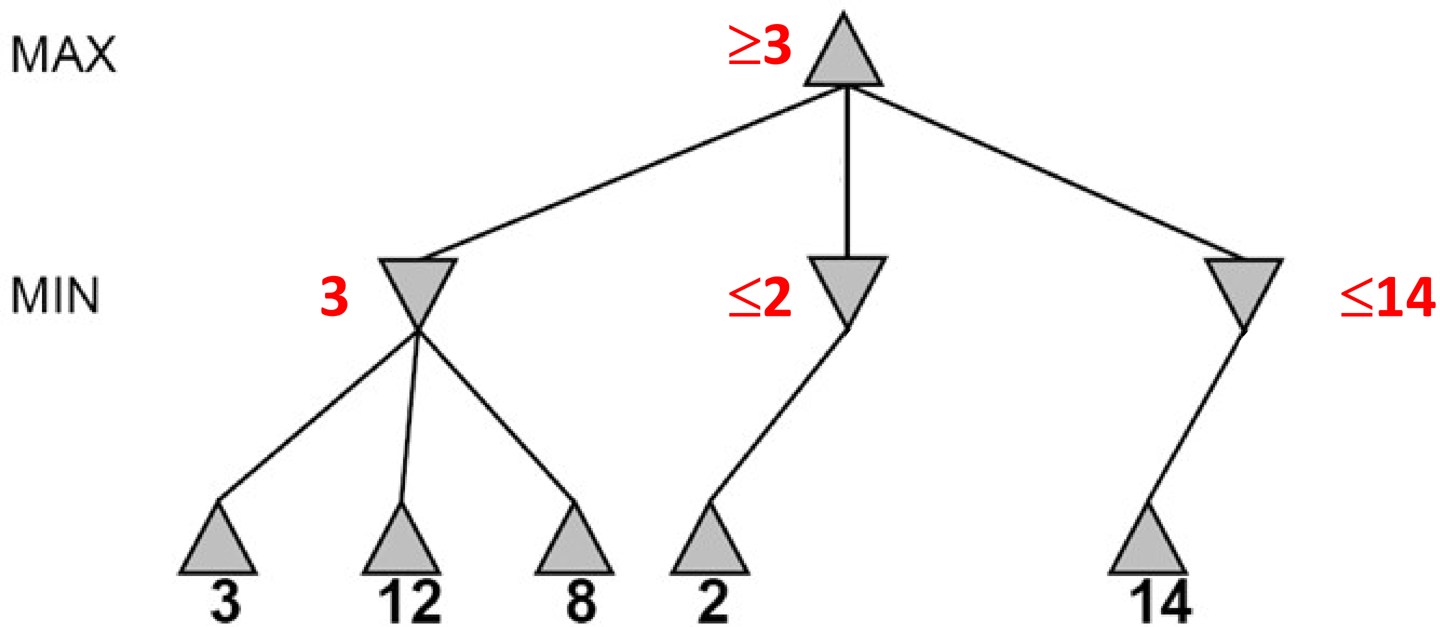
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



