2021sep07

Tuesday, March 24, 2020 16:02

$$\begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 5-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 1 = 0$$

$$10 - 2\lambda - 5\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 7\lambda + 9 = 0$$

$$\lambda = \frac{7 \pm \sqrt{49 - 36}}{2} = \frac{7}{2} \pm \frac{\sqrt{13}}{2}$$

$$\left(\frac{7 \pm \sqrt{13}}{2}\right) \begin{bmatrix} 1 \\ u \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ u \end{bmatrix}$$

$$\begin{matrix} \lambda \\ \lambda u \end{matrix} = \begin{bmatrix} 5+u \\ 1+2u \end{bmatrix}$$

$$\lambda = 5 + u \qquad \vec{v}_1 = \begin{bmatrix} 1 \\ \lambda - 5 \end{bmatrix}$$

$$u = \lambda - 5$$

$$\lambda_1 = \frac{7 \pm \sqrt{13}}{2}$$

$$\lambda u = 1 + 2u \qquad \|v\| = \sqrt{1 + (\lambda - 5)^2}$$

$$a_1 = \frac{1}{\sqrt{1 + (\lambda_1 - 5)^2}}$$

$$a_2 = \frac{1}{\sqrt{1 + (\lambda_2 - 5)^2}}$$

$$\Lambda = V^T A V$$

$$= \begin{bmatrix} a_1 & a_1(\lambda_1 - 5) \\ a_2 & a_2(\lambda_2 - 5) \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a_1 & a_2 \\ a_1\lambda_1 - 5a_1 & a_2\lambda_2 - 5a_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_1(\lambda_1 - 5) \\ a_2 & a_2(\lambda_2 - 5) \end{bmatrix} \begin{bmatrix} a_1 5 + a_1\lambda_1 - 5a_1 & a_1 + 2a_1\lambda_1 - 10a_1 \\ 5a_2 + a_2\lambda_2 - 5a_2 & a_2 + 2a_2\lambda_2 - 10a_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1^2\lambda_1 + a_1a_2\lambda_2 - a_1a_2(\lambda_1 - 5) & \\ a_1a_2\lambda_1 + a_2^2\lambda_2 - a_1a_2(\lambda_2 - 5) & \end{bmatrix}$$

$$V\Lambda V^T = A$$

$$\begin{bmatrix} a_1 & a_1 \\ a_1(\lambda_1 - 5) & a_2(\lambda_2 - 5) \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} a_1 & a_1(\lambda_1 - 5) \\ a_2 & a_2(\lambda_2 - 5) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}$$