Rebirth of a 60's Icon: A Brighter, Safer, Faster Heating Lava Lamp

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1 Introduction

1.1 Motivation and Purpose

The lava lamp is an iconic piece of technology. They have been around since the 60's and have a very large initial "wow" factor, however very few people own a lava lamp. Many lava lamp owners are even scared to admit that they own one. Over the years the lava lamp has become kitsch technology. This aversion could possibly stem from the unchanged form factor, safety issues, or even the amount of time required before the lamp actually starts to function.

A lava lamp works by heating a two liquids with similar densities inside of a sealed glass container with a light bulb. One of the liquids heats up faster than the other and as a result expands thereby lowering its density. The now less dense liquid floats to the top. As it gets to the top it cools, raising its density, and falls back to the bottom to be heated again. This all occurs very slowly and has a very interesting visual effect [1].

Unfortunately, current models can take upwards of an hour to sufficiently heat the lava material enough for it to start moving. Not only that but they also lack adequate heat safety features. The temperature continues to rise, increasing the risk of severe burns if touched. Additionally, as the temperature increases unregulated, the density dynamics will change and the lava globules will be become smaller, more numerous, and stay near the top of the lava lamp. This is not aesthetically pleasing.

Our project will tackle these problems starting with the heat safety features. We will include temperature regulation that will keep the lava lamp temperature only as hot as it needs to be. We can do this because our light and heat source will be separated. Reducing heat will not reduce light. Conversly, adding more heat, such as during the startup phase, will not change the lighting. This will reduce the start up time. As a bonus we will improve the ambient light produced and add some interactive features such as multicolor lighting modes and brightness adjustment to help bring the lava lamp into the modern era.

1.2 Objectives

User Requirements

- Reaches operational temperature within 40 minutes
- Thermal regulation prevents the product from reaching unsafe temperatures
- Provides more ambient light than traditional lava lamps
- Dimmer switch and mode button cycles distinct pre-programmed color modes

Product Features

- Easily plugs into any standard U.S. outlet
- Consumes less energy than other models
- Starts working 50% faster than other models
- Dynamic multicolored lighting
- Interesting and attractive conversation piece
- User can adjust light settings: brightness and color

2 Design

2.1 Block Diagram

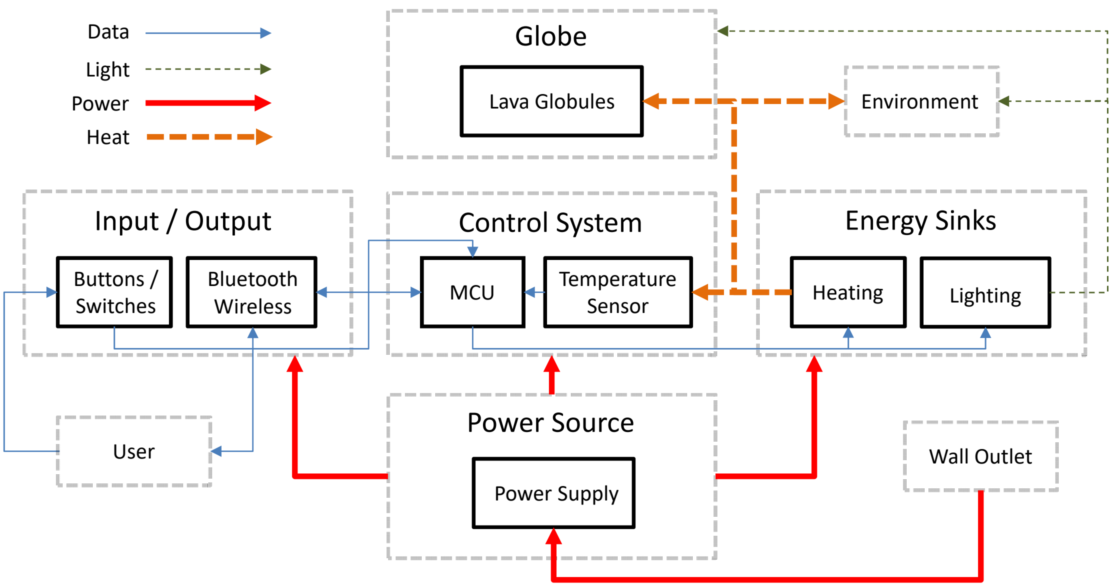

Figure 1: Lava globe block diagram

2.2 Block Descriptions

The elements within Figure 1 will be explained.

Power Source

Power Supply

The power supply receives input power at 20V from a AC/DC power supply. The power is then transformed to appropriate levels and distributed, to 6V via a buck converter (LM2677), and 3.3V via a linear voltage regulator (LM317), to the various components throughout the lava globe that require high power, i.e the lighting (2.8A @ 6V) and the heating (5A @ 20V), as well low power 3.3V for the control components: temperature sensors, microcontroller, on-unit input, and wireless.

Energy Sinks

Heating

The heat source receives a PWM signal from the microcontroller (MCU) and power from the power supply. The heat source consists of an n-terminal MOSFET (PSMN022-30PL) and an array of four 16 Ω/25W power resistors with heat sinks linked to the glass container by thermal epoxy. This heat source will have a max output of 100W. The PWM signal from the MCU is used to control how much heat is produced by the power resistor array for both safety and functionality. The power, and thus heat, can range from 0W to 50W depending on the duty cycle. The primary purpose for this generated heat is to create an environment conducive for the ”lava” to move within the glass container.

Lighting

The light source receives data from the MCU and power from the power supply. The MCU data interacts with the CREE XLamp XM-L Color LEDs through a PWM signals. These LEDs have four channels (RGBW), so the per-channel PWM signals can be leveraged to change the brightness and color of the lights. This light is then used to illuminate the globules moving within the lava globe sphere and output light to the room. At our expected operating temperature of 150°F the RGBW light out should be 74%, 91%, 97%, and 91% respectively.

Control System

Microcontroller

This module, the ATmega328P microcontroller, receives power from the power supply, analog voltage signals from the temperature sensors, analog (slider) and digital (button) signals from the input/output module, and a digital signal from the bluetooth wireless component. The temperature sensor information is used in a feedback loop to determine how much power to give the heating element thereby adjusting the heat output. The input/output information tells the microcontroller to either switch modes or adjust the brightness of the LEDS. The wireless data allows the user to interact with the lava globe. Additionally, the ATmega328P sends data to both the heating and lighting elements as explained in their corresponding block descriptions.

Temperature Sensor

The TMP36 temperature sensors measure heat from both the lava globe glass container as well as the heating element directly. This heat is converted into an analog voltage and sent to the MCU for processing. These sensors are linked to their corresponding measurand by thermal epoxy in order to increase measuring accuracy.

Globe

Lava Globules

The lava globe sphere receives light from the light source and heat from the heating element. The light is used for the illumination of the globules within the sphere. The heat is used to give motion to these globules, but it is also dispersed throughout the rest of the sphere. The sphere will radiate this heat and it will be measured by its temperature sensor.

Input / Output

On-Unit Input

The on-unit input receives power from the power supply and input from the user. The user provides the input by pressing a button (to change the lighting color mode) or by sliding a potentiometer (to change the overall brightness of the lamp). This input is then sent to the MCU for processing.

Wireless

This block receives power from the power supply and a Bluetooth signal from a user-controlled smartphone. It connects to the MCU via serial communication. The user communication is relayed to the MCU, and any feedback is transmitted back user back via Bluetooth. This module will be implemented by the HC-06 chip.

2.3 Circuit Schematics

Overview

Schematic of everything connected together

Power Supply

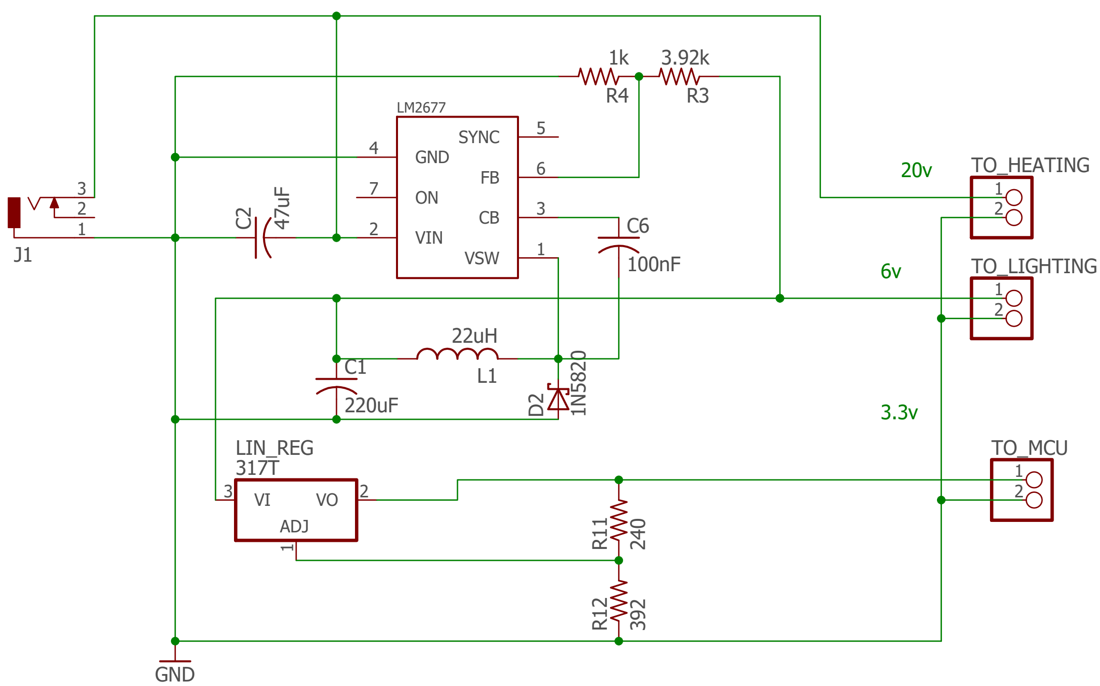

Figure 2: Power board schematic [2]

Figure 2 shows the power board. The switching buck converter is an LM2677, which is set with appropriate feedback resistors to regulate a 6V for the LED board. The sync and on inputs to the converter are intentionally left floating. From the 6V output, the linear regulator (LM317) is set with appropriate feedback resistors to regulate a 3.3V output.

Heating Element

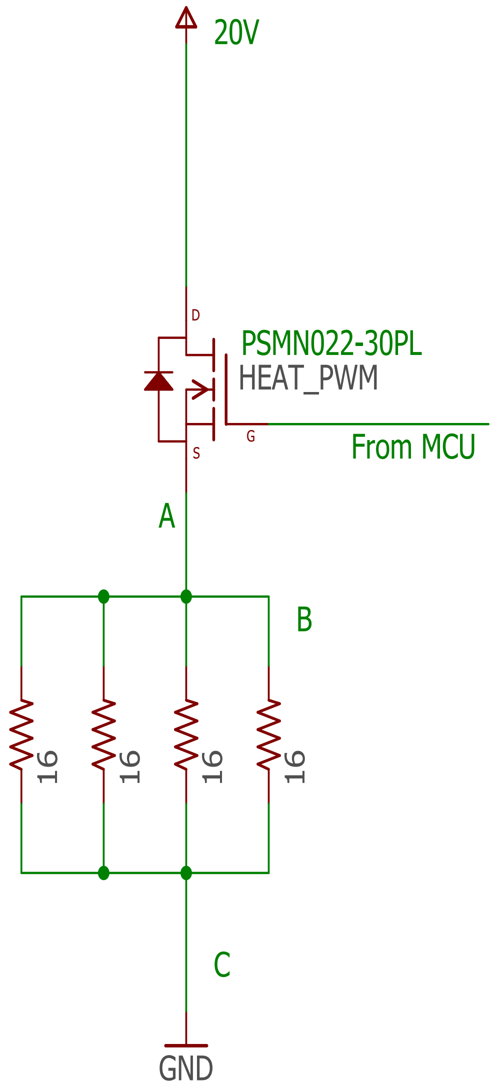

Figure 3: Heating element schematic

Figure 3 shows the setup for the heating element. Powered by 20V, it is an array of four 16Ω/25W power dissipating resistors surrounded by a heat sink generating a calculated maximum power of 100W. This array is controlled by the PWM from the microcontroller via an NMOS transistor. Points A and C represent test points for checking the current, voltage, and resistances across the entire array. Points B and C are used for testing the same characteristics over individual resistors.