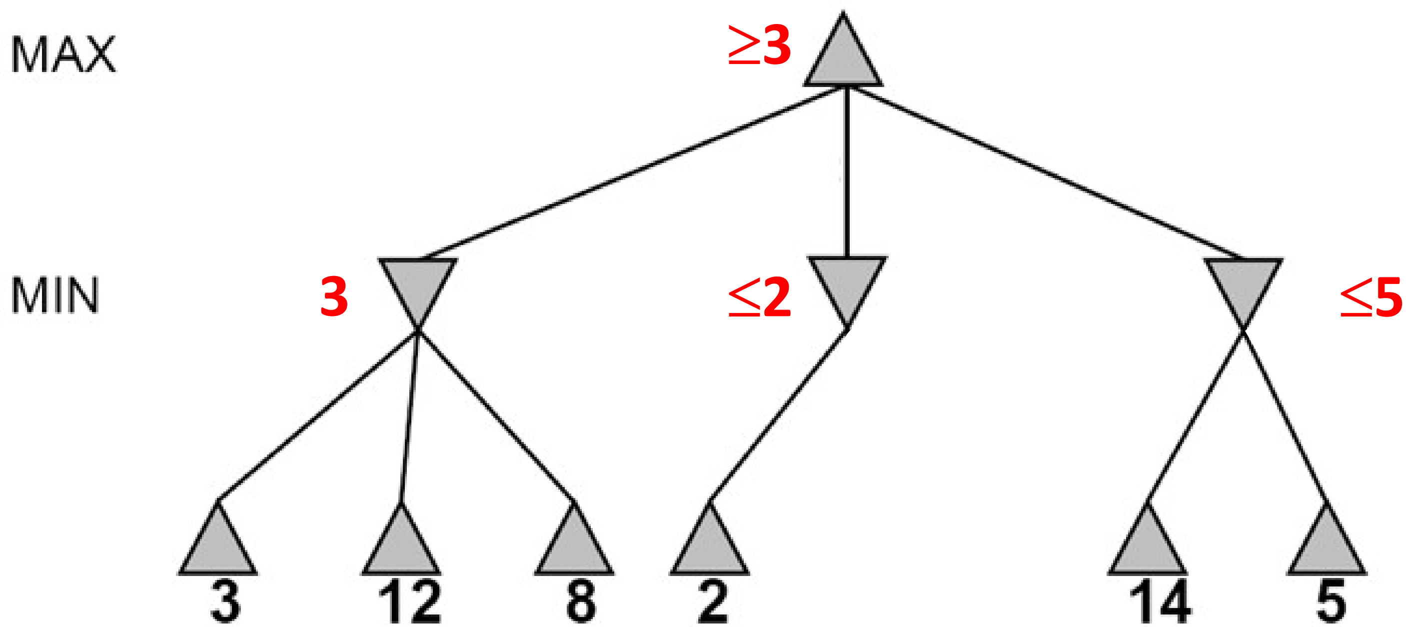
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



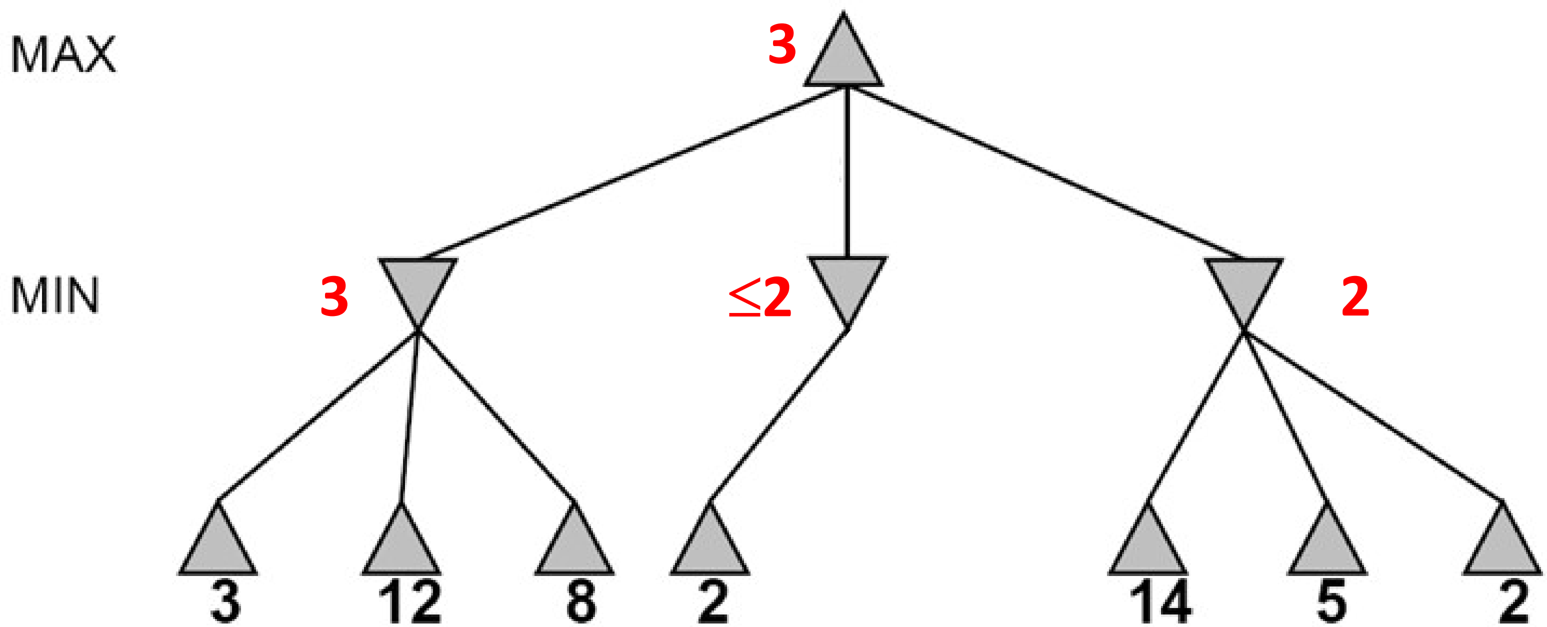
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



