Lecture 1: Intro; DSP Review

Mark Hasegawa-Johnson

ECE 417: Signal and Image Analysis, Fall 2021

Outline

Intro: What is Signal Processing?

- Linear Time Invariant systems: Use Fourier analysis
  - Advantages: analytic solutions, instantaneously, no training
  - Disadvantages: only optimal for LTI systems
- Nonlinear & Time-Varying systems: Use machine learning
  - Advantages: learns the optimal solution for any problem
  - Disadvantages: training takes time, often fails; analytic solutions provide only loose upper bounds

Where can I learn more?

There are many excellent conferences and publications related to particular areas in signal processing. There is also one really excellent conference and one really excellent magazine dedicated specifically to signal processing:

- ICASSP (International Conference on Acoustics, Speech and Signal Processing)
- IEEE Signal Processing Magazine

Outline

1. Intro: Multimedia Signal Processing
2. Overview: Course Policies and Administration
3. Review: Fourier Transforms
4. Review: Poles and Zeros
5. Summary
6. Other Possibly Useful Review Videos

Overview: Course Topics

There are six MPs, covering
$\{\text{speech}, \text{video}\} \times \{\text{DSP methods}, \text{Bayesian methods}, \text{ML methods}\}$:

- DSP methods: MP1 (speech synthesis), MP2 (video synthesis)
- Bayesian methods: MP3 (image recognition), MP4 (speech recognition)
- Neural networks: MP5 (image recognition), MP6 (voice conversion)

Overview: Course Policies and Administration

About here, visit the course webpage.
Notice: if you haven't had a DSP course yet, you might want to consider taking ECE 401 instead of this course.

Outline

Review: Fourier Transforms

About here, watch the movie called fourier_review.mov.

Outline

Z Transform

You might remember that the Z-transform is, basically, a funny way to write the DTFT. The DTFT is

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

. . . and the Z transform is

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

There are some limits on the values of z for which $H(z)$ is finite. Let's talk about those.

FIR and IIR Filters

- An **FIR** (finite impulse response) filter is one whose impulse response lasts a finite amount of time. Any such filter can be written as

$$y[n] = \sum_{m=0}^{M-1} b_m x[n-m]$$

- An **IIR** (infinite impulse response) filter is one whose impulse response lasts an infinite amount of time. We will be most interested in IIR filters that can be written as

$$\sum_{m=0}^{N-1} a_n y[n-m] = \sum_{m=0}^{M-1} b_m x[n-m]$$

First-Order Feedback-Only Filter

Let's find the general form of $h[n]$, for the simplest possible IIR filter: a filter with one feedback term, and no feedforward terms, like this:

$$y[n] = x[n] + ay[n-1],$$

where a is any constant (positive, negative, real, or complex).

Impulse Response of a First-Order Filter

We can find the impulse response by putting in $x[n] = \delta[n]$, and getting out $y[n] = h[n]$:

$$h[n] = \delta[n] + ah[n-1].$$

Recursive computation gives

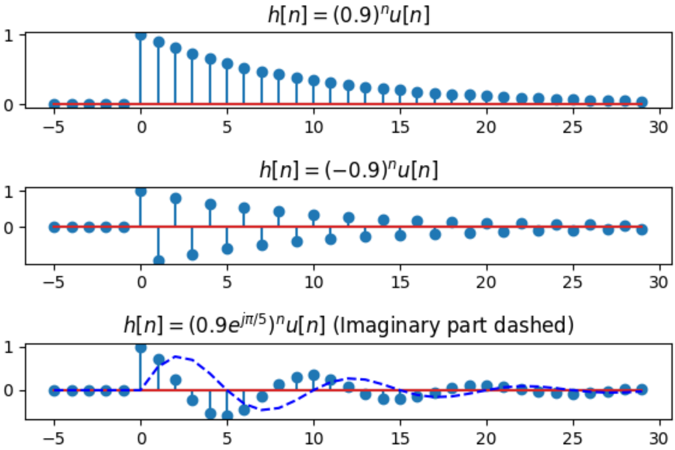
$$\begin{aligned} h[0] &= 1 \\ h[1] &= a \\ h[2] &= a^2 \\ &\vdots \\ h[n] &= a^n u[n] \end{aligned}$$

where we use the notation $u[n]$ to mean the "unit step function,"

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

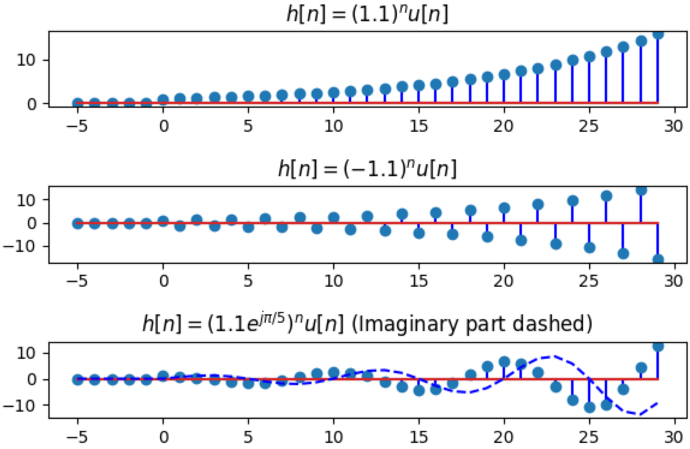
Impulse Response of Stable First-Order Filters

The coefficient, a, can be positive, negative, or even complex. If a is complex, then $h[n]$ is also complex-valued.