LED Lighting

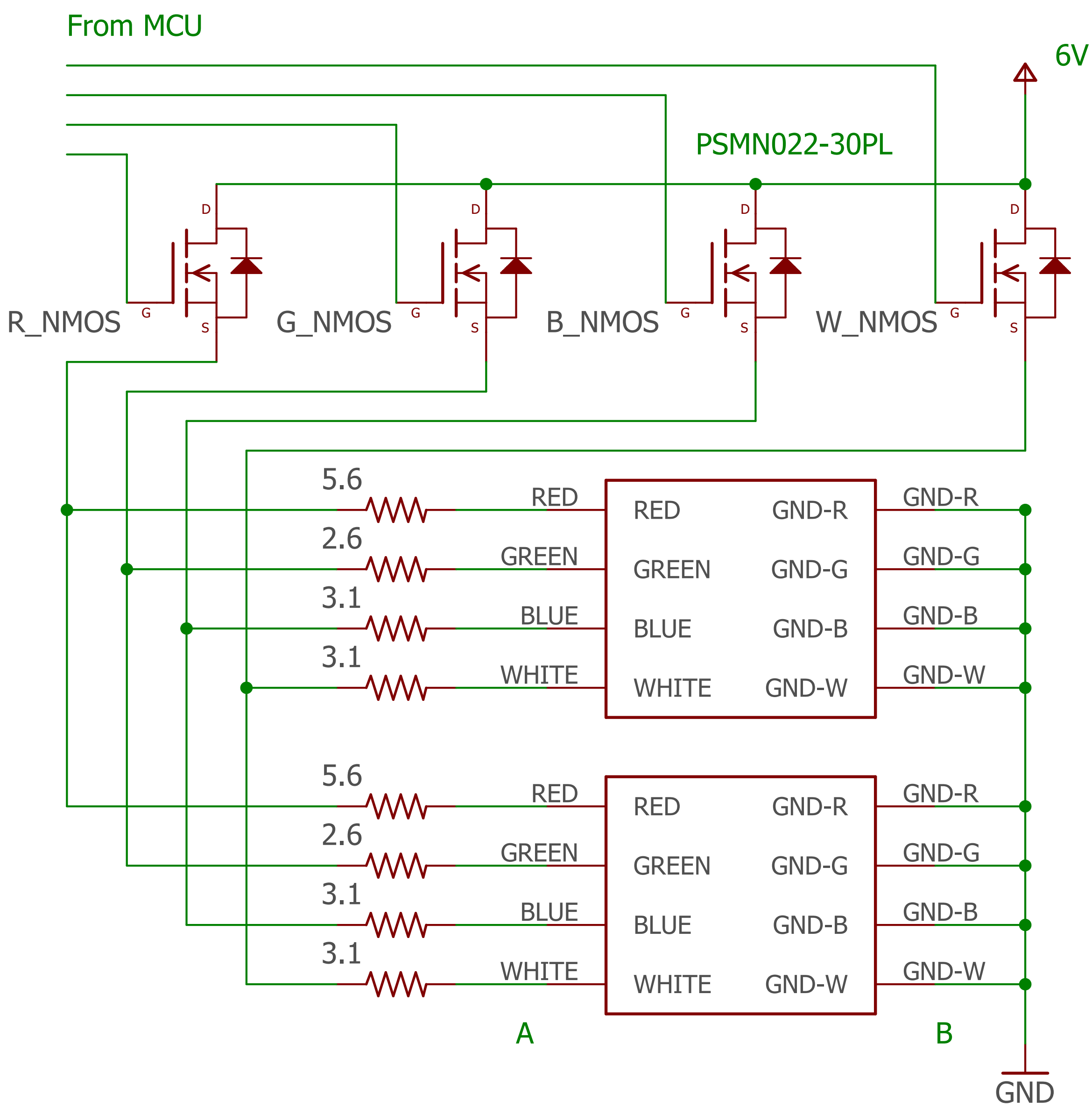

Figure 4: LED Lighting Schematic

In Figure 4, the RGBW NMOS chips control the respective RGBW channels for the two LEDs i.e. R_NMOS only controls the red channels G_NMOS only controls the green channels, etc. by utilizing the PWM functionality of the microcontroller. Each color channel has a current protection resistor. Points A and B illustrate the test points for checking the current stability through the LEDs.

Temperature Sensors

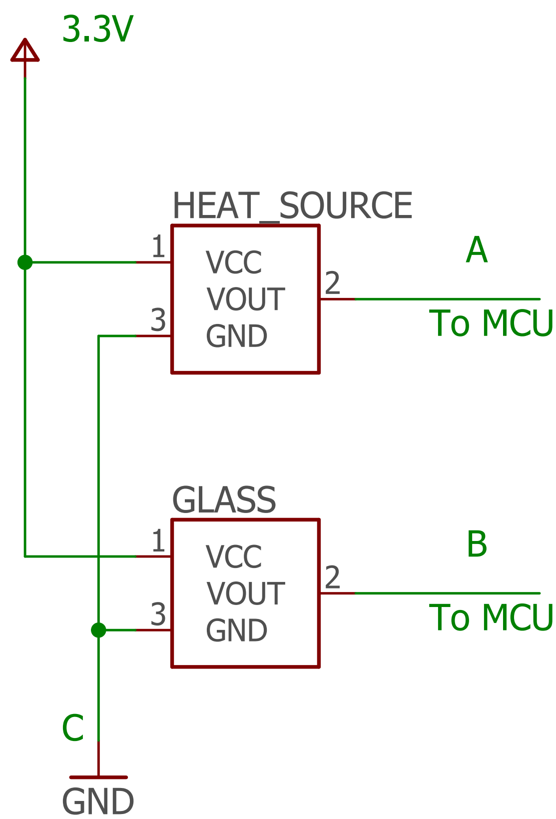

Figure 5: Temperature sensor schematic

Figure 5 shows the two TMP36 temperature sensors. One will be thermally epoxied to the heat source and the other will be thermally epoxied to the the glass container. Points A, B, and C illustrate the test points for testing the output voltages. These voltages are sent to the microcontroller for use in the temperature control feedback loop.

Microcontroller

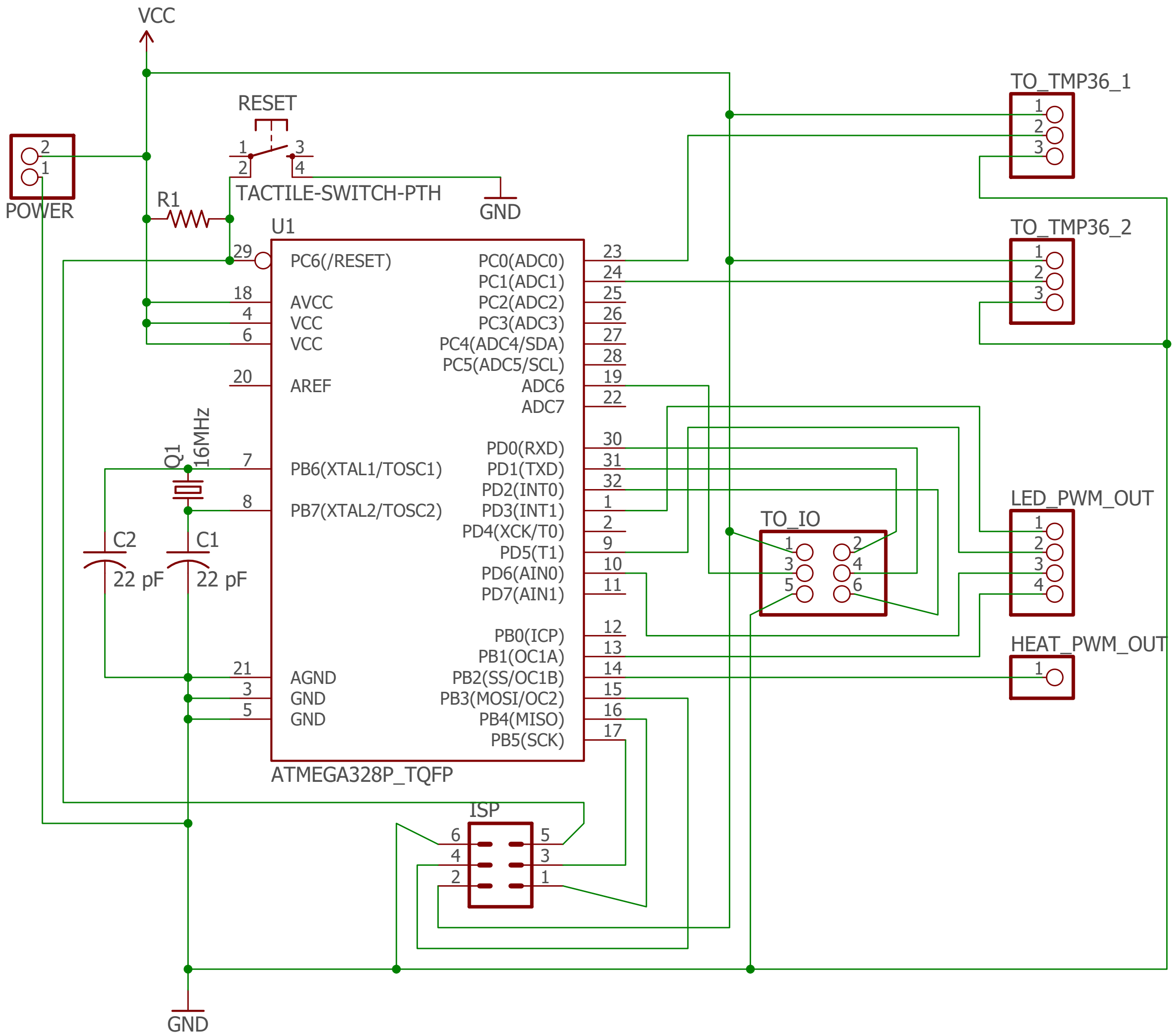

Figure 6: Microcontroller board schematic

In figure 6, the microcontroller board is shown. The primary component is the Atmega 328P, which controls the logic of the lamp. 5 of the 6 available PWM outputs are in use, to drive the LED channels and the heating element. An ISP header and reset button is made accessible for programming, and headers for connections to other boards are present.

IO and Wireless Board

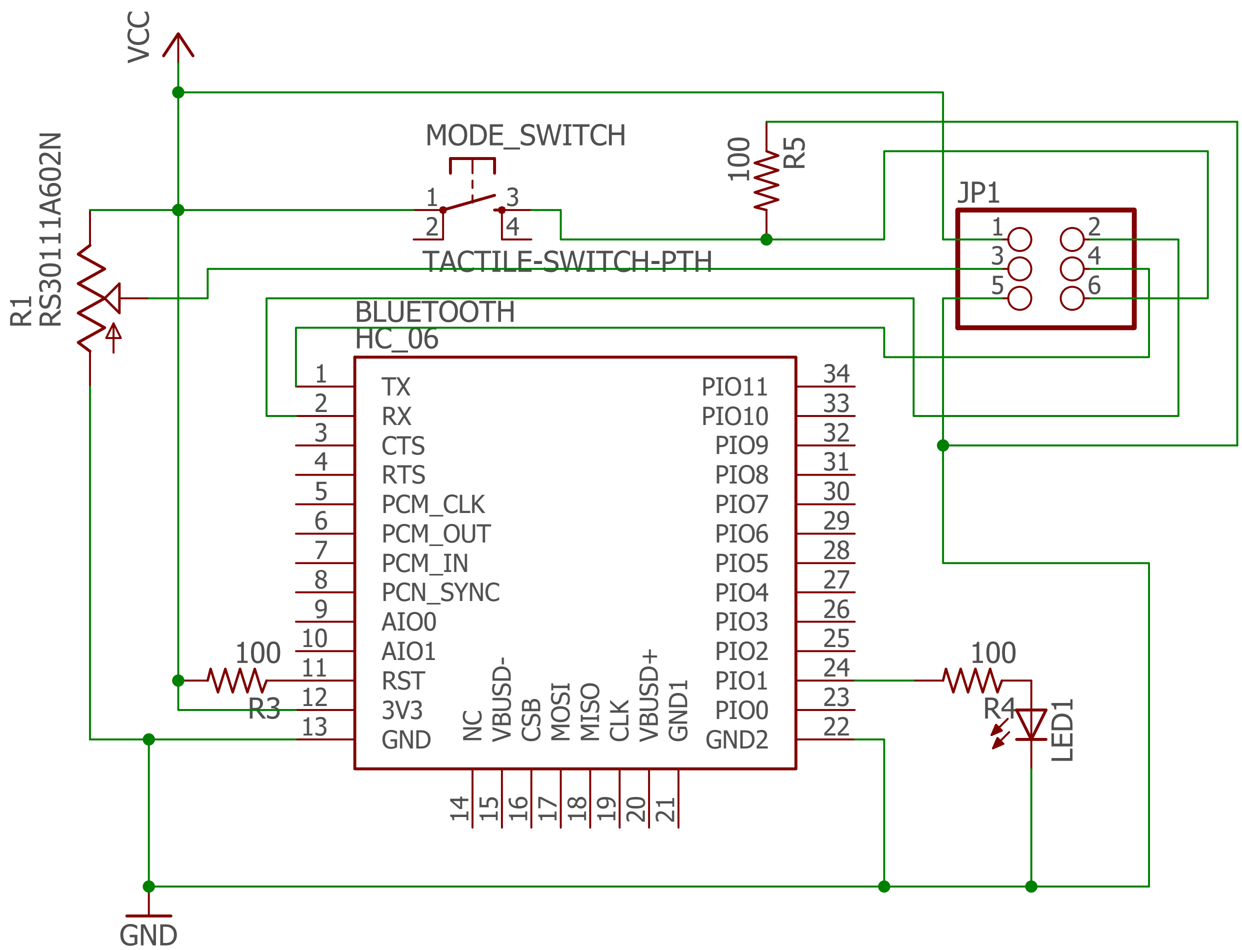

Figure 7: I/O and wireless board schematic

In figure 7, the I/O board is shown. This board contains a tactile switch and sliding potentiometer, which will be externally accessible through cut-outs in the lamp housing. It also contains the HC_06 module, which allows a bluetooth host to connect to the lamp and issue serial commands to the MCU. The board has one header for connection to the MCU board, which supplies it with power, reads the switch state, reads the potentiometer voltage, and communicates with the HC_06 via serial communication.

