Problem 2.1

Suppose that you have a zero-mean unit-variance Gaussian random signal, \( x[n] \), whose samples are perfectly periodic \( (x[n + P] = x[n] \) for all \( n \)), but are otherwise completely unpredictable \( (x[n + k] \) and \( x[n] \) are independent for \( 1 \leq k < P \)). What is the expected autocorrelation of this signal?

Solution:
\[
E[r_{xx}[n]] = \begin{cases} 
1 & n = \ell P \text{ for any integer } \ell \\
0 & \text{otherwise}
\end{cases}
\]

Problem 2.2

Suppose that \( y[n] = x[n] * h[n] \), where \( x[n] \) is zero-mean Gaussian white noise with variance \( \sigma^2 \), and \( h[n] = a^n u[n] \) for some real constant \( 0 < a < 1 \). What is \( E[r_{yy}[n]] \), the autocorrelation of \( y[n] \)?

Solution:
\[
E[r_{yy}[n]] = \frac{a^{|n|}}{1 - a^2}
\]

Problem 2.3

Suppose that \( y[n] = x[n] * h[n] \), where \( x[n] \) is zero-mean Gaussian white noise with variance \( \sigma^2 \), and \( h[n] = a^n u[n] \) for some real constant \( 0 < a < 1 \). What is \( E[R_{yy}(|\omega|)] \), the power spectrum of \( y[n] \)? Would you call \( y[n] \) pink noise, green noise, blue noise, or white noise?

Solution:
\[
E[R_{xx}(|\omega|)] = \frac{1}{|1 - ae^{-j\omega}|^2}
\]
This is a lowpass power spectrum, so we’d call it pink noise.

Problem 2.4
Suppose that $x[n]$ is a zero-mean Gaussian noise signal with the following DTFT power spectrum:

$$E[R_{xx}(\omega)] = \begin{cases} \sigma^2 & |\omega| < \frac{\pi}{3} \\ 0 & \frac{\pi}{3} < |\omega| < \pi \end{cases}$$

What is the expected autocorrelation, $E[r_{xx}[n]]$?

**Solution:**

$$E[r_{xx}[n]] = \frac{\sigma^2 \sin(\pi n/3)}{\pi n}$$