Impulse Response of Unstable First-Order Filters

If $|a| > 1$, then the impulse response grows exponentially. If $|a| = 1$, then the impulse response never dies away. In either case, we say the filter is "unstable."



Instability

- A **stable** filter is one that always generates finite outputs ($|y[n]|$ finite) for every possible finite input ($|x[n]|$ finite).
- An **unstable** filter is one that, at least sometimes, generates infinite outputs, even if the input is finite.
- A first-order IIR filter is stable if and only if $|a| < 1$.

Transfer Function of a First-Order Filter

We can find the transfer function by taking the Z-transform of each term in this equation:

$$y[n] = x[n] + ay[n-1].$$

Using the transform pair $y[n-1] \leftrightarrow z^{-1}Y(z)$, we get

$$Y(z) = X(z) + az^{-1}Y(z),$$

which we can solve to get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - az^{-1}}$$

Frequency Response of a First-Order Filter

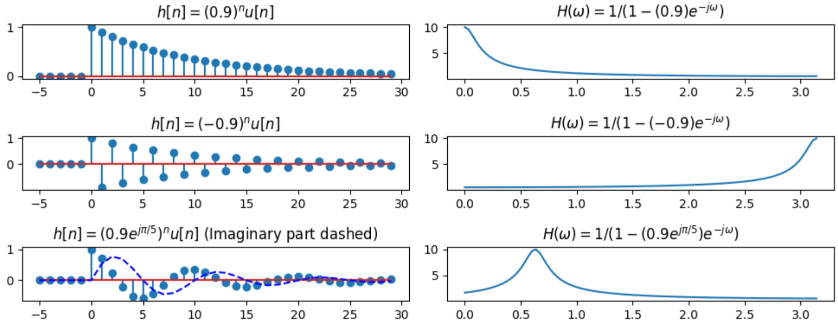
If the filter is stable ($|a| < 1$), then we can find the frequency response by plugging in $z = e^{j\omega}$:

$$H(\omega) = H(z)|_{z=e^{j\omega}} = \frac{1}{1 - ae^{-j\omega}} \quad \text{if } |a| < 1$$

This formula only works if $|a| < 1$.

Frequency Response of a First-Order Filter

$$H(\omega) = \frac{1}{1 - ae^{-j\omega}} \quad \text{if } |a| < 1$$



Transfer Function ↔ Impulse Response

Notice that $H(z)$ is actually the Z-transform of $h[n]$. We can prove that as follows:

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$
$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

This is a standard geometric series, with a ratio of az^{-1}. As long as $|a| < 1$, we can use the formula for an infinite-length geometric series, which is:

$$H(z) = \frac{1}{1 - az^{-1}},$$

So we confirm that $h[n] \leftrightarrow H(z)$ for both FIR and IIR filters, as long as $|a| < 1$.

First-Order Filter

Now, let's find the transfer function of a general first-order filter, including BOTH feedforward and feedback delays:

$$y[n] = x[n] + bx[n-1] + ay[n-1],$$

where we'll assume that $|a| < 1$, so the filter is stable.

Transfer Function of a First-Order Filter

We can find the transfer function by taking the Z-transform of each term in this equation equation:

$$y[n] = x[n] + bx[n-1] + ay[n-1],$$
$$Y(z) = X(z) + bz^{-1}X(z) + az^{-1}Y(z),$$

which we can solve to get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + bz^{-1}}{1 - az^{-1}}.$$

Treating $H(z)$ as a Ratio of Two Polynomials

Notice that $H(z)$ is the ratio of two polynomials:

$$H(z) = \frac{1 + bz^{-1}}{1 - az^{-1}} = \frac{z + b}{z - a}$$

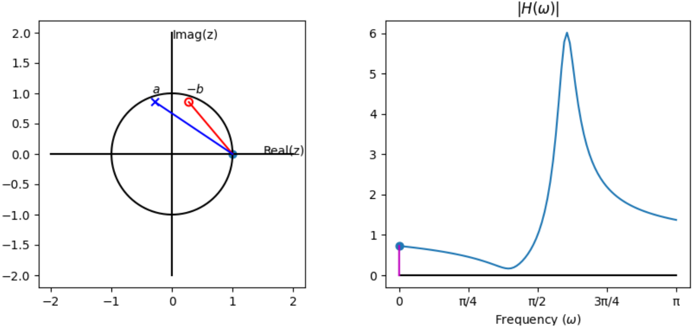
- $z = -b$ is called the **zero** of $H(z)$, meaning that $H(-b) = 0$.
- $z = a$ is called the **pole** of $H(z)$, meaning that $H(a) = \infty$