2.4 Calculations

Power Supply

We know that the the LEDs are designed to draw 350mA per channel. With 8 channels at 6V supply to the LED module, we must provide $16.8W$. Since we are using a switching regulator with efficiencies of about 80%, this adds about $16.8W/.8 = 21W$ to our total system demand. Our heating element is designed to output an absolute maximum of 100W, drawn directly from the 20V supply. The other low-power components are negligible in terms of their load. Thus, we need at least 121W 20V DC supply. We chose a 135W laptop power adapter as a common product that meets these requirements.

$$P = IV = (8 \times 350mA) \times 6V = 16.8W \tag{1}$$

To provide a 6V supply, we use a switching regulator. It has adjustable voltage regulation, set by feedback resistors. From the datasheet [2], we see that a common R1 value is $1k\Omega$, and the following equation can be used to calculate R2:

$$R2 = R1(\frac{V_{out}}{V_{FB}} - 1) = 1k\Omega(\frac{6v}{1.21V} - 1) = 3.96k\Omega \approx 3.92k\Omega \tag{2}$$

To provide a 3.3V supply, we use a linear regulator. It has adjustable voltage regulation, set by feedback resistors. From the datasheet [3], we see that a common R1 value is 240Ω, and the following equation can be used to calculate R2:

$$V_O = V_{ref}(1 + \frac{R_2}{R_1}) + I_{ADJ} \times R_2 \tag{3}$$

The datasheet also explains that in most cases, I_ADJ can be ignored as negligible. Since the 3.3V output is consumed by parts with large tolerances, we reduce equation 3 and solve for R2:

$$R_2 = R_1(\frac{V_O}{V_{ref}} - 1) = 240\Omega(\frac{3.3V}{1.25V}) = 393.6\Omega \approx 392\Omega \tag{4}$$

Heating Element

We want our heating element to produce at least 50W of power. This means that each resistor must supply at least 15W of power to heat independently. If we use the Riedon UAL-25 we can achieve 25W per resistor with the aluminum heat sink. This brings us to a grand total of 100W. To find the resistance we used Equation 5--the power equation:

$$P = \frac{V^2}{R} \tag{5}$$

At our maximum provided voltage of 20V the resistance should be 16Ω to achieve 25W of power. The calculated maximum current traveling through each resistor would thus be 1.25A found by using the Equation 6:

$$V = IR \tag{6}$$

Since the four heating resistors are in parallel, the current would need to be quadrupled to 5A. Our power supply can offer almost 7A max current so this is achievable with room to spare for the LED array.

LED Lighting

The CREE XLamp XM-L Color LEDs operate at 350mA and the RGBW channels use 2.25V, 3.3V, 3.1V, and 3.1V respectively. Our NMOS drops 1.8V across its terminals. And since we will be providing 6V to our LED circuit we can find the necessary resistance for the LED protection resistors by using Ohm's Law and Equation 6.

$$V_R = 6 - 1.8 - Channel\,Voltage \tag{7}$$

Equation 7 tells us that voltage across the RGBW resistors will be 1.95V, 0.9V, 1.1V, and 1.1V respectively. Therefore, the RGBW protection resistor resistances should be 5.6Ω, 2.6Ω, 3.1Ω, and 3.1Ω respectively. At these resistances Equation 1 tells us that the RGBW protection resistor power dissipation requirements will be 0.7W, 0.3W, 0.4W, and 0.4W respectively.

3 Control

3.1 System Model

We will model our system as a Cylindrical Lava Lamp and a Cylindrical Heat Source as seen in Figure 8.

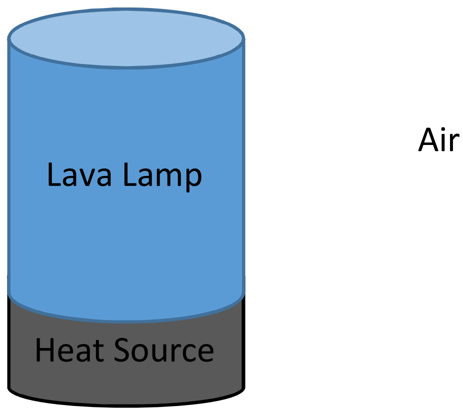

Figure 8: Physical Model of Lava Lamp System as two cylinders

To simplify our control of the system, we will consider each block in Figure 8 (Lava Lamp, Heat Source, and Air) as having a uniform internal temperature ($T_{Lava}, T_{Heat}, and T_{Air} respectively$). Also, we will model the heat flow between blocks using equation 8 [7].

$$q = \frac{1}{R}(T_1 - T_2), \tag{8}$$

Where,

q is the heat energy flow ($\frac{J}{sec}$)

R is the thermal resistance ($\frac{^\circ C sec}{J}$)

T is the temperature ($^\circ C$)

And the net heat-energy flow into a substance affects the temperature of the substance according to the relation in equation 9 [7].

$$\dot{T} = \frac{1}{C}q \tag{9}$$

Where,

C is the thermal capacity ($\frac{J}{^\circ C}$)

q is the sum of heat flows obeying equation 8

and $\dot{T}$ is the rate of change of the temperature ($\frac{^\circ C}{sec}$)

Using equations 8 and 9 and our model we can derive the differential equations that determine the temperature in each block in figure 8. Note, that we will consider the air block to have an infinite thermal capacity, allowing it to exchange heat without changing its temperature.

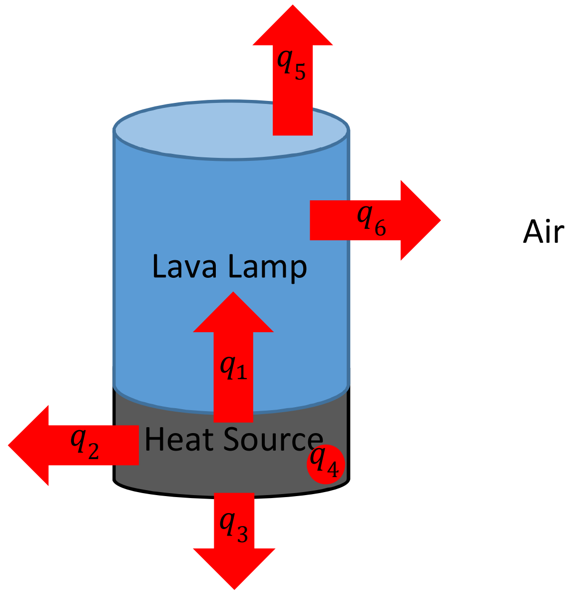

Figure 9: Heat Flow Illustrated on Lava Lamp System Model

In figure 9 you can see the heat flow between every surface defined. A positive heat flow will indicate the flow of thermal energy from the base of the arrow to the tip. The definitions are as follows:

q_1: From the Heat Source to the Lava Lamp

q_2: From the Heat Source to the Air (side of cylinder)

q_3: From the Heat Source to the Air (bottom of cylinder)

q_4: From the Power Resistor to the Heat Source

q_5: From the Lava Lamp (top of cylinder) to the Air

q_6: From the Lava Lamp (side of cylinder) to the Air

So,

$$q_{Heat} = q_4 - q_1 - q_2 - q_3$$

$$q_{Lava} = q_1 - q_5 - q_6$$

If we assign a thermal Resistance R_i to each of the q_i and plug in equation 8 (besides q_4 which we will assume we can control directly), then plug in equation 9 and simplify we have the following:

$$\dot{\tilde{T}}_{Heat} = (\frac{-1}{R_{1,2,3}C_{Heat}})\tilde{T}_{Heat} + (\frac{1}{R_1 C_{Heat}})\tilde{T}_{Lava} + (\frac{1}{C_{Heat}})q \tag{10}$$

$$\dot{\tilde{T}}_{Lava} = (\frac{1}{R_1 C_{Lava}})\tilde{T}_{Heat} + (\frac{-1}{R_{1,5,6} C_{Lava}})\tilde{T}_{Lava} \tag{11}$$

Where

$$R_{1,2,3} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}, \frac{R_1 R_5 R_6}{R_1 R_5 + R_1 R_6 + R_5 R_6}, q = q_4$$

$$\tilde{T}_{Heat} = T_{Heat} - T_{Air}, \text{ and } \tilde{T}_{Lava} = T_{Lava} - T_{Air}$$

3.2 Theoretical Estimation of Parameters

In order to simulate our model, we must first estimate the parameters of our model. Namely, we must estimate:

$$R_1, R_2, R_3, R_4, R_5, C_{Heat}, \text{ and } C_{Lava}$$

In our formulation of thermal resistance from equation 8, we never specified the method of heat transfer between our system blocks. We merely gave a constant of proportionality and assumed that the majority of the change in temperature would happen at the surfaces. At this point it will be useful to extend our model by specifying dimensions. As we can see in figure 10, the dimensions of the lava lamp cylinder are (height H, diameter D), and of the heat source cylinder (height h, diameter h).

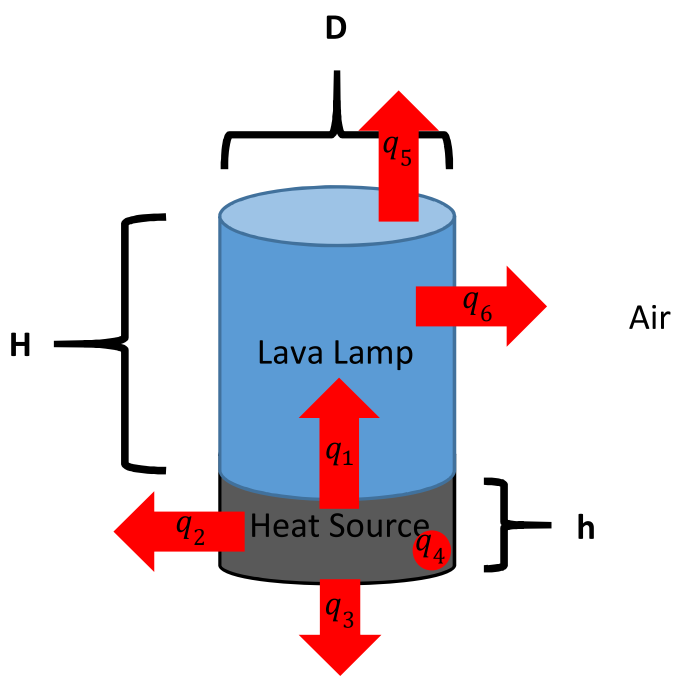

Figure 10: Heat Flow Illustrated on Model with Dimensions Specified

For the lava lamp and heat sources we have chosen the values are:

$$H = 0.69m, D = 0.115m, \text{ and } h = 0.03m$$

Through some derivation (included in Appendix) we can obtain a theoretical estimate for the parameters [3][4][5][6].

$$R_1 = 0.9628(\frac{K}{W})$$

$$R_2 = 184.5(\frac{K}{W})$$

$$R_3 = 192.6(\frac{K}{W})$$

$$R_5 = 192.6(\frac{K}{W})$$

$$R_6 = 8.022(\frac{K}{W})$$

$$C_{Lava} = 29,990(\frac{J}{^\circ C})$$

$$C_{Heat} = 757.2(\frac{J}{^\circ C})$$

3.3 Controller Design and Simulation

From the previous section we have the differential equations that govern our system. We can transform this into a state space representation for easier analysis of the form

$$\dot{x} = Ax + Bu$$

For our system we have:

$$\begin{bmatrix} \dot{\tilde{T}}_{Heat} \\ \dot{\tilde{T}}_{Lava} \end{bmatrix} = \begin{bmatrix} (\frac{-1}{R_{1,2,3}C_{Heat}}) & (\frac{1}{R_1 C_{Heat}}) \\ (\frac{1}{R_1 C_{Lava}}) & (\frac{-1}{R_{1,5,6}C_{Lava}}) \end{bmatrix} \begin{bmatrix} \tilde{T}_{Heat} \\ \tilde{T}_{Lava} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{Heat}} \\ 0 \end{bmatrix} q \tag{12}$$

The system matrix A contains information about the stability of the system. The eigenvalues of this matrix are the poles in the complex plane. If these poles are all in the Left Hand Plane (real part less than 0), then we can say our system is stable. So for our system, the eigenvalues and thus poles are:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -0.00142001 \\ -0.0000045667 \end{bmatrix} \tag{13}$$

Therefore, we can say that the open-loop system is stable. Also, from the state space representation we can easily obtain the system block diagram as seen in figure 11.

