

ECE/CS 439  
Homework 1: Math Basics  
Due 27 September 2017

**1 Write True/False with a brief justification (around 1 sentence).  
Make assumptions where necessary and state them clearly: (6 X 5  
= 30)**

**Q1**

A is a  $m \times n$  matrix with  $m < n$ . The null space  $N(A)$  is always 0.

**Q2**

The matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  when left multiplied with a second matrix  $B$  subtracts twice of the first column of  $B$  from the second column of  $B$ .

**Q3**

For an orthonormal matrix  $Q$  (Columns of  $Q$  are orthogonal to each other and the length of each column is 1),  $Q^{-1} = Q^T$

**Q4**

A signal  $S1$  is composed of frequencies  $f_1, f_2, f_3$  and  $S2$  consists of frequencies  $f_2, f_3, f_4$ . Then, a signal  $S3 = S1 \star S2$  (Convolution of  $S1$  and  $S2$ ), would consist of union of these frequencies i.e.,  $f_1, f_2, f_3, f_4$

**Q5**

If  $b_1, b_2, b_3$  form the basis of a space, then  $c_1b_1 + c_2b_2 + c_3b_3 = 0$  implies that all  $c_1, c_2$  and  $c_3$  are zero.

**Q6**

The length of the vector  $\frac{V}{\|V\|}$  is 0 ( $\|V\|$  is the  $l_2$  norm of  $V$ ).

**2 Symmetric matrices**

**(5)**

Prove that  $A^T A$  is a symmetric matrix

### 3 Column spaces (10)

Choose  $b$  which gives no solution and another  $b$  which gives infinitely many solutions. What are two of those solutions?

$$3x + 2y = 10 \tag{1}$$

$$6x + 4y = b \tag{2}$$

### 4 Least Squares (10)

Consider the following system of equations.

$$x - y = 2 \tag{3}$$

$$x + y = 4 \tag{4}$$

$$2x + y = 8 \tag{5}$$

How many solutions exist for the above system of equations? If a solution exists find one, if not, determine the least squares solution for  $x$  and  $y$ .

### 5 FFT (10)

Compute the  $4 \times 4$  FFT matrix  $W$ . Use  $W$  to transform  $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$  into the frequency domain. Also, find the

inverse of  $W$  and use it to transform  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$  into the time domain.

### 6 Interference Nulling (10)

Figure 1(a) shows a setup in which your neighbor AP  $A$  is communicating with her laptop  $B$ . Both the AP and laptop use 2 antennas to communicate using  $2 \times 2$  MIMO. The received signals at two antennas of  $B$  can be represented as  $Y^{ab} = H^{ab} X^a$ , where  $Y^{ab}, H^{ab}$  and  $X^a$  are expanded below

$$\begin{bmatrix} y_1^{ab} \\ y_2^{ab} \end{bmatrix} = \begin{bmatrix} h_{11}^{ab} & h_{21}^{ab} \\ h_{12}^{ab} & h_{22}^{ab} \end{bmatrix} \begin{bmatrix} x_1^a \\ x_2^a \end{bmatrix} \tag{6}$$

$x_1^a, x_2^a$  are transmit signals from antennas 1 and 2 of  $A$  respectively.  $y_1^{ab}, y_2^{ab}$  are received signals at antennas 1 and 2 of  $B$ .  $h_{ij}^{ab}$  indicate channels between various transmit-receive antenna pairs as shown in Figure1(a). Under ideal conditions,  $B$  can decode two simultaneous streams from  $A$ .