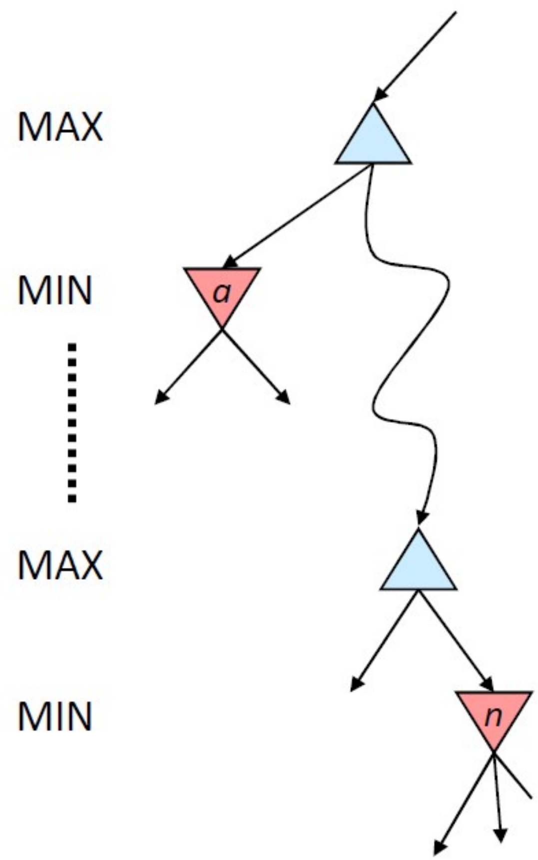
Alpha-Beta Pruning

Key point that I find most counter-intuitive:

- MIN needs to calculate which move MAX will make.
- MAX would never choose a suboptimal move.
- So if MIN discovers that, at a particular node in the tree, she can make a move that's REALLY REALLY GOOD for her…
- She can assume that MAX will never let her reach that node.
- … and she can prune it away from the search, and never consider it again.

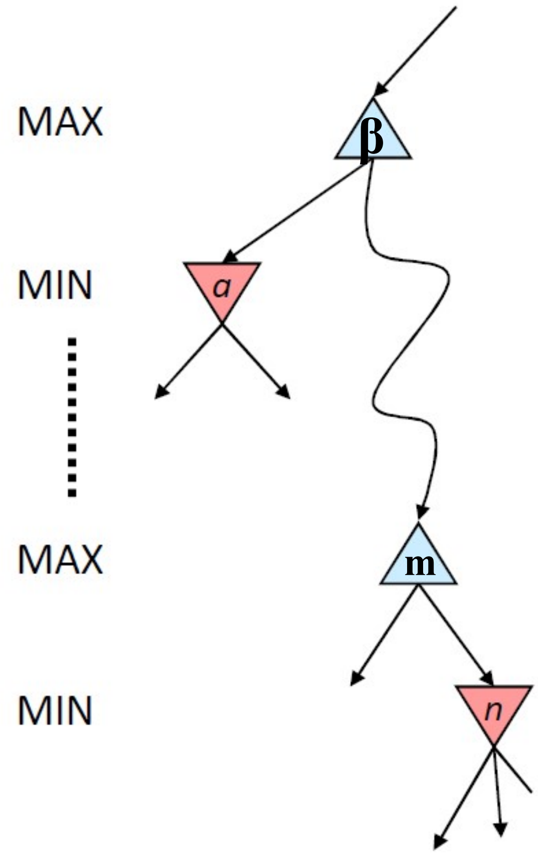
Alpha-beta pruning

- α is the value of the best choice for the MAX player found so far at any choice point above node *n*
- More precisely: α is the highest number that MAX knows how to force MIN to accept
- We want to compute the MIN-value at *n*
- As we loop over *n*'s children, the MIN-value decreases
- If it drops below α, MAX will never choose *n*, so we can ignore *n*'s remaining children