Where

$$C_1 = (\frac{1}{C_{Heat}}), C_2 = (\frac{1}{R_{1,2,3}C_{Heat}}), C_3 = (\frac{1}{R_1 C_{Lava}}), C_4 = (\frac{1}{R_{1,5,6}C_{Lava}}), C_5 = (\frac{1}{R_1 C_{Heat}})$$

A traditional lava lamp would at this point give a constant control input to the system and perform open-loop control. In figure 12 we can see the results from our Simulink simulation with the theoretical parameters from the previous section. The input power was chosen to reflect two common power levels for lava lamp

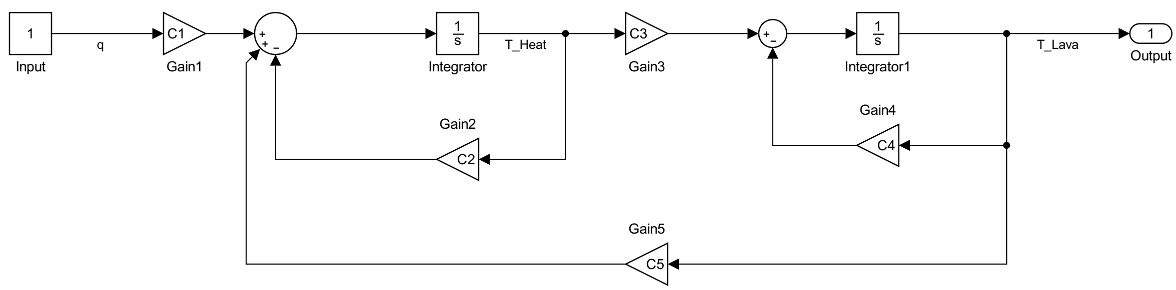

Figure 11: System Block Diagram

light bulbs (25W and 40W) as well as our maximum power output (64W).

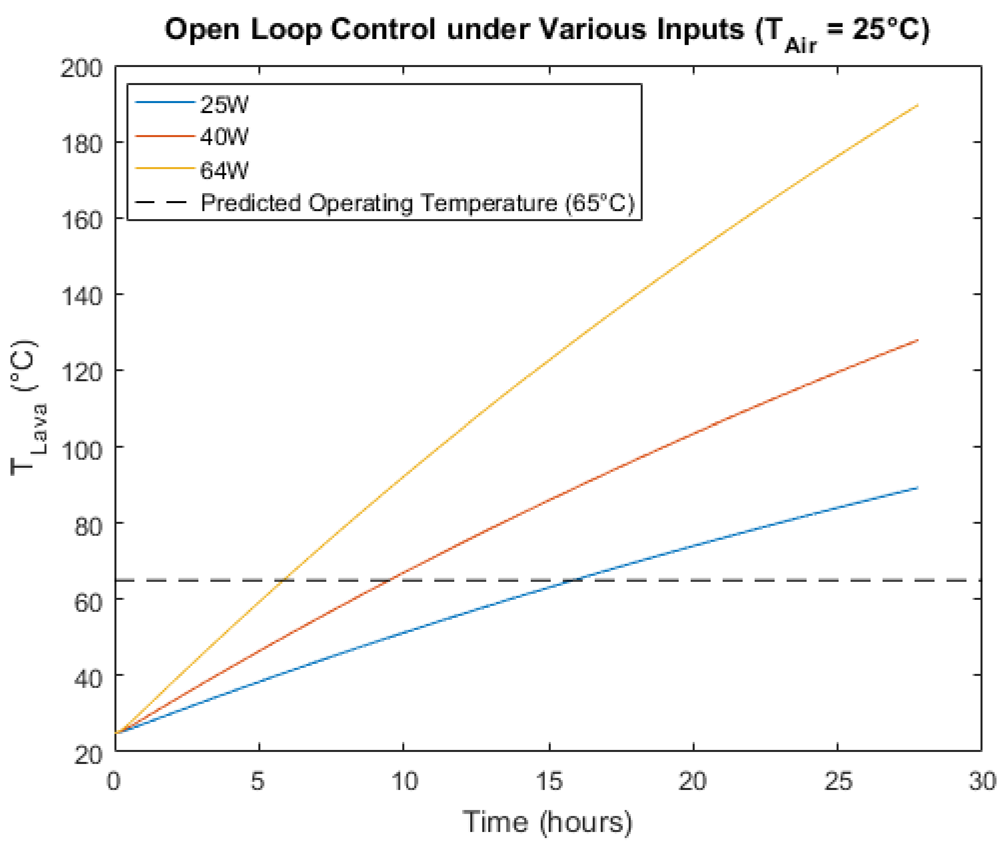

Figure 12: Open Loop Control Under Various Inputs

Although the parameters may not be completely accurate and our model is not perfect, we can see the intrinsic problem with this control scheme. Using a large input power allows the system to get to the operating temperature faster, however, it also causes the system to get to dangerous levels faster. Using a small input power would get to the dangerous levels slower, but it would also get to the operating temperature at a slower rate.

We propose a simple proportional feedback controller that addresses both of these problems. In the figure below, we can see the new block diagram with an error and gain added. We then simulated this controller using various gains to see the effects.

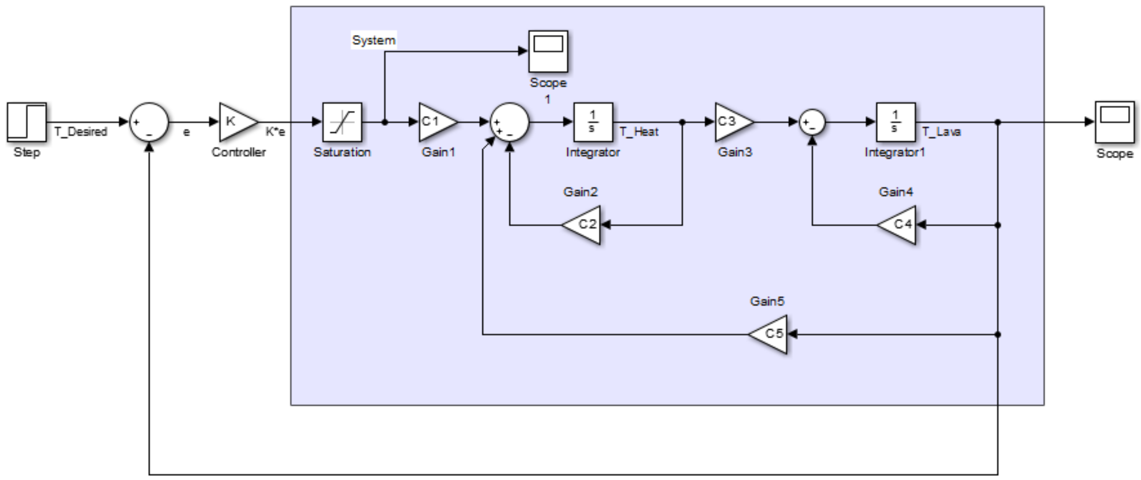

Figure 13: Proportional Control System Model

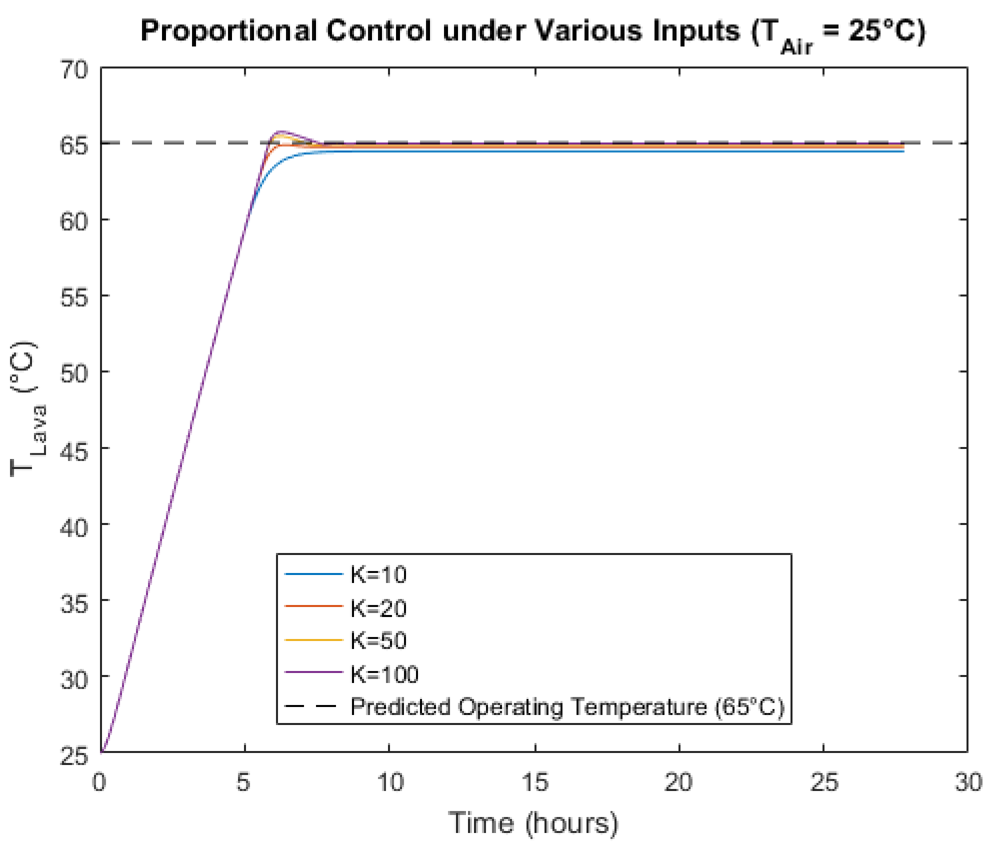

Figure 14: Proportional Control Under Various Gains

In the figure above we can see the results. The first thing to notice is that as K increases the amount of damping decreases. Ideally, the system is critically damped and there is no overshoot. When K is 20, it appears to be very close to this. The second thing to see is the steady state error in the previous plot. The values for the errors are listed in the table below. As K increases the error decreases. So, to design our controller there is a trade-off between steady state error and overshoot. Also, in the second figure below, we can see the control effort. Because it is impossible in our system to input a negative heat, we can see the controller flat-line at 0 for the underdamped values. Based on these plots we decided to assign the theoretical value as (K=20). However, the specific parameters of our real system will most likely change this value of K. In the next section we will describe how to obtain the parameters experimentally, and then later we would re-design our controller using the same methodology.

K	Steady State Error ($^{\circ}C$)
10	0.5592
20	0.2816
50	0.1131
100	0.0566

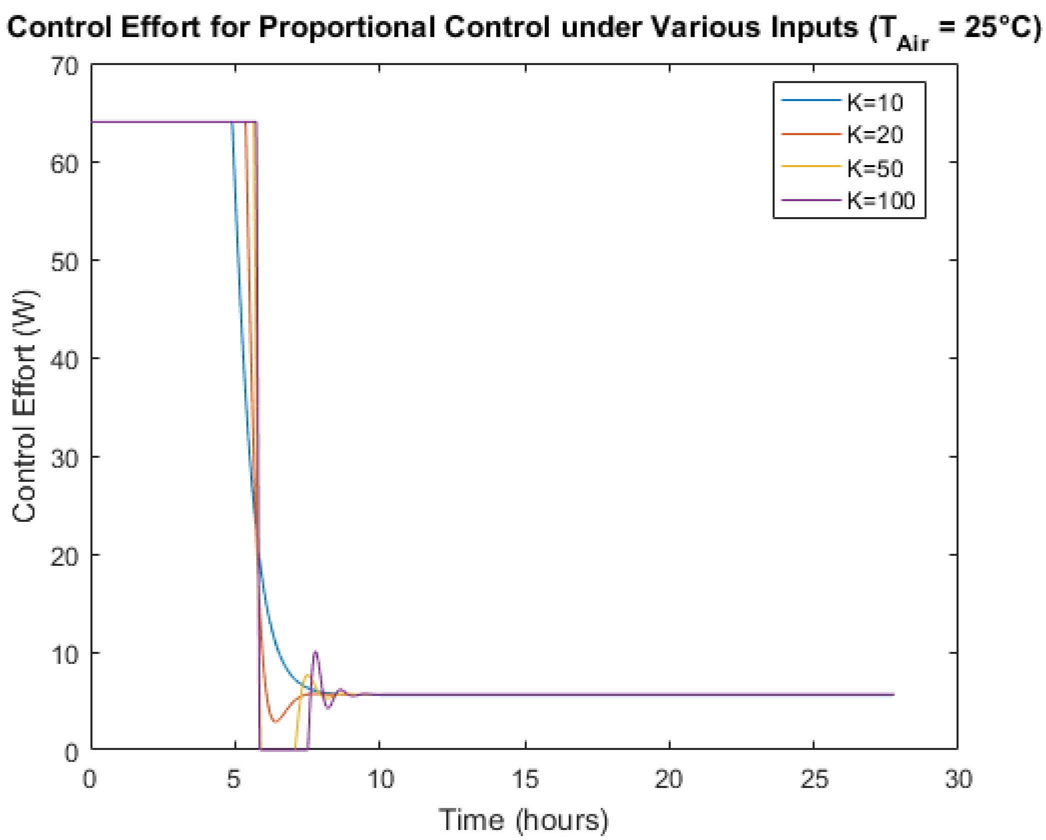

Figure 15: Control Effort for Proportional Control Under Various Gains

3.4 Experimental Estimation of Parameters

With the controller in its current form, there is actually no need for the parameters to be estimated for the system to run using our simple proportional controller. It is stable and an effective K can be chosen such that it meets most of the requirements.

Suppose that we were to attach temperature sensors to the Heat Source and Lava Lamp, and also measure the heat going into the Heat Source (q) by measuring the current and voltage across the resistor. We could then heat up the system from the air temperature all the way up to its max operating point, taking data points along the way. From this we could construct a system of equations and use a Least Squares Method to obtain all of our parameters. For a full treatment see Appendix:Lava Lamp Control.

