## Homework 1

CS425/ECE428 Spring 2019

## Due: Thursday, Feb 7 at 11:55 p.m.

1. (a) (3 points) Consider an asynchronous distributed system with unbounded message delay (though guaranteed message delivery), and perfectly synchronized clocks. $p_{i}$ and $p_{j}$ implement a heartbeat protocol, where $p_{j}$ sends a heartbeat every $T$ time units, and $p_{i}$ declares $p_{j}$ as failed when it does not hear a heartbeat for $T+\Delta$ time units. What is the worst-case detection time for $p_{j}$ having failed? (Hint: it will depend on the number of heartbeats sent.)

Solution: After $n$ heartbeats, the worst-case detection time is $(n+1) \Delta+T$.
Suppose the first heartbeat is sent at time $T ; p_{i}$ will wait for this heartbeat until time $T+\Delta$. If it arrives at exactly this time, $p_{i}$ will wait for the next heartbeat, which will be sent at time $T$, until time $2 T+2 \Delta$. Therefore, the $k$-th heartbeat, sent at time $k T$, can arrive as late as $k(T+\Delta)$ without $p_{i}$ declaring $p_{j}$ as having failed. If a failure occurs right after the $n$-th heartbeat is sent at $n T, p_{i}$ can receive that heartbea at $n(T+\Delta)$ and then wait until $(n+1)(T+\Delta)$ before declaring $p_{j}$ as failed. $(n+1)(T+\Delta)-n T$ gives us the worst-case detection time.
If we assume the first heartbeat is sent at time $t=0$ this value is $(n+1) \Delta+2 T$.
(b) (2 points) Can you suggest a modification to the protocol to make the worst-case detection time bounded regardless of the number of heartbeats?

Solution: The delays will be accumulated in the scenario of (a). Instead of resetting the wait period every time $p_{i}$ hears from $p_{j}$, it should expect a heartbeat every $T$ time units within a timeout of $\Delta$. That is, $p_{i}$ declares $p_{j}$ as failed when it does not hear a heartbeat within $k T$ and $k T+\Delta$ time units, when $k \in \mathbb{N}$. In this case, the worst-case detection time is $T+\Delta$.

Other answers may be accepted as long as it does not make modifications on the problem setting or hurt the accuracy too much.
(c) (4 points) Consider $N$ processes using a ring ping-ack protocol. (I.e., $p_{i}$ sends a ping to $p_{i+1}$ who sends an acknowledgment back.) Pings are sent every $T$ time units, and a timeout is set to $\Delta$. Assume again we are in an asynchronous network, and let the probability that a round-trip time exceeds $\Delta$ be $p$.
What is the probability that at least one alive process will be declared as having failed within a time period $T$ ? Calculate the value for $p=0.01$ and $N=100$.

Solution: 0.6340 .
$P$ [at least one alive processes declared as failed $]=1-P$ all alive processes are safe $]=1-(1-$ $0.01)^{100} \approx 0.6340$.

What if we declare failure only after $k$ missed acks?
Solution: $1-\left(1-(0.01)^{k}\right)^{100}$.
The protocol makes false decisions for $k$ times on the same process.
(d) (2 points) Consider a ping-ack protocol in a synchronous network with a minimum one-way delay
of 10 ms and a maximum delay of 100 ms . Let $T=1 \mathrm{~s}$ and assume no processing delays and perfectly synchronized clocks.
What should your timeout value be? What is the maximum detection time?
Solution: 200 ms .1190 ms .

The timeout should be the travel time of a complete ping-ack process in the worst case.

The maximum detection time happens when the pinged process fails immediately after it sends the last ack. It is as early as 10 ms after the last ping message is sent. The detection time is $1000-10+200=1190 \mathrm{~ms}$.
2. (a) (6 points) Consider a hierarchical NTP synchronization between four processes, A, B, C, and D as shown in Figure 1a. Each arrow is labeled with the round-trip delay betwen two processes. What is the bound on the clock skew between every pair of processes?

Solution: Given the accuracy of NTP prediction (with no drift): $o=o_{i} \pm \frac{d_{i}}{2}$ the bound on the clock skew between every pair of the nodes will be (in $m s$ ):

|  | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| A | - | 25 | 25.5 | 26 |
| B | - | - | 0.5 | 1 |
| C | - | - | - | 1.5 |
| D | - | - | - | - |

Note $d_{i}=R T T$. Also note that we are summing up the skew between each level because we are measuring the bounds (i.e. worst case scenario).
(b) (4 points) Consider a maximum drift rate of $0.01 \%$. How frequent should each synchronization be to ensure that no two processes have a skew larger than 50 ms ?

