Homework 2

CS425/ECE428 Spring 2019

Due: Wednesday, Feb 20 at 11:55 p.m.

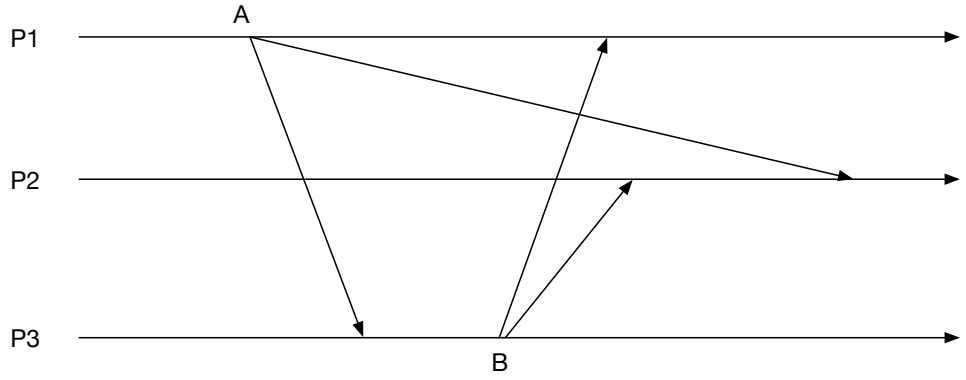
1. For this question, consider a distributed system where each pair of processes is connected by a first-in, first-out (FIFO) reliable channel. Assume that there are no process failures.

   (a) (4 points) In this system, if the basic multicast scheme (`B-multicast`) is used, what ordering guarantees would be satisfied? FIFO, causal, and/or total? Justify your answer: either argue why a guarantee *would* be satisfied, or present a counterexample.

   Assume that a process immediately delivers a message to itself before sending it out to any other processes.

   Solution: Messages would be FIFO: if p_i multicasts m and then m', then at every p_j, m will arrive on the channel from p_i to p_j before m'.

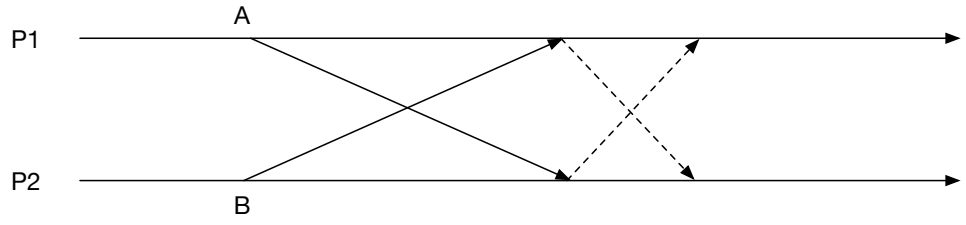
   Neither causal nor total ordering would *not* be satisfied, see counterexample below. $A \rightarrow B$ but P2 delivers B before A.



   (b) (4 points) What if the reliable multicast scheme (`R-multicast`) were used instead? Assume again that the process immediately delivers a message to itself before sending it out.

   Solution: Causal ordering would be satisfied. When p_i sends message X to p_j (either because p_i initiated the message or during a rebroadcast), any message already delivered by p_i before X will have already been sent from p_i to p_j.

   Total ordering would *not* be satisfied, see counterexample below. The dashed lines represent the rebroadcast of messages A and B by the receiver. (The implementation of R-multicast on the slides re-multicasts the received message to all other, including the sender, though a real implementation could optimize this out.)



   (c) (2 points) Do your answers for the above two parts change in a system with only two processes?

Solution: For B-multicast, with 2 processes the ordering is causal. Consider two events A and B with $A \rightarrow B$. Assume without loss of generality that A was sent by $P1$. If B was also sent by $P1$ then $P2$ must deliver A before B due to FIFO ordering. If B was sent by $P2$, then B must follow the delivery of A by $P2$. (If A is delivered after B is sent, then any messages sent by $P1$ after A are also delivered after B is sent, which would make A and B concurrent.) In both cases, A is delivered before B by both $P1$ and $P2$.

B-multicast is not total following a counterexample similar to the previous part.

R-multicast is causal but not total, since the arguments and counterexample in part (b) apply in a system with only 2 processes.

2. (a) (3 points) Consider a synchronous system with $N = 100$ processes running the synchronous consensus protocol. When sending a message, each value $v_i = z$ is encoded using two bytes. Assuming there are no failures, how many total bytes are sent by the system over five rounds of the protocol, starting from round 1?

Solution: Each process in the 1st round multicasts only its own value, therefore each process will send $2(N-1)$ bytes in the first round. Assuming a synchronous system with no failure, every process will receive and communicate all the values in the next rounds which means $2N(N-1)$ bytes will be communicated by each process. Therefore, in 5 rounds there will be $2(N-1) + 4 \times 2N(N-1)$ communicated bytes by each process in total. Since $N = 100$, total number of bytes will be $N[2(N-1) + 4 \times 2N(N-1)] = 7,939,800$.

Assuming that we are also interested in counting the messages that each process sends to itself the result changes to $N[2N + 4 \times 2N^2] = 8,020,000$.

(b) (3 points) Consider a synchronous system where processes can only fail in pairs, so that in any round, an even number of proceses fail (e.g., 0, 2, 4, etc.) How many rounds would you need to ensure consensus among $N = 100$ processes if there is no bound on the number of failures?

Solution: After a failure-free round, the synchronous consensus algorithm ensures that all processes have the same set of values and can therefore terminate. If we run the protocol for 50 rounds, then either there has been at least one failure-free round, in which case we can terminate the protocol, or all processes have failed, in which case the protocol has terminated anyway. So therefore 50 rounds are sufficient. (You can verify that 49 rounds are *not* sufficient since it is possible that two processes with remain with different values.)