4 Heating Module Simulation

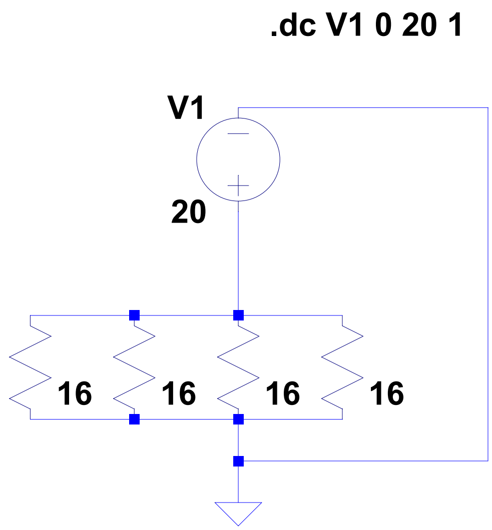

Figure 16: LTSpice schematic for simulation

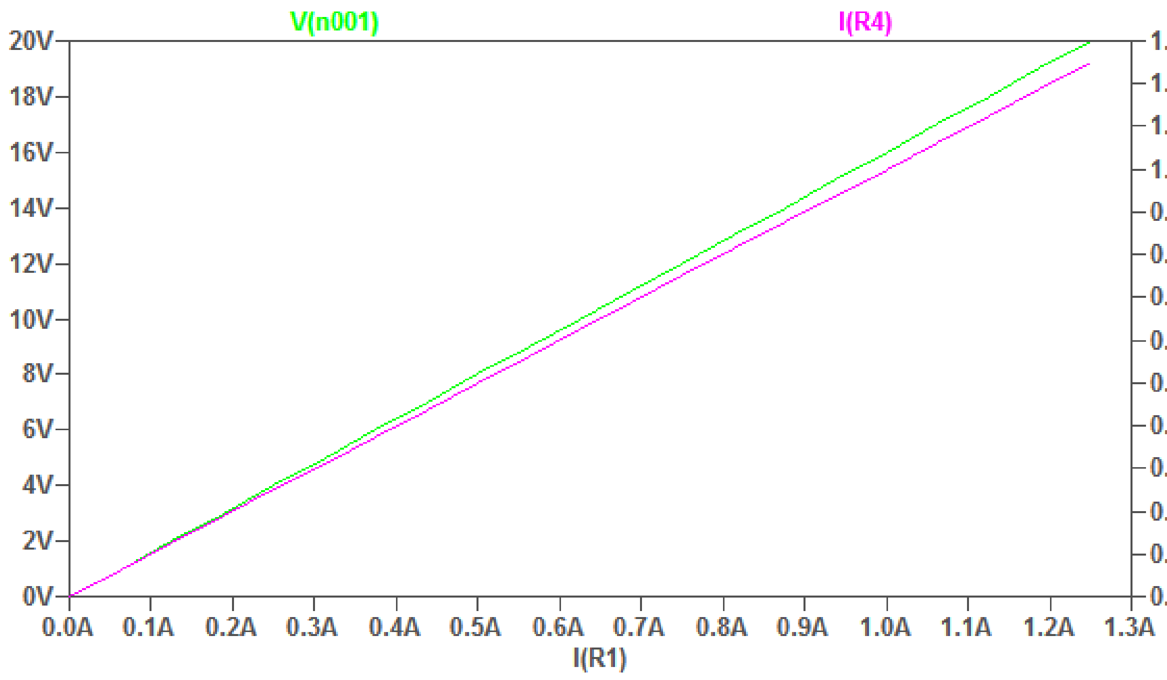

Figure 17: LTSpice simulation plot Amps vs. Voltage

Figure 16 illustrates a test case for the heating element where we omit the NMOS transistor due to its mΩ resistance and thus very small affect on the voltage. The simulation shown in Figure 17 shows that the current going through each resistor is indeed 1.25A.

5 Requirements and Verifications

<table>
<tr><th>Requirements</th><th>Verification</th></tr>
<tr><td>Power Supply
1. Accept 100-130V 60Hz AC input and supply at least 5A at 19 - 20V (7.5 points)
2. Provide at least 2A at 4.7 - 5V (7.5 points)</td><td>Power Supply
1. Verification for item 1
(a) Assemble a 4Ω resistor network that can dissipate 100W
(b) Connect the 20V output across the resistor network
(c) Use a multimeter to measure the voltage across the 20V output
(d) It should remain between 19V - 20V
2. Verification for item 2
(a) Assemble a 1Ω resistor network that can dissipate 25W
(b) Connect the 5V output across the resistor network
(c) Use a multimeter to measure the voltage across the 5V output
(d) It should remain between 4.7V - 5V</td></tr>
<tr><td>Light Source
1. Must produce at least 300 lumens of light. (5 points)
2. Must be able to control power in increments of 30 lumens or less (5 points)
3. Provide red, green, blue, purple, and pink lighting (5 points)</td><td>Light Source
1. Verification for item 1
(a) Use a light meter 1m away from the lava lamp, in three directions 60° apart and horizontal to the lamp
(b) Measure at least 300 lux
2. Verification for item 2
(a) From minimum power, step power up by small increments until the light meter registers a change in output.
(b) Record the flux as above and repeat
(c) At least 10 flux levels should have been recorded, each differing by less than 30 lux
3. Verification for item 3
(a) Use a cell phone camera to capture the light
(b) Use a color picker tool to confirm that the average hue and saturation of the lamp differs from the target hue by no more than 20° and target saturation by no more than 0.1 in either direction in an HSV color space [0-360, 0-1, 0-1].</td></tr>
</table>

Heat Source 1. Must dissipate between 100W ± 0.5W of power when operated at 20V ± 0.1V for at least 45 minutes (15 points) 2. Must be able to have power adjusted in increments of at least 1W (10 points)	*Heat Source* 1. Verification for item 1 (a) Connect an ammeter between heat source test points A and C. (b) Supply a 20V ± 0.5V DC signal between test points A and C. (c) Measure the system for 45 minutes (d) Confirm that the heat source current remained 5A ± 0.5A (e) Confirm that the voltage remained 20V ± 0.5V 1. Verification for item 2 (a) Connect ammeter in the same configuration as verification one (b) Use MCU PWM to step up current by smallest increments starting from the NMOS off state to 5A (c) Measure the current for each step (d) Find at least six consecutive levels that differ from each other by no more than 1W.
Temperature Sensor 1. Able to take a temperature measurement at least once every minute. (2.5 points) 2. Able to track temperature to at least one degree Fahrenheit. (2.5 points)	*Temperature Sensor* 1. Verification for item 1 (a) Attach the temperature sensor to an object (b) Heat the object (c) Use the temperature sensor and an independent heat measurement device to track the heat change (d) If the temperature matches the independent thermometer every minute then the requirement is met 2. Verification for item 2 (a) Look at the data sheets

<table>
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Microcontroller

1. Needs at three analog and and two digital input channels (2 points)
2. Needs at least five PWM and one digital output channels (2 points)
3. Needs at least 20kB of memory (2 points)
4. Needs a clock of at least 20 MHz (2 points)
5. Needs to be able to run on 5V±1V (2 points)

</td>
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Microcontroller

1. Verification for item 1
    (a) Verify with the datasheet
2. Verification for item 2
    (a) Verify with the datasheet
3. Verification for item 3
    (a) Verify with the datasheet
4. Verification for item 4
    (a) Hook up the clock pin to the oscilloscope positive terminal and ground to the ground terminal
    (b) Measure the clock frequency
5. Verification for item 5
    (a) Verify with the datasheet

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</table>

On-Unit Input 1. Able to accept user input from the mode button and transmit the correct state(2.5 points) 2. Able to accept user input from the mode button and transmit the correct state(2.5 points)	*On-Unit Input* 1. Verification for item 1 (a) From the output of the On-Unit Input, hook up header pins 1 and 6 to 3.3V and ground, respectively, and header pin 3 and ground to a voltmeter. respectively (b) Push and hold the the mode button (c) the voltmeter must read a potential of at least 3V (d) release the the mode button (e) the voltmeter must read a potential of less than 0.25V 2. Verification for item 2 (a) From the output of the On-Unit Input, hook up header pins 1 and 6 to 3.3V and ground, respectively, and header pin 3 and ground to a voltmeter. respectively (b) Move the potentiometer to its lowest point (c) The voltmeter must read a potential of less than 0.5V (d) Move the potentiometer to its highest point (e) The voltmeter must read a potential of at least than 3.0V (f) Move the potentiometer to its midpoint (g) The voltmeter must read a potential of between 1V and 2.5V.

<table>
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<td>Wireless

1. Be able to transmit and receive commands as a Bluetooth client device to a standard smartphone from 20ft in open air. (2.5 points)

2. Be able to transmit and receive at a rate of at least 9600 baud within a radius of 20ft from the lava lamp to the microcontroller. (2.5 points)</td>
<td>Wireless

1. Verification for item 1
(a) stand 20ft from the lamp
(b) With the lamp on, search for the device with a smartphone and pair.
(c) Open the companion app
(d) Increase or decrease the brightness. The brightness of the lamp must noticeably change

2. Verification for item 2
(a) With the lamp on, search for the device with a smartphone and pair.
(b) Open the companion app
(c) The app must report a 9600 baud connection to the MCU</td>
</tr>
</table>

Control	*Control*
1. Fundamentally BIBO Stable. For any bounded input the system should have a bounded output. (7.5 points) 2. Rise time of 40 minutes for T_{Lava} (from an Air temperature of between 23°C and 27°C to a temperature in which the lava globules are moving). (5 points) 3. Steady State Error of Less than 0.2 °C between the desired T_{Lava} and the sensor readings (at an Air temperature of between 23°C and 27°C). (7.5 points)	1. Verification for item 1 (a) Obtain the System Parameters Experimentally as described in the Appendix: Lava Lamp Control (b) Compute the eigenvalues of the closed-loop system. If they are in the LHP of the complex plane then the system is stable. 2. Verification for item 2 (a) Measure the air temperature to make sure it is in the range between 23°C and 27°C (b) Start the controller and begin a stopwatch. (c) Gaze aimlessly into the lava lamp until the lava globules begin moving, at which point stop the stopwatch. (d) If the time reads 40 minutes or less the requirement is met. 3. Verification for item 3 (a) Measure the air temperature to make sure it is in the range between 23°C and 27°C (b) Connect to the debugging port of the system and read the desired temperature and system temperature from the sensor. (c) Start the controller and wait until the system reaches operating temperature. (d) Then, wait 30 minutes and measure the difference between the desired temperature and sensor reading for T_{Lava}. (e) If this difference is less than 0.2 it passes the criterion.

6 Tolerance Analysis

The most important module within this project is the heating source. Correct regulation of heat is critical to both the function and safety of the lava globe. Our operational temperature is 150ºF±2ºF. With the exception of startup (room temperature-150ºF), keeping the temperature of the heating source within this operational range will ensure both optimal lava globule conditions and thermal safety requirements are met.

We will confirm this tolerance is met by turning on the lava globe from room temperature, allow the lava globe to reach its optimal temperature of 150ºF, and then continue to monitor the temperature for six consecutive hours. If the temperature does not leave the 148ºF-152ºF range then the tolerance will be confirmed. This test will compensate for the tolerances of any components along power chain. This final temperature range is the most critical.

7 Cost and Schedule

7.1 Cost Analysis

Personnel	
Average Starting Salary	$67,000 [8]
Per Hour Rate	$32
Total Hours (20 / Week)	280
Personnel Cost (3 members)	$27,056
Total Cost (with overhead)	**$67,641**

Parts				
Function	**Part**	**Amt.**	**Unit Price**	**Total Price**
Power Supply	AC $\rightarrow$ 20V, 6.75A with Power Plug	1	$13.95	$13.95
Linear regulator	LM317	1	$0.53	$0.53
Switching regulator	LM2677	1	$5.96	$5.96
Mode Switch	Tactile Button	1	$0.89	$0.89
Sliding Potentiometer	RS30111A602N	1	$2.85	$2.85
Kill Switch	PRASA1-16F-BB0BW	1	$0.93	$0.93
16Ω/25W Heating Resistors w/ heat sink	RE70	4	$4.00	$16
LED	CREE XLamp XM-L Color LED	2	$15.29	$30.58
NMOS Transistor	PSMN022-30PL	5	$0.58	$2.90
Temperature Sensor	TMP36	2	$2.27	$4.54
Microcontroller	ATmega328P	1	$2.00	$2.00
Testing	Lava Lamp	1	$29.99	$29.99
Bluetooth Chip	HC-06	1	$4.00	$4.00
Thermal Adhesive	Arctic Alumina Thermal Adhesive 5g	1	$7.77	$7.77
Oscillator	FOXSLF/160-20	1	$0.33	$0.33
Total Parts Cost				**$123.22**

7.2 Schedule Summary

Week	Matt	Devin	Daniel
09/11 - 09/17	Proposal	Proposal	Proposal
09/18 - 09/24	Research Control Problem	Research Lighting MCU Control	Begin conversations with ID (Industrial Design) contact
09/25 - 10/01	Research Cntrl Prob. (cont) PCB Design	Research Wireless PCB Design Update ID contact	Research Heating Element Temperature Sensor
10/02 - 10/08	Design Review Update ID contact	Design Review	Design Review
10/09 - 10/15	Design PWM Circuit Design power supply	Conduct heating tests Update ID contact	Begin coding heating element logic
10/16 - 10/22	Test PWM Test power supply	Begin coding light logic	Test heating element logic Update ID contact
10/23 - 10/29	Revise PCB	Test light logic Finalize machine shop design Update ID contact	Finalize machine shop design
10/30 - 11/05	Finalize PCB Design Assemble and test prototype Update ID contact	Configure wireless Assemble and test prototype	Assemble and test prototype
11/06 - 11/12	Revise R&V Continue testing prototype	Revise R&V Continue testing prototype Update ID contact	Revise R&V Continue testing prototype
11/13 - 11/19	Final Testing	Final Testing	Final Testing Update ID contact
11/20 - 11/26	Break	Break	Break
11/27 - 12/03	Create Videos / Demo	Create Videos / Demo	Create Videos / Demo
12/04 - 12/10	Final Paper	Final Paper	Final Paper

8 Safety Statement

The safety issues of our device are electrical, chemical, and thermal in nature. During the development phase team members will be at risk of strong electrical shock and possibly even death. We recognize this fact and will minimize the risk by utilizing proper lab electrical safety measures such as removing power from circuits currently not in use and testing circuits for power with an ammeter before we start using them. Moreover, we will seek the the help of those more experienced in these issues.

