Lecture 24: Barycentric Coordinates

ECE 417: Multimedia Signal Processing
Mark Hasegawa-Johnson

University of Illinois

11/14/2018

1. Overview of MP4
2. Barycentric Coordinates
3. Conclusion

Outline

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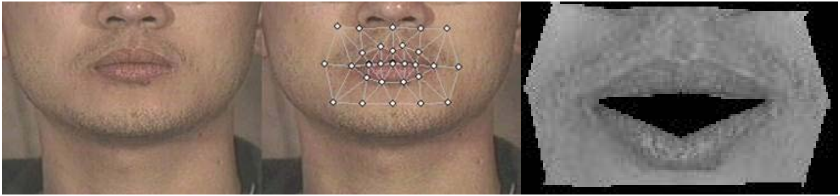
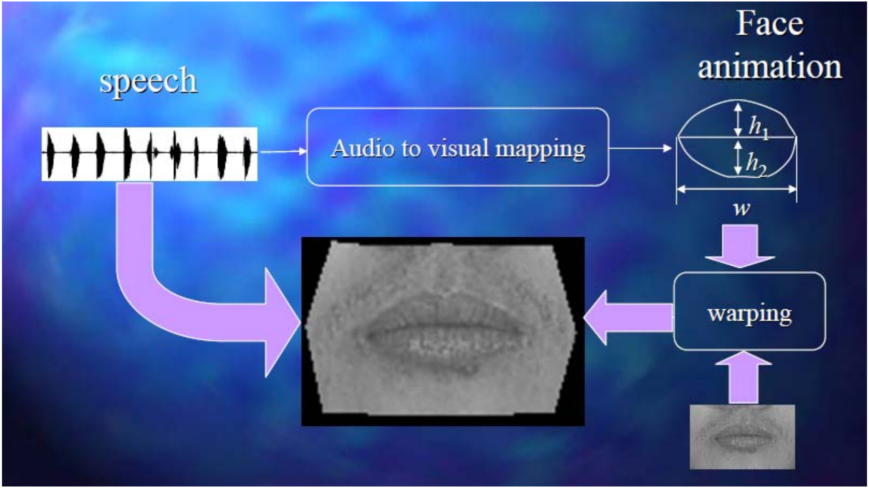
2. Barycentric Coordinates
3. Conclusion



Goal of MP4: Generate video frames (right) by warping a static image (left).

MP4 full outline



How it is done (Full walkthrough: Tuesday November 27)

$$\begin{aligned}
\text{lip_height,width} &= \text{NeuralNet}\left(\text{MFCC}\right)\\
\text{out_triangs} &= \text{LinearlyInterpolate}\left(\text{inp_triangs,lip_height,width}\right)\\
\text{inp_coord} &= \text{BaryCentric}\left(\text{out_coord,inp_triangs,out_triangs}\right)\\
\text{out_image} &= \text{BilinearInterpolate}\left(\text{inp_coord,inp_image}\right)
\end{aligned}$$

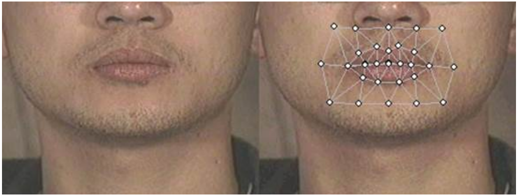
Outline


Affine Transformations
Affine Transformations
* Combines linear transformations, and Translations
TECH
TECH
the ones we looked at, that were
the you know the rotation scaling and
0:26 / 1:19
HD

Piece-wise affine transform

- OK, so somebody's given us a lot of points, arranged like this in little triangles.
- We know that we want a DIFFERENT AFFINE TRANSFORM for EACH TRIANGLE. For the k^{th} triangle, we want to have

$$A_k = \begin{bmatrix} a_k & b_k & c_k \\ d_k & e_k & f_k \\ 0 & 0 & 1 \end{bmatrix}$$

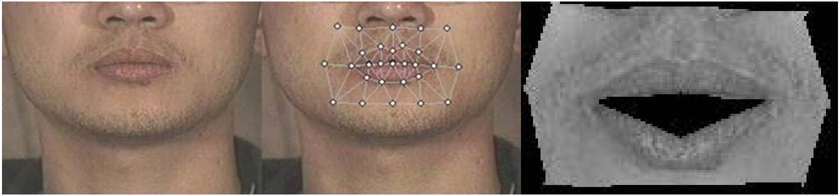


Piece-wise affine transform

$$\text{output point: } \vec{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \text{input point: } \vec{u} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Definition: if $\vec{x}$ is in the k^{th} triangle in the **output image**, then we want to use the k^{th} affine transform:

$$\vec{x} = A_k \vec{u}, \quad \vec{u} = A_k^{-1} \vec{x}$$



If **it is known that** $\vec{u} = A_k^{-1}\vec{x}$ for some unknown affine transform matrix A_k,

then

the method of barycentric coordinates finds $\vec{u}$

without ever finding A_k.