Alpha-beta pruning

- **β** is the value of the best choice for the **MIN** player found so far at any choice point above node *n*
- More precisely: **β** is the lowest number that **MIN** know how to force **MAX** to accept
- We want to compute the MAX-value at *m*
- As we loop over *m*'s children, the MAX-value increases
- If it rises above **β**, MIN will never choose *m*, so we can ignore *m*'s remaining children



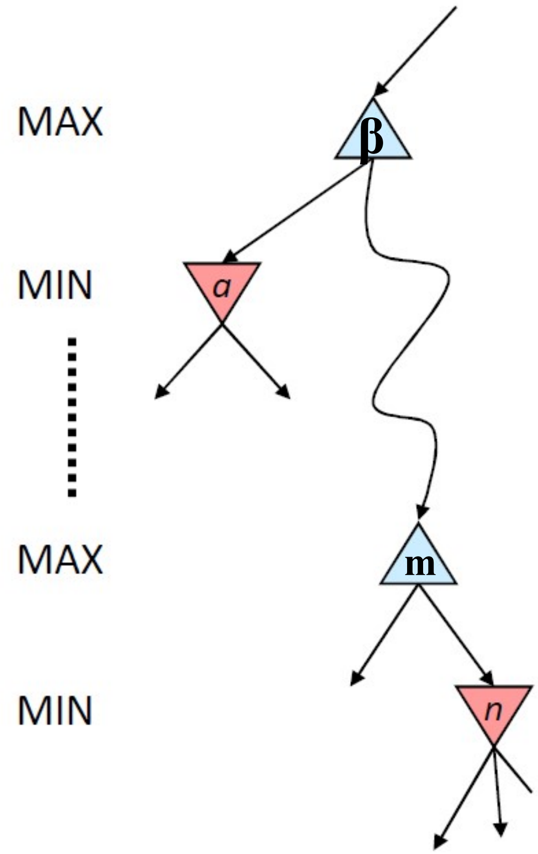
Alpha-beta pruning

An unexpected result:

- **α** is the highest number that MAX knows how to force MIN to accept
- **β** is the lowest number that **MIN** know how to force **MAX** to accept

So

$$\alpha \leq \beta$$



Alpha-beta pruning

Function *action* = **Alpha-Beta-Search**(*node*)

$\quad v$ = **Min-Value**(*node*, $-\infty$, ∞)

$\quad$ return the *action* from *node* with value *v*

α: best alternative available to the Max player

β: best alternative available to the Min player

Function *v* = **Min-Value**(*node*, α, β)

$\quad$ if Terminal(*node*) return Utility(*node*)

$\quad v = +\infty$

$\quad$ for each *action* from *node*

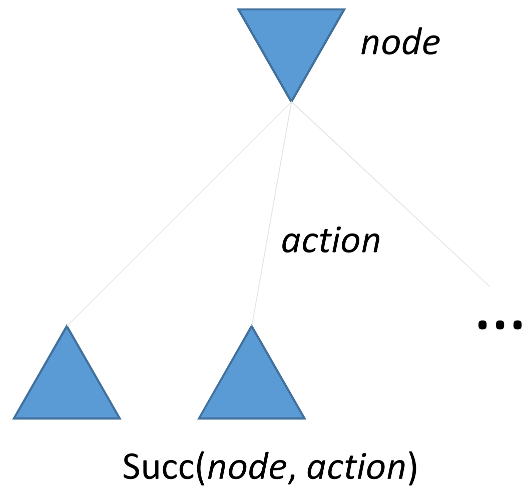
$\quad\quad v$ = Min(*v*, **Max-Value**(Succ(*node*, *action*), α, β))

$\quad\quad$ if $v \le \alpha$ return *v*

$\quad\quad \beta$ = Min(β, *v*)

$\quad$ end for

$\quad$ return *v*



Alpha-beta pruning

Function *action* = **Alpha-Beta-Search**(*node*)

  v = **Max-Value**(*node*, $-\infty$, ∞)

  return the *action* from *node* with value v

α: best alternative available to the Max player

β: best alternative available to the Min player

Function v = **Max-Value**(*node*, α, β)

  if Terminal(*node*) return Utility(*node*)