Additionally, we intend to use the lab bench as our primary power source for the majority of our development to avoid the hazards posed by using an wall socket directly. The lab bench will offer us more precise control over what is going in and out of our circuits. Finally, we will not bathe with the lava globe, as electricity and water are a dangerous combination, and we will advise others to do the same as suggested by the first point in the IEEE Code of Ethics regarding informing the public of hazards [9].

The chemical hazard stems from using possibly unsafe materials inside of the lava sphere. The hazard presents itself if the glass integrity is compromised and a leak occurs. We do not intend to use chemicals that are unsafe to the touch, but they may be unfit for human consumption. Accordingly, we will not try to eat the chemicals if they escape the sphere, and we will also provide a warning label to warn others to also not eat the chemicals within the sphere.

The most significant and likely safety issue is the threat of thermal burns. We expect the lava globe sphere to require a temperature of around 150°F to function correctly. These temperatures are enough to cause contact burns within a second of contact [10]. Therefore, we will do our best to not touch the sphere when it is operational and we will create a warning label to warn others to do the same.

Another issue related to thermal safety is that as the temperature inside of the sphere increases the pressure will also increase, thereby creating an explosion hazard. Our lava globe will have thermal control features to prevent the temperatures from rising above the 150°F operating temperature. We intend to make the temperature control circuit robust enough to ensure that the risk of explosion is negligible.

9 References

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[4] The Engineering Toolbox (2016, September 29). Convective Heat Transfer [Online]. Available: http://www.engineeringtoolbox.com/convective-heat-transfer-d_430.html

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[10] IEEE. (2016, September 20). 7.8 IEEE Code of Ethics [Online]. Available: http://www.ieee.org/about/corporate/governance/p7-8.html

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Appendix: Lava Lamp Control

System Model

We will model our system as a Cylindrical Lava Lamp and a Cylindrical Heat Source as seen in figure 1.

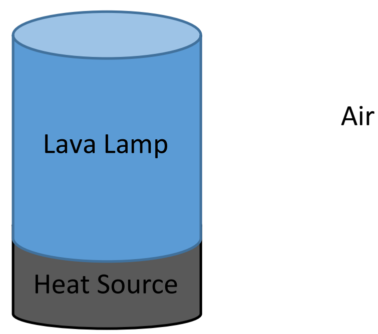


Figure 1: Physical Model of Lava Lamp System as two cylinders

To simplify our control of the system, we will consider each block in Figure 1 (Lava Lamp, Heat Source, and Air) as having a uniform internal temperature ($T_{Lava}, T_{Heat}, and\ T_{Air}\ repectively$).

Also, we will model the heat flow between blocks using equation 1.

$$q = \frac{1}{R}(T_1 - T_2), \tag{1}$$

Where

$q = heat\ energy\ flow, joules\ per\ second\ (\frac{J}{sec})$

$R = thermal\ resistance, \left(\frac{°C\ sec}{J}\right)$

$T = temperature, (°C)$

And the net heat-energy flow into a substance affects the temperature of the substance according to the relation in equation 2:

$$\dot{T} = \frac{1}{C}q, \tag{2}$$

Where

$C\ is\ the\ thermal\ capacity\ \left(\frac{J}{°C}\right),$

$q\ is\ the\ sum\ of\ heat\ flows\ obeying\ equation\ (1)$

$and\ \dot{T}\ is\ the\ rate\ of\ change\ of\ the\ temperature\ \left(\frac{°C}{sec}\right)$

Using equations (1) and (2) and our model we can derive the differential equations that determine the temperature in each block in figure 1. Note, that we will consider the “air” block to have an infinite thermal capacity, allowing it to exchange heat without changing its temperature.

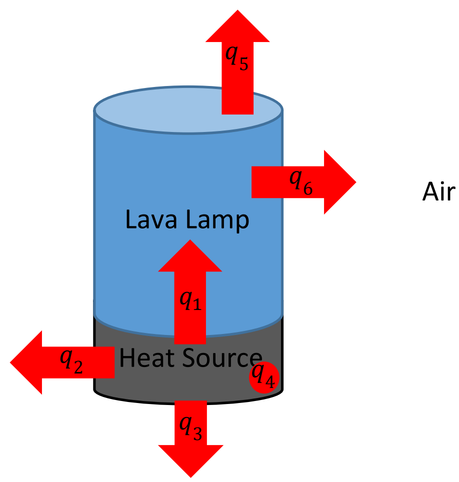


Figure 2: Heat Flow Illustrated on Lava Lamp System Model

In figure 2 you can see the heat flow between every surface defined. A positive heat flow will indicate the flow of thermal energy from the base of the arrow to the tip. The definitions are as follows

q_1: *From the Heat Source to the Lava Lamp*

q_2: *From the Heat Source to the Air (side of cylinder)*

q_3: *From the Heat Source to the Air (bottom of cylinder)*

q_4: *From the Power Resistor to the Heat Source*

q_5: *From the Lava Lamp (top of cylinder) to the Air*

q_6: *From the Lava Lamp (side of cylinder) to the Air*

So,

$$q_{Heat} = q_4 - q_1 - q_2 - q_3$$

$$q_{Lava} = q_1 - q_5 - q_6$$

And, if we assign a thermal Resistance R_i to each of the q_i and plug in eq. 1 (besides q_4 which we will assume we can control directly) we have

$$q_{Heat} = q_4 - \frac{1}{R_1}(T_{Heat} - T_{Lava}) - \frac{1}{R_2}(T_{Heat} - T_{Air}) - \frac{1}{R_3}(T_{Heat} - T_{Air})$$

$$q_{Lava} = \frac{1}{R_1}(T_{Heat} - T_{Lava}) - \frac{1}{R_5}(T_{Lava} - T_{Air}) - \frac{1}{R_6}(T_{Lava} - T_{Air})$$

And plugging the heat flow into equation 2 we have

$$\dot{T}_{Heat} = \frac{1}{C_{Heat}}\left[q_4 - \frac{1}{R_1}(T_{Heat} - T_{Lava}) - \frac{1}{R_2}(T_{Heat} - T_{Air}) - \frac{1}{R_3}(T_{Heat} - T_{Air})\right]$$

$$\dot{T}_{Lava} = \frac{1}{C_{Lava}}\left[\frac{1}{R_1}(T_{Heat} - T_{Lava}) - \frac{1}{R_5}(T_{Lava} - T_{Air}) - \frac{1}{R_6}(T_{Lava} - T_{Air})\right]$$

Since T_{Air} isn't changing, we can make all of our variables deviations from that point as follows:

$$\boxed{\tilde{T}_{Heat} = T_{Heat} - T_{Air}}$$

$$\boxed{\dot{\tilde{T}}_{Heat} = \dot{T}_{Heat}}$$

$$\boxed{\tilde{T}_{Lava} = T_{Lava} - T_{Air}}$$

$$\boxed{\dot{\tilde{T}}_{Lava} = \dot{T}_{Lava}}$$

$$\tilde{T}_{Heat} - \tilde{T}_{Lava} = (T_{Heat} - T_{Air}) - (T_{Lava} - T_{Air}) = T_{Heat} - T_{Lava} - T_{Air} + T_{Air}$$

$$\boxed{\tilde{T}_{Heat} - \tilde{T}_{Lava} = T_{Heat} - T_{Lava}}$$

So, substituting in the deviation versions we have

$$\dot{\tilde{T}}_{Heat} = \frac{1}{C_{Heat}}\left[q_4 - \frac{1}{R_1}(\tilde{T}_{Heat} - \tilde{T}_{Lava}) - \frac{1}{R_2}(\tilde{T}_{Heat}) - \frac{1}{R_3}(\tilde{T}_{Heat})\right]$$

$$\dot{\tilde{T}}_{Lava} = \frac{1}{C_{Lava}}\left[\frac{1}{R_1}(\tilde{T}_{Heat} - \tilde{T}_{Lava}) - \frac{1}{R_5}(\tilde{T}_{Lava}) - \frac{1}{R_6}(\tilde{T}_{Lava})\right]$$

Simplifying,

$$\boxed{\dot{\tilde{T}}_{Heat} = \left(\frac{-1}{R_{1,2,3}C_{Heat}}\right)\tilde{T}_{Heat} + \left(\frac{1}{R_1 C_{Heat}}\right)\tilde{T}_{Lava} + \left(\frac{1}{C_{Heat}}\right)q}$$

$$\boxed{\dot{\tilde{T}}_{Lava} = \left(\frac{1}{R_1 C_{Lava}}\right)\tilde{T}_{Heat} + \left(\frac{-1}{R_{1,5,6}C_{Lava}}\right)\tilde{T}_{Lava}}$$

Where

$$R_{1,2,3} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \quad and \quad R_{1,5,6} = \frac{R_1 R_5 R_6}{R_1 R_5 + R_1 R_6 + R_5 R_6} \; and \; q = q_4$$

Theoretical Estimation of Parameters

In order to simulate our model, we must first estimate the parameters of our model. Namely, we must estimate:

$$R_1, R_2, R_3, R_4, R_5, C_{Heat}, C_{Lava}$$

In our formulation of thermal resistance from equation 1, we never specified the method of heat transfer between our system blocks. We merely gave a constant of proportionality and assumed that the majority of the change in temperature would happen at the surfaces.

At this point it will be useful to extend our model by specifying dimensions. As we can see in figure 3, the dimensions of the lava lamp cylinder are (height H, diameter D), and of the heat source cylinder (height h, diameter h).

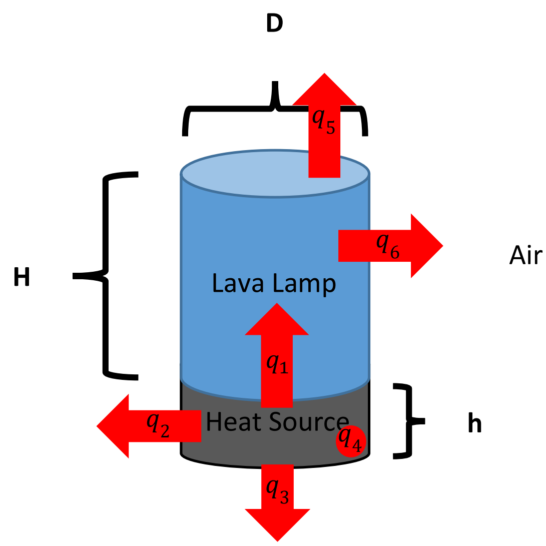


Figure 3: Heat Flows Illustrated on Model with Dimensions Specified

For the lava lamp and heat sources we have chosen the values are:

$$H = 0.69m, D = 0.115m, and\ h = 0.03m$$

To estimate these parameters, we must now consider what method of heat transfer is dominant and derive the thermal resistance from the appropriate equations.