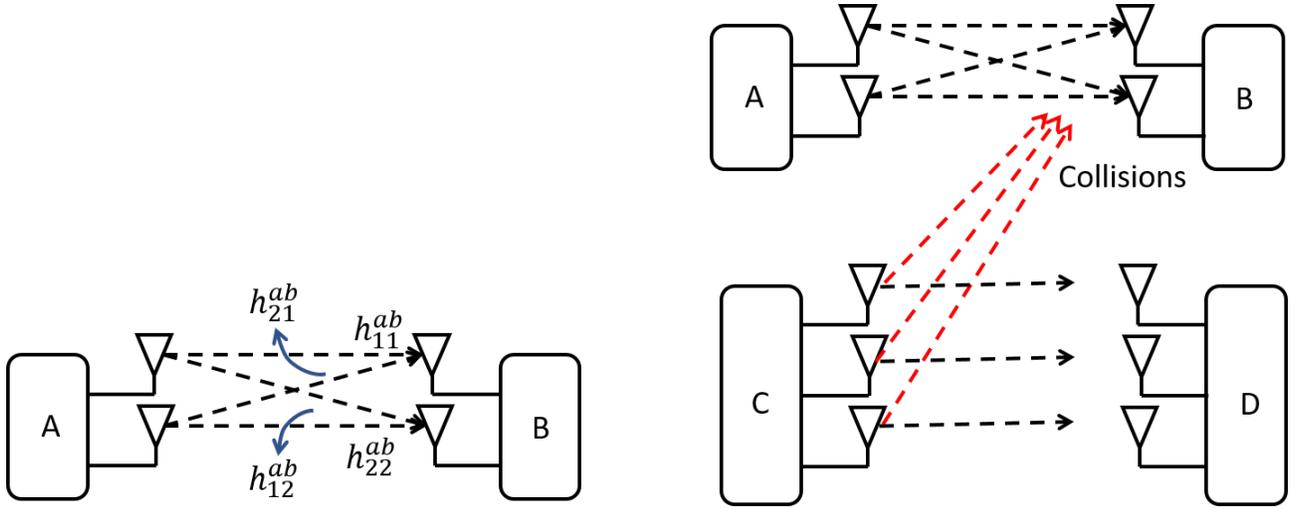


Figure 1: Interference Nulling (a) 2x2 MIMO communication (b) 3x3 MIMO communication interferes with 2x2 MIMO

Now consider your own AP,  $C$  communicating with your laptop  $D$  as in Figure 1(b) using 3x3 MIMO. However, your communication would interfere with your neighbor's 2x2 MIMO system. The interference at the neighbor laptop  $B$  due to  $C$  can be represented as  $Y^{cb} = H^{cb}X^c$ , where  $Y^{cb}$ ,  $H^{cb}$  and  $X^c$  are expanded below.

$$\begin{bmatrix} y_1^{cb} \\ y_2^{cb} \end{bmatrix} = \begin{bmatrix} h_{11}^{cb} & h_{21}^{cb} & h_{31}^{cb} \\ h_{12}^{cb} & h_{22}^{cb} & h_{32}^{cb} \end{bmatrix} \begin{bmatrix} x_1^c \\ x_2^c \\ x_3^c \end{bmatrix} \quad (7)$$

$Y^{cb}$  is the interference at the two antennas of  $B$  and  $X^c$  is the transmitted signals from the three antennas of  $C$ . If the channel matrix  $H^{cb}$  is defined as follows,

$$H^{cb} = \begin{bmatrix} 2 & 3 & 7 \\ 1 & 8 & 9 \end{bmatrix} \quad (8)$$

Determine the null space  $X_{null}^c$  of  $H^{cb}$  such that if  $C$  transmits in the null space,  $H^{cb}X_{null}^c = 0$ . Thus,  $C$  can completely eliminate its interference at  $B$ . If  $C$  transmits in its null space, how many simultaneous streams of data can it communicate to  $D$ .

## 7 Multipath Convolution (15)

Consider a wireless transmitter and receiver placed as shown in Figure 2. Two copies of the transmitted signal arrive at the receiver. A direct path 1 and a reflected path 2. The Channel Impulse Response (CIR)  $h$  of a such a system can be represented as  $[0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ . Here, the CIR is represented in the granularity of the sampling interval of 0.1 seconds. This indicates that the direct path arrives at a delay of 0.2 seconds whereas the reflected path arrives at a delay of 0.4 seconds.

For any transmitted signal  $x$ , the receiver would receive a convolved signal  $y = x \star h$ . Suppose a sine wave is transmitted at frequency  $2.5Hz$ . Convolve a 16 sample long (sampling interval of 0.1 seconds)

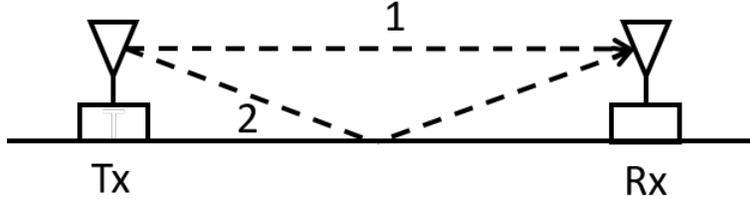


Figure 2: Multipath Channel

sine wave of frequency  $2.5Hz$  with the impulse response and observe the output. Now perform convolution with a sine wave of frequency  $1.875Hz$  and observe the output. To maximize the average amplitude of reception, which of the above two frequencies would you transmit a signal at? Now, compute the FFT of the impulse response and observe the frequency response at the above two frequencies. Based on the FFT, what frequency would provide the best amplitude for reception. (Hint: You need to know the frequency of each point on an FFT output in  $Hz$ )

## 8 MIMO Channel Matrix (10)

A list of  $n \times n$  MIMO channel matrices are given below. Determine which of the following channels allow decoding all  $n$  streams of data in parallel

$$\begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 3 \\ 6 & 5 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -13 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 3 & 6 & 9 \\ 8 & 2 & 4 & 6 \\ 4 & 1 & 2 & 3 \\ 20 & 5 & 10 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 4 & 11 & 6 \\ 8 & 7 & 29 & -9 \\ 4 & 9 & 42 & -21 \\ 20 & 10 & 35 & 0 \end{bmatrix}$$