ECE 417 Multimedia Signal Processing
Homework 1

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Tuesday, 8/25/2020; Due: Monday, 8/31/2020
Reading: **Strang, Section 6.1** and **Gallager, pp. 33-34, 36, 39-43, 45**

Problem 1.1

Suppose that $x[n]$ is the following time-shifted rectangle function:

$$x[n] = u[n-15] - u[n-31] \tag{1.1-1}$$

Find $X(\omega)$.

Solution:

$$X(\omega) = e^{-j15\omega}\left(\frac{1-e^{-j16\omega}}{1-e^{-j\omega}}\right) = e^{-j22.5\omega}\left(\frac{\sin(8\omega)}{\sin(\omega/2)}\right)$$

Problem 1.2

Suppose that $\vec{x} = [x_1, x_2]^T$ is a Gaussian random vector, with mean vector $\vec{\mu}$ and covariance matrix Σ given by:

$$\vec{\mu} = \left[\begin{array}{c} 2 \\ 5 \end{array}\right], \quad \Sigma = \left[\begin{array}{cc} 9 & 0 \\ 0 & 4 \end{array}\right] \tag{1.2-1}$$

Remember that the standard normal CDF is defined to be:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt \tag{1.2-2}$$

In terms of $\Phi(z)$, find $\Pr\{x_1 > 4\}$, the probability of the event that x_1 is greater than 4.

Solution:

$$\Pr\{x_1 > 4\} = 1 - \Pr\{x_1 \le 4\} = 1 - \Phi\left(\frac{4-2}{3}\right)$$

Problem 1.3

Let A be a 2×2 matrix, and let x be one of its elements. All of its other elements are known, and are given as:

$$A = \left[\begin{array}{cc} x & 3 \\ -1 & 2 \end{array}\right] \tag{1.3-1}$$

The eigenvalues of A are given by

$$\lambda = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \tag{1.3-2}$$

for some particular values of a, b, and c. Find a, b, and c, in terms of x, such that Equation (1.3-2) gives the eigenvalues of A.

Solution:

$$\begin{aligned} a &= 1 \\ b &= -(x+2) \\ c &= 2x+3 \end{aligned}$$

Problem 1.4

Let A be a 2×2 matrix, and let x be one of its elements. All of its other elements are known, and are given as:

$$A = \begin{bmatrix} x & 3 \\ -1 & 2 \end{bmatrix} \tag{1.4-1}$$

Suppose that you are given one of its eigenvalues, λ, and you want to find the corresponding eigenvector. As you know, the scale of an eigenvector is arbitrary, so let's arbitrarily set its first element to 1: $\vec{v} = [1, v_2]^T$. Solve for its second element, v_2, in terms of λ.

Solution: Setting $A\vec{v} = \lambda\vec{v}$ gives two equations in one unknown: $x + 3v_2 = \lambda$, and $-1 + 2v_2 = \lambda v$. These will give the same answer if λ is an eigenvalue:

$$v_2 = \frac{\lambda - x}{3} = \frac{1}{2 - \lambda}$$