3. (3 points) Consider the following sequence of events, using the model from the Fisher-Lynch-Patterson proof:

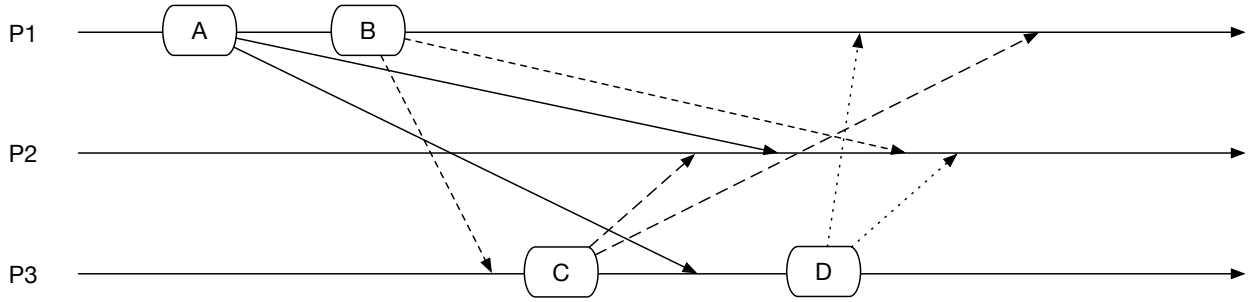
$e_1 @ p_1$	$\mathsf{receive}(m_1), \mathsf{send}(p_2, m_2), \mathsf{send}(p_3, m_3)$
$e_2 @ p_2$	$\mathsf{receive}(m_2), \mathsf{send}(p_1, m_4)$
$e_3 @ p_3$	$\mathsf{receive}(m_3), \mathsf{send}(p_2, m_5), \mathsf{send}(p_1, m_6)$
$e_4 @ p_1$	$\mathsf{receive}(m_4), \mathsf{send}(p_3, m_7)$
$e_5 @ p_1$	$\mathsf{receive}(m_6), \mathsf{send}(p_2, m_8)$
$e_6 @ p_2$	$\mathsf{receive}(m_8)$

Which events can commute with e_2?

Solution: e_3 and e_5.

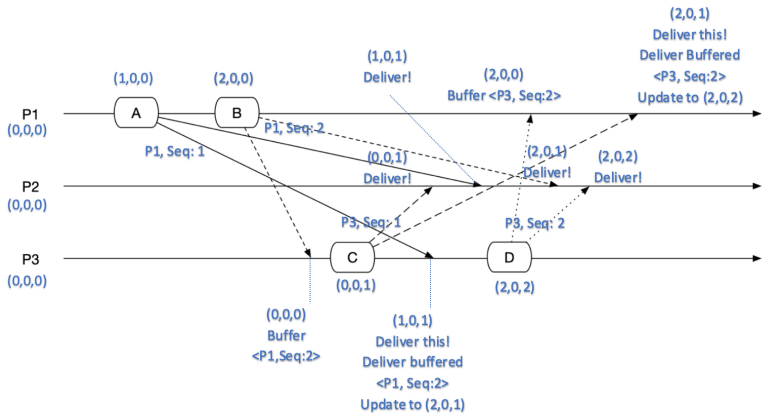
e_1 sends a message that e_2 receives and e_4 receives a message that e_2 sends. e_6 happens on the same process as e_2.

4. For this question, consider the diagram below. It shows four messages, A, B, C, D, being multicast by processes $P1$ and $P3$.

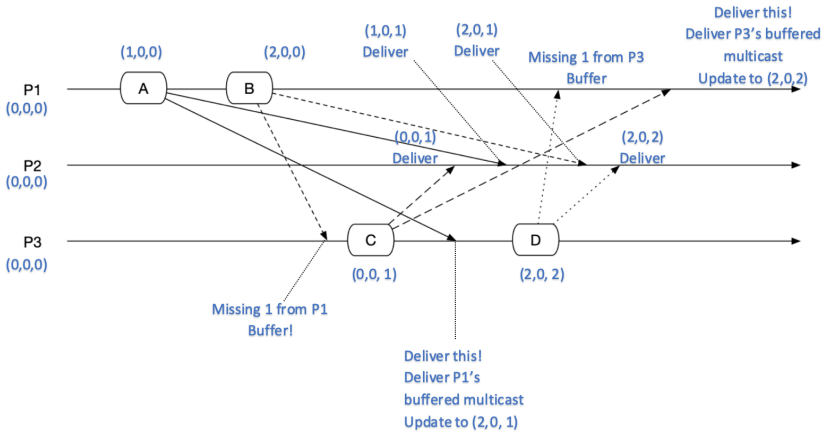


(a) (6 points) Consider a FIFO multicast implementation using sequence numbers, as discussed in class. Write down the sequence number vector for each send and receive event at each process, and the sequence number sent with each message. Also indicate when each message will be delivered at each process. (Do not worry about processes delivering their own message, e.g., $P1$ delivering A.)

Solution:



(b) (5 points) Repeat the exercise with a causal multicast implementation. (Instead of a sequence number for each message you will include its sequence vector).


P1
(0,0,0)
P2
(0,0,0)
P3
(0,0,0)
A
B
C
D
(1,0,0)
(2,0,0)
(1,0,1)
Deliver
(2,0,1)
Deliver
Missing 1 from P3
Buffer
Deliver this!
Deliver P3's buffered
multicast
Update to (2,0,2)
(0,0,1)
Deliver
(2,0,2)
Deliver
(0,0, 1)
(2,0, 2)
Missing 1 from P1
Buffer!
Deliver this!
Deliver P1's
buffered multicast
Update to (2,0, 1)

Solution: