Problem 6.1

Suppose you have a recurrent neural network with input $x[n]$, target $y[n]$, output $h[n]$, and loss function:

$$
\mathcal{L} = -\frac{1}{N} \sum_{n=0}^{N-1} (y[n] \ln h[n] + (1 - y[n]) \ln(1 - h[n]))
$$

where

$$
h[n] = \sigma(\xi[n]),
\xi[n] = x[n] + \sum_{m=1}^{M-1} w[m] h[n - m],
$$

and where $\sigma(\cdot)$ is the logistic sigmoid. Write $d\mathcal{L}/dw[3]$ in terms of the signals $y[n]$ and $h[m]$. You can invent partial back-prop signals if you wish, but you need to define them clearly. You may assume that $h[n] = 0$ for $n < 0$.

Problem 6.2

Suppose that

$$
h_0 = x^3
h_1 = \cos(x) + \sin(h_0)
\hat{y} = \frac{1}{2} (h_1^2 + h_0^2)
$$

What is $d\hat{y}/dx$? Express your answer as a function of $x$ only, without the variables $h_0$ or $h_1$ in your answer.

Problem 6.3

Consider a one-gate recurrent neural net, defined as follows:

$$
c[n] = c[n - 1] + w_c x[n] + u_c h[n - 1] + b_c
h[n] = o[n] c[n]
o[n] = \sigma(w_o x[n] + u_o h[n - 1] + b_o)
$$
where $\sigma(\cdot)$ is the logistic sigmoid, $x[n]$ is the network input, $c[n]$ is the cell, $o[n]$ is the output gate, and $h[n]$ is the output. Suppose that you’ve already completed synchronous back-prop, which has given you the following quantity:

$$\epsilon[n] = \frac{\partial L}{\partial h[n]}$$

Find asynchronous back-prop formulas, i.e., find formulas for the following quantities, in terms of one another, and/or in terms of the other quantities defined above:

$$\delta_h[n] = \frac{dL}{dh[n]}$$
$$\delta_o[n] = \frac{dL}{do[n]}$$
$$\delta_c[n] = \frac{dL}{dc[n]}$$

**Problem 6.4**

Using the CReLU nonlinearity for both $\sigma_h$ and $\sigma_g$ in an LSTM, choose weights and biases,

$$\{b_c, u_c, w_c, b_f, u_f, w_f, b_i, u_i, w_i, b_o, u_o, w_o\},$$

that will cause an LSTM to count the number of nonzero inputs, and output the tally only when the input is zero:

$$h[n] = \begin{cases} 
\sum_{m=0}^{n} 1[x[m] \geq 1] & x[n] = 0 \\
0 & x[n] \geq 1 
\end{cases}$$

where $1[\cdot]$ is the unit indicator function, and you may assume that $x[n]$ is always a non-negative integer.