Solution: A non-zero drift will modify the bound to:

$$
o=o_{i} \pm\left(\frac{d_{i}}{2}+\alpha \Delta T\right)
$$

where $\alpha$ is the maximum drift rate and $\Delta T$ is the delay between each sync. For each pair, the min sync frequency will be:

$$
\begin{align*}
\frac{d_{i}}{2}+\alpha \Delta T \leq 50 & \Rightarrow \alpha \Delta T \leq 50-\frac{d_{i}}{2}  \tag{1}\\
& \Rightarrow \Delta T \leq \frac{1}{\alpha}\left(50-\frac{d_{i}}{2}\right)
\end{align*}
$$

Therefore, the maximum sync time between nodes should be:

$$
\begin{aligned}
& \text { (A) }- \text { B } \leq \frac{1}{0.0001} *(50-25)=250000 \mathrm{~ms} \\
& \text { (B) }- \text { C } \leq \frac{1}{0.0001} *(50-0.5)=495000 \mathrm{~ms} \\
& \text { (B) }- \text { D } \leq \frac{1}{0.0001} *(50-1)=490000 \mathrm{~ms}
\end{aligned}
$$



Figure 1: NTP configuration for Question 2
(c) (2 points) Suppose that there was a flat hierarchy instead where B, C, and D all synchronized with A, with an RTT of 50 ms , as shown in Figure 1b. What would be the clock skew between every pair of processes?

Solution: Similar to part(a), the maximum clock skew between every pair of processes will be equal to RTT therefore it will be 25 ms between A and every other node and 50 ms for all the other pairs.
3. (a) (5 points) Looking at fig. 2, write down the Lamport timestamp of each event.

Solution: Check fig. 3
(b) (5 points) Using fig. 2, write down the vector timestamp for each event

Solution: Check fig. 4
(c) (5 points) List all the concurrent events

Solution: Concurrent Events are: $A \|\{E, F, G, J, K, M, N, O\}$
$B \|\{F, G, J, K, M, N, O\}$
$C \|\{F, G, J, K\}$
$D \|\{F, G, J, K\}$
$E \|\{A, I, M\}$
$F \|\{A, B, C, D, J, K, L, M, N, O, P\}$
$G \|\{A, B, C, D, J, K, L, P\}$

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H|{J,K,L,P}
J|{A,B,C,D,F,G,H,P}
K|{A,B,C,D,F,G,H,P}
L|{F,G,H,P}
M|{A,B,E,F}
N|{A,B,F}
O|{A,B,F}
P|{F,G,H,J,K,L}
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4. (a) (4 points) Looking at fig. 2, suppose that P1 initiates the Chandy-Lamport snapshot algorithm at time 9. Write down all possible consistent cuts that the resulting snapshot could capture. (You can describe each cut by its frontier events.)

Solution: Since P1 initiates the snapshot at time 9, it writes down $B$ as the latest event.
P 2 must receive its marker from P 1 before H due to FIFO channels. It may receive its first marker from P1 (or another process) before or after G, so the frontier would include F or G.
Likewise, P1's marker message to P3 must arrive before L, but P3 could receive its first marker before J , between J and K , or between K and L .
Finally P 4 must receive its first marker before P but after O .
So the set of possible frontiers is $B \times\{F, G\} \times\{I, J, K\} \times O$.
(b) (4 points) Pick an example of a distributed system; feel free to use some of the ones previously mentioned in class. Give one example of each of the following in that system:

- A stable property
- An unstable property
- A liveness property
- A safety property

You may use the same system for all properties or several different ones.
Solution: We offer a few examples here; obviously there are many possible correct answers.
Stable property: this video has been viewed at least 100 times; this video has finished uploading; all correct processes have delivered message $m$.
Unstable property: this video has been viewed exactly 100 times; this account has 100 friends; some process has not yet delivered $m$
Liveness property: all of this account's friends will eventually see a post; a lock will be released Safety property: a restricted video will only be shown to people who are allowed; a video will never have a negative number of views; a message will be delivered at most once


Figure 2: Timeline for questions 3 and 4.


Figure 3: Lamport Timestamp for questions 3.


Figure 4: Vector Clock for questions 3.