  $v = -\infty$

  for each *action* from *node*

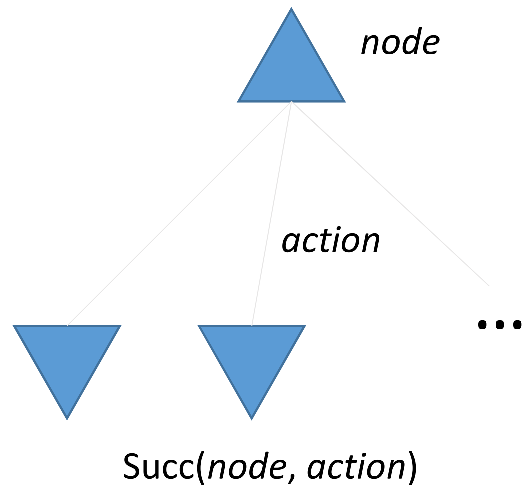
    v = Max(v, **Min-Value**(Succ(*node*, *action*), α, β))

    if $v \geq \beta$ return v

    α = Max(α, v)

  end for

  return v



Alpha-beta pruning

- Pruning does not affect final result
- Amount of pruning depends on move ordering
  - Should start with the "best" moves (highest-value for MAX or lowest-value for MIN)
  - For chess, can try captures first, then threats, then forward moves, then backward moves
  - Can also try to remember "killer moves" from other branches of the tree
- With perfect ordering, the time to find the best move is reduced to $O(b^{m/2})$ from $O(b^m)$
  - Depth of search is effectively doubled

Limited-Horizon Computation

Games vs. single-agent search

- We don't know how the opponent will act
  - The solution is not a fixed sequence of actions from start state to goal state, but a ***strategy*** or ***policy*** (a mapping from state to best move in that state)

Games vs. single-agent search

- We don't know how the opponent will act
  - The solution is not a fixed sequence of actions from start state to goal state, but a ***strategy*** or ***policy*** (a mapping from state to best move in that state)
- Efficiency is critical to playing well
  - The time to make a move is limited
  - The branching factor, search depth, and number of terminal configurations are huge
    - In chess, branching factor ≈ 35 and depth ≈ 100, giving a search tree of 10^{154} nodes
      - Number of atoms in the observable universe ≈ 10^{80}
  - This rules out searching all the way to the end of the game

Evaluation function

- Cut off search at a certain depth and compute the value of an **evaluation function** for a state instead of its minimax value
  - The evaluation function may be thought of as the probability of winning from a given state or the *expected value* of that state
- A common evaluation function is a weighted sum of *features*:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

  - For chess, w_k may be the **material value** of a piece (pawn = 1, knight = 3, rook = 5, queen = 9) and $f_k(s)$ may be the advantage in terms of that piece
- Evaluation functions may be *learned* from game databases or by having the program play many games against itself

Cutting off search

- **Horizon effect:** you may incorrectly estimate the value of a state by overlooking an event that is just beyond the depth limit
  - For example, a damaging move by the opponent that can be delayed but not avoided
- Possible remedies
  - **Quiescence search:** do not cut off search at positions that are unstable – for example, are you about to lose an important piece?
  - **Singular extension:** a strong move that should be tried when the normal depth limit is reached

Advanced techniques

- **Transposition table** to store previously expanded states
- **Forward pruning** to avoid considering all possible moves
- **Lookup tables** for opening moves and endgames

Chess playing systems

- Baseline system: 200 million node evalutions per move (3 min), minimax with a decent evaluation function and quiescence search
  - 5-ply ≈ human novice
- Add alpha-beta pruning
  - 10-ply ≈ typical PC, experienced player
- Deep Blue: 30 billion evaluations per move, singular extensions, evaluation function with 8000 features, large databases of opening and endgame moves
  - 14-ply ≈ Garry Kasparov
- More recent state of the art (Hydra, ca. 2006): 36 billion evaluations per second, advanced pruning techniques
  - 18-ply ≈ better than any human alive?

Summary

- A zero-sum game can be expressed as a minimax tree
- Alpha-beta pruning finds the correct solution. In the best case, it has half the exponent of minimax (can search twice as deeply with a given computational complexity).
- Limited-horizon search is always necessary (you can't search to the end of the game), and always suboptimal.
  - Estimate your utility, at the end of your horizon, using some type of learned utility function
  - Quiescence search: don't cut off the search in an unstable position (need some way to measure "stability")
  - Singular extension: have one or two "super-moves" that you can test at the end of your horizon