ECE 417 Multimedia Signal Processing
Homework 2

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Tuesday, 9/1/2020; Due: Monday, 9/14/2020
Reading: **Rabiner, "On the use of autocorrelation analysis for pitch detection"**

Problem 2.1

Suppose that you have a zero-mean unit-variance Gaussian random signal, $x[n]$, whose samples are perfectly periodic ($x[n+P]=x[n]$ for all n), but are otherwise completely unpredictable ($x[n+k]$ and $x[n]$ are independent for $1 \le k < P$). What is the expected autocorrelation of this signal?

Solution:

$$E\left[r_{xx}[n]\right] = \begin{cases} 1 & n = \ell P \text{ for any integer } \ell \\ 0 & \text{otherwise} \end{cases}$$

Problem 2.2

Suppose that $y[n] = x[n] * h[n]$, where $x[n]$ is zero-mean Gaussian white noise with variance σ^2, and $h[n] = a^n u[n]$ for some real constant $0 < a < 1$. What is $E\left[r_{yy}[n]\right]$, the autocorrelation of $y[n]$?

Solution:

$$E\left[r_{yy}[n]\right] = \frac{a^{|n|}}{1-a^2}$$

Problem 2.3

Suppose that $y[n] = x[n] * h[n]$, where $x[n]$ is zero-mean Gaussian white noise with variance σ^2, and $h[n] = a^n u[n]$ for some real constant $0 < a < 1$. What is $E\left[R_{yy}[\omega]\right]$, the power spectrum of $y[n]$? Would you call $y[n]$ pink noise, green noise, blue noise, or white noise?

Solution:

$$E\left[R_{xx}(\omega)\right] = \frac{1}{|1-ae^{-j\omega}|^2}$$

This is a lowpass power spectrum, so we'd call it pink noise.

Problem 2.4

Suppose that $x[n]$ is a zero-mean Gaussian noise signal with the following DTFT power spectrum:

$$E\left[R_{xx}(\omega)\right] = \begin{cases} \sigma^2 & |\omega| < \frac{\pi}{3} \\ 0 & \frac{\pi}{3} < |\omega| < \pi \end{cases}$$

What is the expected autocorrelation, $E\left[r_{xx}[n]\right]$?

Solution:

$$E\left[r_{xx}[n]\right] = \frac{\sigma^2 \sin(\pi n/3)}{\pi n}$$