Barycentric Coordinates

Barycentric coordinates turns the problem on its head. Suppose $\vec{x}$ is in a triangle with corners at $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$. That means that

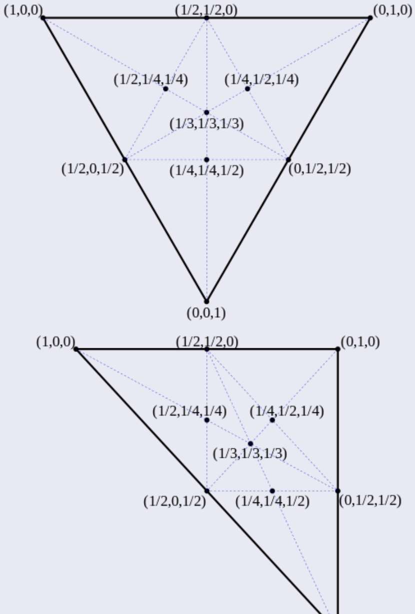
$$\vec{x} = \lambda_1\vec{x}_1 + \lambda_2\vec{x}_2 + \lambda_3\vec{x}_3$$

where

$$0 \leq \lambda_1, \lambda_2, \lambda_3 \leq 1$$

and

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$



Barycentric Coordinates

Suppose that all three of the corners are transformed by some affine transform A, thus

$$\vec{u}_1 = A\vec{x}_1, \quad \vec{u}_2 = A\vec{x}_2, \quad \vec{u}_3 = A\vec{x}_3$$

Then if

$$\text{If: } \vec{x} = \lambda_1\vec{x}_1 + \lambda_2\vec{x}_2 + \lambda_3\vec{x}_3$$

Then:

$$\begin{aligned} \vec{u} &= A\vec{x} \\ &= \lambda_1 A\vec{x}_1 + \lambda_2 A\vec{x}_2 + \lambda_3 A\vec{x}_3 \\ &= \lambda_1\vec{u}_1 + \lambda_2\vec{u}_2 + \lambda_3\vec{u}_3 \end{aligned}$$

In other words, once we know the λ's, we no longer need to find A. We only need to know where the corners of the triangle have moved.

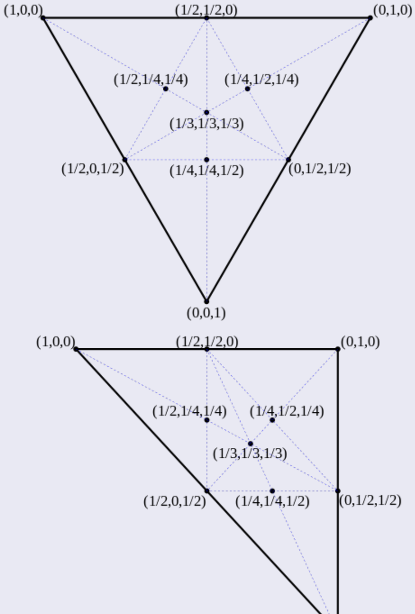
Barycentric Coordinates

If

$$\vec{x} = \lambda_1 \vec{x}_1 + \lambda_2 \vec{x}_2 + \lambda_3 \vec{x}_3$$

Then

$$\vec{u} = \lambda_1 \vec{u}_1 + \lambda_2 \vec{u}_2 + \lambda_3 \vec{u}_3$$



How to find Barycentric Coordinates

But how do you find λ_1, λ_2, and λ_3?

$$\vec{x} = \lambda_1\vec{x}_1 + \lambda_2\vec{x}_2 + \lambda_3\vec{x}_3 = [\vec{x}_1, \vec{x}_2, \vec{x}_3]\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix} = \begin{bmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix}$$

Write this as:

$$\vec{x} = X\vec{\lambda}$$

Therefore

$$\vec{\lambda} = X^{-1}\vec{x}$$

This **always works:** the matrix X is always invertible, unless all three of the points $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$ are on a straight line.

How do you find out which triangle the point is in?

- Suppose we have K different triangles, each of which is characterized by a 3×3 matrix of its corners

$$X_k = [\vec{x}_{1,k}, \vec{x}_{2,k}, \vec{x}_{3,k}]$$

where $\vec{x}_{m,k}$ is the m^{th} corner of the k^{th} triangle.

- Notice that, for any point $\vec{x}$, for ANY triangle X_k, we can find

$$\lambda = X_k^{-1}\vec{x}$$

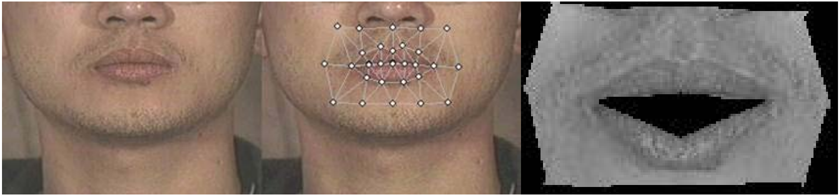
- However, the coefficients λ_1, λ_2, and λ_3 will all be between 0 and 1 **if and only if** the point $\vec{x}$ is inside the triangle X_k. Otherwise, some of the λ's must be negative.

The Method of Barycentric Coordinates

To construct the animated output image frame $O(x, y)$, we do the following things:

- First, for each of the reference triangles U_k in the input image $I(u, v)$, decide where that triangle should move to. Call the new triangle location X_k.
- Second, for each output pixel (x, y):
  - For each of the triangles, find $\vec{\lambda} = X_k^{-1}\vec{x}$.
  - Choose the triangle for which all of the λ coefficients are $0 \leq \lambda \leq 1$.
  - Find $\vec{u} = U_k\vec{\lambda}$.
  - Estimate $I(u, v)$ using bilinear interpolation.
  - Set $O(x, y) = I(u, v)$.

Outline



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