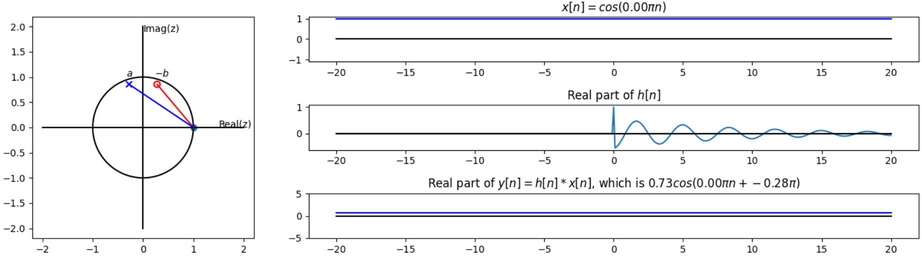
The Pole and Zero of $H(z)$

- The pole, $z = a$, and zero, $z = -b$, are the values of z for which $H(z) = \infty$ and $H(z) = 0$, respectively.
- But what does that mean? We know that for $z = e^{j\omega}$, $H(z)$ is just the frequency response:

$$H(\omega) = H(z)|_{z=e^{j\omega}}$$

  but the pole and zero do not normally have unit magnitude.
- What it means is that:
  - When $\omega = \angle(-b)$, then $|H(\omega)|$ is as close to a zero as it can possibly get, so at that that frequency, $|H(\omega)|$ is as low as it can get.
  - When $\omega = \angle a$, then $|H(\omega)|$ is as close to a pole as it can possibly get, so at that that frequency, $|H(\omega)|$ is as high as it can get.


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Imag(z)
Real(z)
a
−b
−2
−1
0
1
2
|H(ω)|
6
5
4
3
2
1
0
0
π/4
π/2
3π/4
π
Frequency (ω)


Imag(z)
Real(z)
a
−b
x[n] = cos(0.00πn)
Real part of h[n]
Real part of y[n] = h[n] * x[n], which is 0.73cos(0.00πn + −0.28π)

Vectors in the Complex Plane

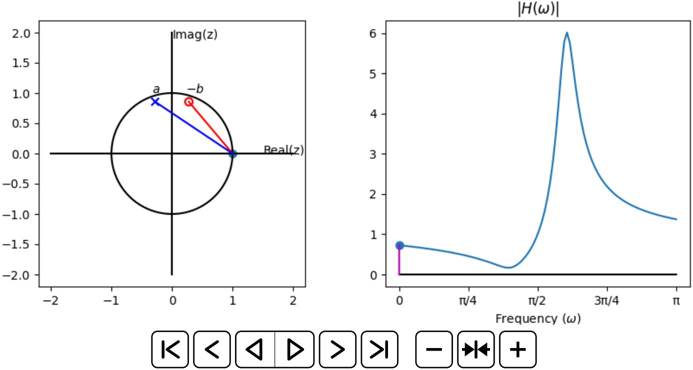
Suppose we write $|H(z)|$ like this:

$$|H(z)| = \frac{|z+b|}{|z-a|}$$

Now let's evaluate at $z = e^{j\omega}$:

$$|H(\omega)| = \frac{|e^{j\omega}+b|}{|e^{j\omega}-a|}$$

What we've discovered is that $|H(\omega)|$ is small when the vector distance $|e^{j\omega}+b|$ is small, but LARGE when the vector distance $|e^{j\omega}-a|$ is small.


Imag(z)
Real(z)
a
−b
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2
−1
0
1
2
|H(ω)|
6
5
4
3
2
1
0
0
π/4
π/2
3π/4
π
Frequency (ω)

Why This is Useful

Now we have another way of thinking about frequency response.

- Instead of just LPF, HPF, or BPF, we can design a filter to have zeros at particular frequencies, $\angle(-b)$, AND to have poles at particular frequencies, $\angle a$,
- The magnitude $|H(\omega)|$ is $|e^{j\omega} + b|/|e^{j\omega} - a|$.
- Using this trick, we can design filters that have much more subtle frequency responses than just an ideal LPF, BPF, or HPF.

Outline

Summary: DSP Review

In summary, you should remember how to do these things:

- Fourier Series, CTFT, DTFT, and DFT.
- Z-transform.
- Z-transform and frequency-response of FIR and IIR filters.

Outline

Other Possibly Useful Review Videos

Here are some other videos that might be useful.

- This one, I think, is a review of material from ECE 310, so I kind of expect you to know it: filtering review
- These three provide more details about noise, and about speech. This material is optional; if it helps you understand the LPC material, that's great, but if not, you can ignore it.
  - Noise.
  - Speech.
  - Windwed Speech.