For solid to fluid interfaces, convection will be dominant. This will apply for $R_2, R_3, R_4, and\ R_5$. A convective heat transfer coefficient can describe this interaction, which would be equivalent to our original formulation through:

$$R = \left(\frac{1}{hA}\right)$$

Where

$$R\ is\ the\ thermal\ resistance\ as\ described\ in\ equation\ 1$$

$$h\ is\ the\ convective\ heat\ transfer\ coefficient$$

$$A\ is\ the\ Area$$

The calculation of the convective heat transfer coefficient involves a large number of equations, is very complicated, and is out of the scope of this design. The actual thermal resistances will be obtained through experimentation so this theoretical value only needs to be somewhat reasonable. According to a few sources [4][5][6] a good estimate for the convective heat transfer coefficient of un-forced air is

$$h_{Air} = 0.5\ \left(\frac{W}{m^2K}\right)$$

Therefore, by calculating the Areas of each of the surfaces we can compute the respective thermal resistances

$$R_2 = \left(\frac{1}{h_{Air}A_2}\right) = \left(\frac{1}{h_{Air}(\pi D h)}\right) = \left(\frac{1}{\left(0.5\frac{W}{m^2K}\right)(\pi)(0.115m)(0.03m)}\right)$$

$$\boxed{R_2 = 184.5\left(\frac{K}{W}\right)}$$

$$R_3 = \left(\frac{1}{h_{Air}A_3}\right) = \left(\frac{1}{h_{Air}\left(\pi(\frac{D}{2})^2\right)}\right) = \left(\frac{1}{\left(0.5\frac{W}{m^2K}\right)(\pi)(\frac{0.115m}{2})^2}\right)$$

$$\boxed{R_3 = 192.6\left(\frac{K}{W}\right)}$$

$$R_5 = \left(\frac{1}{h_{Air}A_5}\right) = \left(\frac{1}{h_{Air}\left(\pi(\frac{D}{2})^2\right)}\right) = \left(\frac{1}{\left(0.5\frac{W}{m^2K}\right)(\pi)(\frac{0.115m}{2})^2}\right)$$

$$\boxed{R_5 = 192.6\left(\frac{K}{W}\right)}$$

$$R_6 = \left(\frac{1}{h_{Air}A_6}\right) = \left(\frac{1}{h_{Air}(\pi DH)}\right) = \left(\frac{1}{\left(0.5\frac{W}{m^2K}\right)(\pi)(0.115m)(0.69m)}\right)$$

$$\boxed{R_6 = 8.022\left(\frac{K}{W}\right)}$$

For solid to solid interfaces conduction will be dominant. This will apply for R_1. We can model this resistance with this equation

$$R = \frac{l}{kA}$$

Where

$k\ is\ the\ thermal\ conductivity$

$A\ is\ the\ cross-sectional\ area$

$and\ l\ is\ the\ length\ of\ the\ heat\ flow\ path.$

We initially assumed that the lava lamp was one entity with one temperature, and the heat source was another entity with another temperature. However, when considering the effects of conduction, it will be useful to consider the lava lamp to be a material encased in glass since the glass will be the dominant resistance (can be verified by looking at the thermal conductivities).

$$k_{glass} = 1\left(\frac{W}{mK}\right),\ \ k_{Aluminum} = 237\ \left(\frac{W}{mK}\right)$$

Therefore, our assumption that the heat source will be the same temperature is very good, and we need to model the heat flow from the surface of the heat source, through the glass of the lava lamp.

And lastly, if we assume that the bottom of the glass is 1 cm thick we can calculate an estimate for the resistance.

$$R_1 = \frac{l}{k_{glass}A_1} = \frac{l}{k\left(\pi(\frac{D}{2})^2\right)} = \frac{(0.01m)}{\left(1\frac{W}{mK}\right)\left(\pi(\frac{0.115m}{2})^2\right)}$$

$$\boxed{R_1 = 0.9628\left(\frac{K}{W}\right)}$$

Now, we need to calculate the thermal capacities. Given the mass and specific heat of one of our blocks we can achieve this

$$C = mc_v$$

Where

$$C\ is\ the\ thermal\ capacity$$

$$m\ is\ the\ mass$$

$$c_v\ is\ the\ specific\ heat\ at\ constant\ volume$$

If we are considering the Lava Lamp to be mostly water, and the heat source to be mostly aluminum then, the relevant specific heats are:

$$c_{water} = 4.184\left(\frac{J}{g°\mathrm{C}}\right)$$

$$c_{Al} = 0.9\left(\frac{J}{g°\mathrm{C}}\right)$$

However, to obtain the mass we need to obtain the volume and densities first

$$m = \rho V$$

$$\rho_{water} = 1\left(\frac{g}{cm^3}\right)$$

$$\rho_{Al} = 2.7$$

$$V_{Lava} = \pi\left(\frac{D}{2}\right)^2 H = \pi\left(\frac{0.115m}{2}\right)^2 (.69m)\left(\frac{10^6 cm^3}{m^3}\right)$$

$$V_{Heat} = \pi\left(\frac{D}{2}\right)^2 h = \pi\left(\frac{0.115m}{2}\right)^2 (.03m)\left(\frac{10^6 cm^3}{m^3}\right)$$

Finally, we can obtain the thermal capacities

$$C_{Lava} = m_{Lava}c_{water} = \rho_{Water}V_{Lava}c_{water} = \rho_{Water}\pi\left(\frac{D}{2}\right)^2 Hc_{water}$$

$$= \left(1\frac{g}{cm^3}\right)\pi\left(\frac{0.115m}{2}\right)^2 (.69m)\left(\frac{10^6 cm^3}{m^3}\right)\left(4.184\frac{J}{g°\mathrm{C}}\right)$$

$$\boxed{C_{Lava} = 29{,}990\left(\frac{J}{°\mathrm{C}}\right)}$$

$$C_{Heat} = m_{Heat}c_{Al} = \rho_{Al}V_{Heat}c_{Al} = \rho_{Al}\pi\left(\frac{D}{2}\right)^2 hc_{Al}$$

$$= \left(2.7\frac{g}{cm^3}\right)\pi\left(\frac{0.115m}{2}\right)^2 (.03m)\left(\frac{10^6 cm^3}{m^3}\right)\left(0.9\frac{J}{g℃}\right)$$

$$\boxed{C_{Heat} = 757.2\left(\frac{J}{℃}\right)}$$

Controller Design and Simulation

From the previous section we have the differential equations that govern our system. We can transform this into a state space representation for easier analysis of the form

$$\dot{x} = Ax + Bu$$

For our system we have:

$$\begin{bmatrix} \dot{\tilde{T}}_{Heat} \\ \dot{\tilde{T}}_{Lava} \end{bmatrix} = \begin{bmatrix} \left(\frac{-1}{R_{1,2,3}C_{Heat}}\right) & \left(\frac{1}{R_1 C_{Heat}}\right) \\ \left(\frac{1}{R_1 C_{Lava}}\right) & \left(\frac{-1}{R_{1,5,6}C_{Lava}}\right) \end{bmatrix} \begin{bmatrix} \tilde{T}_{Heat} \\ \tilde{T}_{Lava} \end{bmatrix} + \begin{bmatrix} \left(\frac{1}{C_{Heat}}\right) \\ 0 \end{bmatrix} q$$

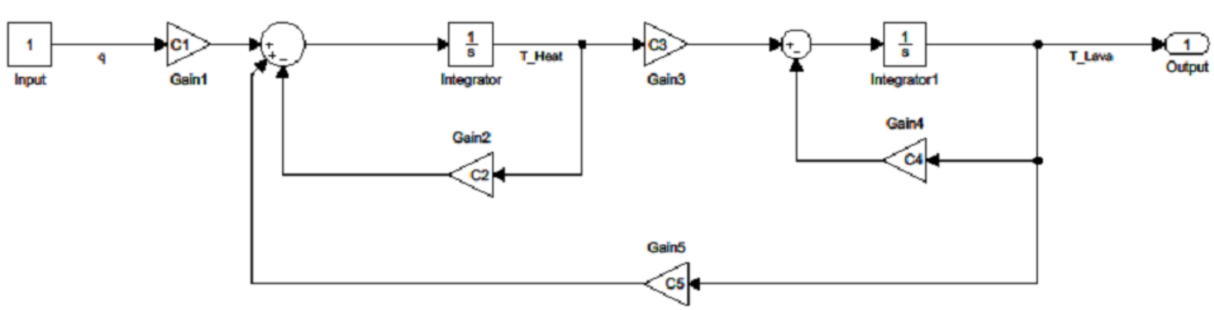
The system matrix A contains information about the stability of the system. The eigenvalues of this matrix are the poles in the complex plane. If these poles are all in the Left Hand Plane (real part less than 0), then we can say our system is stable. So for our system, the eigenvalues and thus poles are:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -0.00142001 \\ -0.0000045667 \end{bmatrix}$$

Therefore, we can say that the open-loop system is stable.

Also, from the state space representation we can easily obtain the system block diagram.

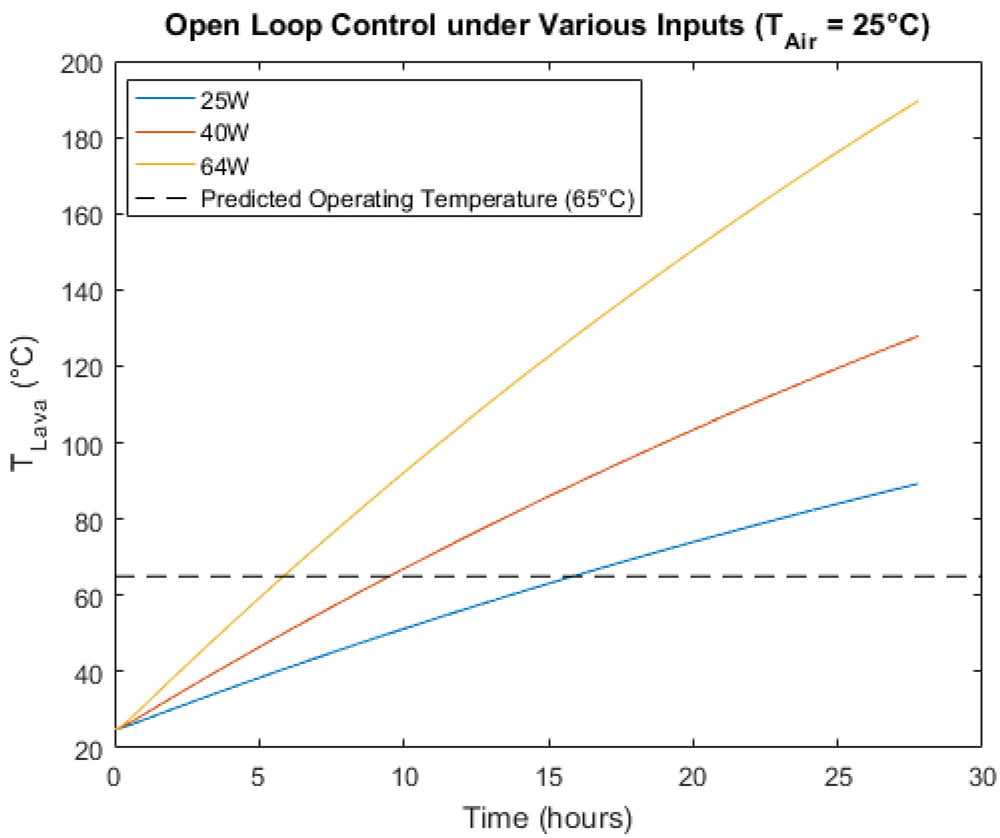
System Block Diagram



Where

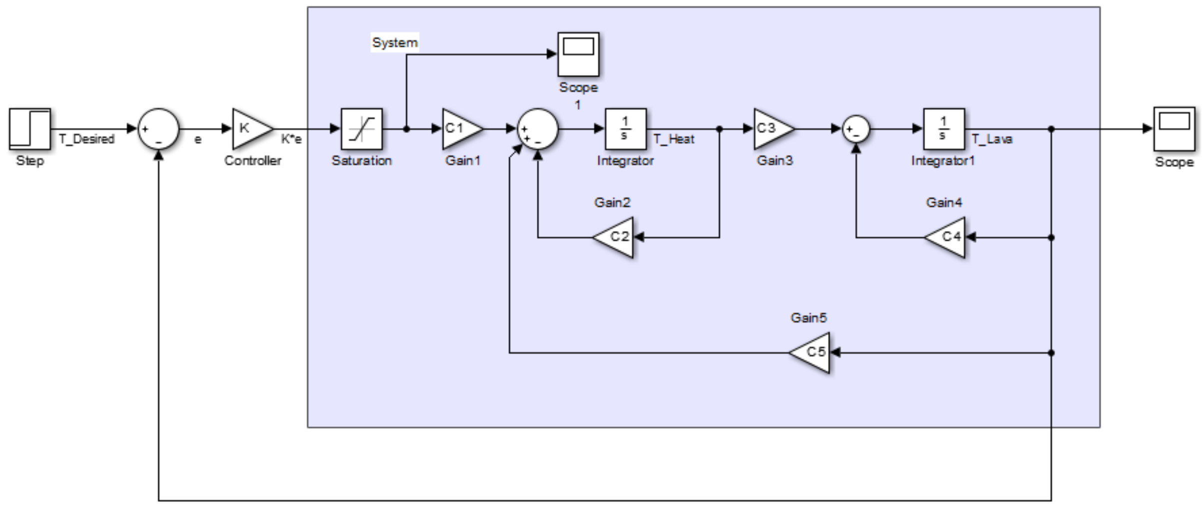
$$C_1 = \left(\frac{1}{C_{Heat}}\right), C_2 = \left(\frac{1}{R_{1,2,3}C_{Heat}}\right), C_3 = \left(\frac{1}{R_1 C_{Lava}}\right), C_4 = \left(\frac{1}{R_{1,5,6}C_{Lava}}\right), C_5 = \left(\frac{1}{R_1 C_{Heat}}\right)$$

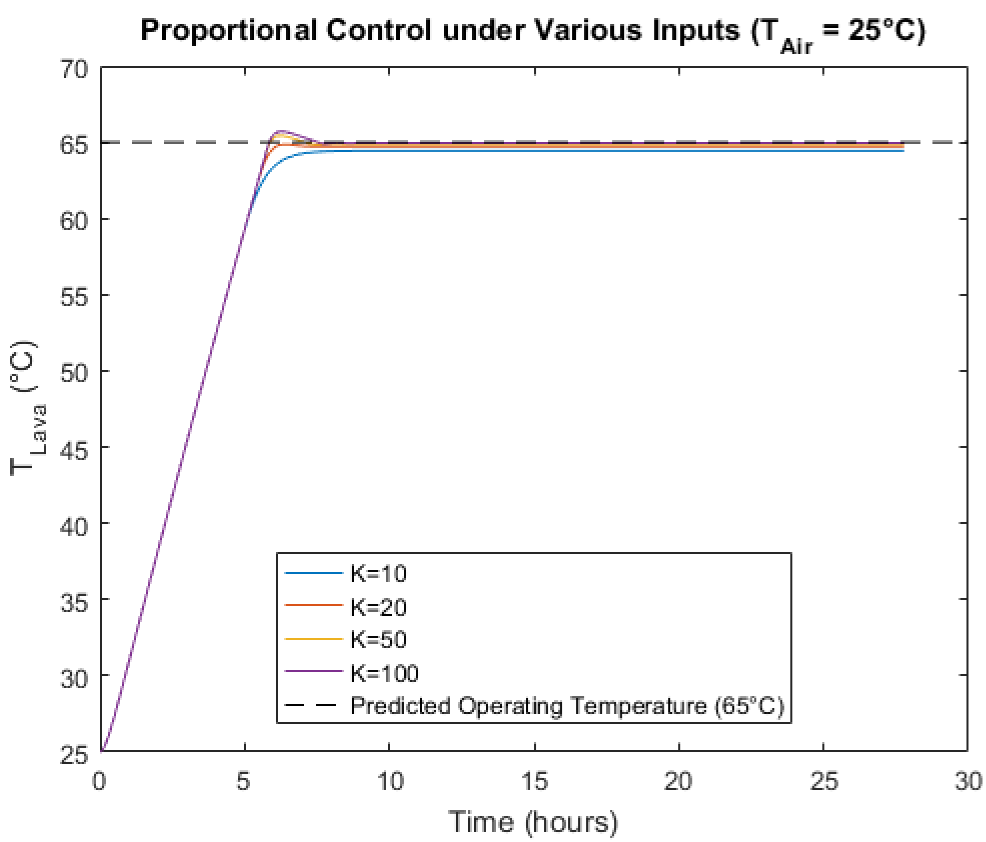
A traditional lava lamp would at this point give a constant control input to the system and perform open-loop control. In the figure below we can see the results from our Simulink simulation with the theoretical parameters from the previous section. The input power was chosen to reflect two common power levels for lava lamp light bulbs (25W and 40W) as well as our maximum power output (64W).



Although the parameters may not be completely accurate and our model is not perfect, we can see the intrinsic problem with this control scheme. Using a large input power allows the system to get to the operating temperature faster, however, it also causes the system to get to dangerous levels faster. Using a small input power would get to the dangerous levels slower, but it would also get to the operating temperature at a slower rate.

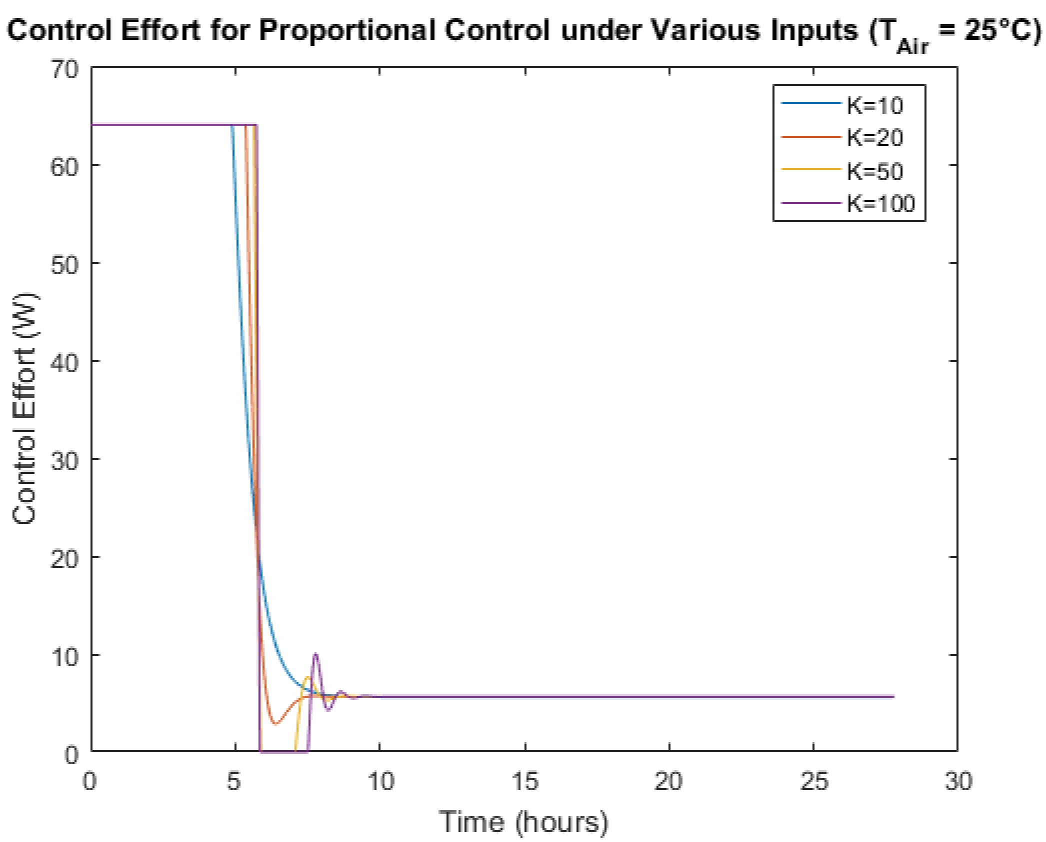
We propose a simple proportional feedback controller that addresses both of these problems. In the figure below, we can see the new block diagram with an error and gain added. We then simulated this controller using various gains to see the effects.





In the figure above we can see the results. The first thing to notice is that as K increases the amount of damping decreases. Ideally, the system is critically damped and there is no overshoot. When K is 20, it appears to be very close to this. The second thing to see is the steady state error in the previous plot. The values for the errors are listed in the table below. As K increases the error decreases. So, to design our controller there is a trade-off between steady state error and overshoot. Also, in the second figure below, we can see the control effort. Because it is impossible in our system to input a negative heat, we can see the controller flat-line at 0 for the underdamped values. Based on these plots we decided to assign the theoretical value as $\boxed{K = 20}$. However, the specific parameters of our real system will most likely change this value of K. In the next section we will describe how to obtain the parameters experimentally, and then later we would re-design our controller using the same methodology.

K	**Steady State Error (℃)**
10	0.5592
20	0.2816
50	0.1131
100	0.0566



Estimating Parameters Experimentally

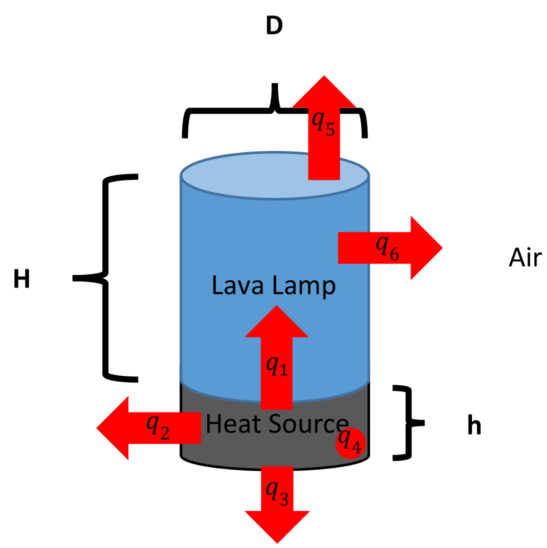


Figure 4: System Model with Heat Flows

We have the system equations as derived above.

$$\boxed{\dot{\tilde{T}}_{Heat} = \left(\frac{-1}{R_{1,2,3}C_{Heat}}\right)\tilde{T}_{Heat} + \left(\frac{1}{R_1 C_{Heat}}\right)\tilde{T}_{Lava} + \left(\frac{1}{C_{Heat}}\right)q}$$

$$\boxed{\dot{\tilde{T}}_{Lava} = \left(\frac{1}{R_1 C_{Lava}}\right)\tilde{T}_{Heat} + \left(\frac{-1}{R_{1,5,6}C_{Lava}}\right)\tilde{T}_{Lava}}$$

However, the individual terms within the coefficients are not directly observable by measuring the states. Therefore, we have:

$$\dot{\tilde{T}}_{Heat} = X\tilde{T}_{Heat} + Y\tilde{T}_{Lava} + Zq$$

$$\dot{\tilde{T}}_{Lava} = U\tilde{T}_{Heat} + V\tilde{T}_{Lava}$$

Where

$$X, Y, Z, U, and\ V\ are\ constants$$

Now, we can construct an experiment to solve for these 5 unknowns. And later we will see that we can even separate the coefficients into their individual terms. And lastly, we will add on a method to associate operating temperatures with Lava Lamp Globule Behavior.

Experiment Description

Suppose that we were to attach temperature sensors to the Heat Source and Lava Lamp, and also measure the heat going into the Heat Source (q) by measuring the current and voltage across the resistor. We could then heat up the system from the air temperature all the way up to its max operating point, taking data points along the way.

We could numerically determine $\dot{\tilde{T}}_{Heat}\ and\ \dot{\tilde{T}}_{Lava}$ using the finite difference method, for example of the first order:

$$f'(a) \approx \frac{f(a+h) - f(a)}{h}$$

But, we could always use a higher order and more accurate approximation.

Now that we are able to measure all of our states, our two equations would now each become an overdetermined system of the form:

$$Ax = b$$

Where

$$x\ is\ a\ vector\ containing\ all\ of\ the\ variables$$

$$(\#\ rows\ of\ A) > (\#\ of\ columns\ of\ A)$$

For our 2 equations the systems are as follows:

$$\begin{bmatrix} \tilde{T}_{Heat,1} & \tilde{T}_{Lava,1} & q_1 \\ & \cdots & \\ & \cdots & \\ \tilde{T}_{Heat,n} & \tilde{T}_{Lava,n} & q_n \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \dot{\tilde{T}}_{Heat,1} \\ \cdots \\ \cdots \\ \dot{\tilde{T}}_{Heat,n} \end{bmatrix}$$

And also

$$\begin{bmatrix} \tilde{T}_{Heat,1} & \tilde{T}_{Lava,1} \\ \cdots & \\ \cdots & \\ \tilde{T}_{Heat,n} & \tilde{T}_{Lava,n} \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} \dot{\tilde{T}}_{Lava,1} \\ \cdots \\ \cdots \\ \dot{\tilde{T}}_{Lava,n} \end{bmatrix}$$

In the general case, there is no exact solution. However, using the Least Squares method we can find a solution that minimizes the error, specifically:

$$\min_x ||Ax - b|| \tag{17}$$

For simplicity, we can then use a MATLAB function to solve for the coefficients of each of our equations using the Least Squares Method.

Obtaining actual parameters from Coefficients

Now that we have the coefficients we need to obtain the original parameters. Below are the coefficients as they were implicitly described above for the first equation.

$$X = \left(\frac{-1}{R_{1,2,3}}\right), \qquad Y = \left(\frac{1}{R_1 C_{Heat}}\right), \qquad and \qquad Z = \left(\frac{1}{C_{Heat}}\right)$$

We can write each of our parameters as a function of only the derived coefficients.

$$\boxed{R_{1,2,3} = \left(\frac{-1}{X}\right)}$$

$$\boxed{C_{Heat} = \left(\frac{1}{Z}\right)}$$

$$R_1 = \left(\frac{1}{Y C_{Heat}}\right)$$

$$\boxed{R_1 = \left(\frac{Z}{Y}\right)}$$

From the second equation we have:

$$U = \left(\frac{1}{R_1 C_{Lava}}\right) \quad and \quad V = \left(\frac{-1}{R_{1,5,6} C_{Lava}}\right)$$

And again we can solve for the rest of the parameters

$$C_{Lava} = \left(\frac{1}{R_1 U}\right)$$

$$\boxed{C_{Lava} = \left(\frac{Y}{ZU}\right)}$$

$$R_{1,5,6} = \left(\frac{-1}{V C_{Lava}}\right)$$

$$\boxed{R_{1,5,6} = \left(\frac{ZU}{VY}\right)}$$

Associating Lava Lamp Behavior with Operating Temperature

In the above experiment, we were able to obtain the parameters for our simplified model. However, we never considered the actual behavior of the lava lamp globules inside of the Lamp block. We were just assuming that there was an operating temperature we needed to keep the system at. We are going to maintain this assumption but we will still need to obtain this operating temperature.

Suppose we added a camera to our test setup. We could then take pictures of the globules, timestamp them, and also do the same for our temperature measurements.

After the experiment we could evaluate the image data to obtain the first frame and the last frame that we consider is at operating temperature visually. Then we would be able to read the temperature of these points and have an association between the visual effects and the temperature. Using this information, we could set our desired temperature for our control loop.