Lecture 20: Rotating, Scaling, Shifting and Shearing an Image

ECE 417: Multimedia Signal Processing

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1. Modifying an Image by Moving Its Points

2. Image Interpolation

3. Affine Transformations

4. Conclusions

Outline

1 Modifying an Image by Moving Its Points

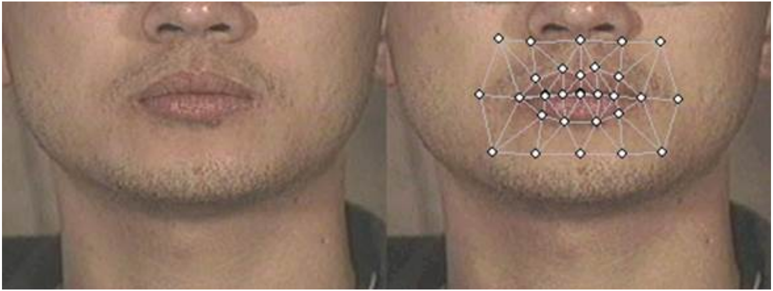
2 Image Interpolation

3 Affine Transformations

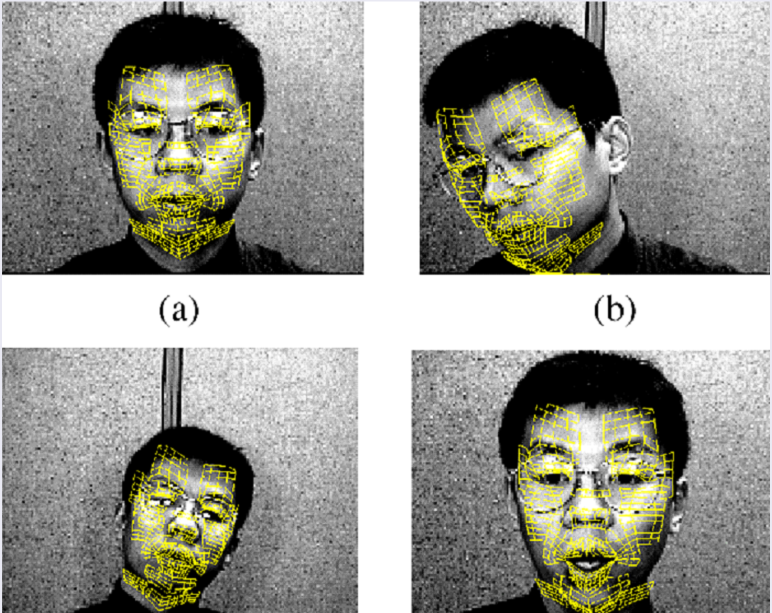
4 Conclusions

Moving Points Around

First, let's suppose that somebody has given you a bunch of points:



. . . and let's suppose you want to move them around, to create new images. . .



Moving One Point

- Your goal is to synthesize an output image, $J[x, y]$, where $J[x, y]$ might be intensity, or RGB vector, or whatever, x is **row** number (measured from top to bottom), y is **column** number (measured from left to right).
- What you have available is:
  - An input image, $I[m, n]$, sampled at integer values of m and n.
  - Knowledge that the input point at $I(u, v)$ has been **moved** to the output point at $J[x, y]$, where x and y are integers, but u and v might not be integers.

$$J[x, y] = I(u, v)$$

Integer Output Points

You want to create the output image as

```
for x in range(0,M):
  for y in range(0,N):
    (u,v) = input_pixels_corresponding_to(x,y)
    J[x,y] = compute_pixel(I,u,v)
```

Non-Integer Input Points

If $[x, y]$ are integers, then usually, (u, v) are not integers.

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Image Interpolation

The function compute_pixel performs image interpolation. Given the pixels of $I[m,n]$ at integer values of m and n, it computes the pixel at a non-integer position $I(u,v)$ as:

$$I(u,v) = \sum_m \sum_n I[m,n]h(u-m, v-n)$$

Piece-Wise Constant Interpolation

$$I(u,v) = \sum_m \sum_n I[m,n]h(u-m, v-n) \tag{1}$$

For example, suppose

$$h(u,v) = \begin{cases} 1 & 0 \le u < 1, \quad 0 \le v < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then Eq. (1) is the same as just truncating u and v to the next-lower integer, and outputting that number:

$$I(u,v) = I\left[\lfloor u \rfloor, \lfloor v \rfloor\right]$$

where $\lfloor u \rfloor$ means "the largest integer smaller than u".

Bi-Linear Interpolation

$$I(u,v)=\sum_m\sum_n I[m,n]h(u-m,v-n)$$

For example, suppose

$$h(u,v)=\max\left(0,(1-|u|)(1-|v|)\right)$$

Then Eq. (1) is the same as piece-wise linear interpolation among the four nearest pixels. This is called **bilinear interpolation** because it's linear in two directions.

$$\begin{aligned}
m&=\lfloor u\rfloor,\quad e=u-m\\
n&=\lfloor v\rfloor,\quad f=v-m\\
I(u,v)&=(1-e)(1-f)I[m,n]+(1-e)fI[m,n+1]\\
&\quad+e(1-f)I[m+1,n]+efI[m+1,n+1]
\end{aligned}$$

Sinc Interpolation

$$I(u,v) = \sum_m \sum_n I[m,n]h(u-m, v-n)$$

For example, suppose

$$h(u,v) = \text{sinc}(\pi u)\text{sinc}(\pi v)$$

Then Eq. (1) is an ideal band-limited sinc interpolation. It guarantees that the continuous-space image, $I(u,v)$, is exactly a band-limited D/A reconstruction of the digital image $I[m,n]$.

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How do we find (u, v)?

Now the question: how do we find (u, v)?
We're going to assume that this is a piece-wise affine transformation.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}$$

How do we find (u, v)?

An affine transformation is defined by:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}$$

A much easier to write this is by using extended-vector notation:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

It's convenient to define $\vec{u} = [u, v, 1]^T$, and $\vec{x} = [x, y, 1]^T$, so that for any $\vec{x}$ in the output image,

$$\vec{u} = A\vec{x}$$

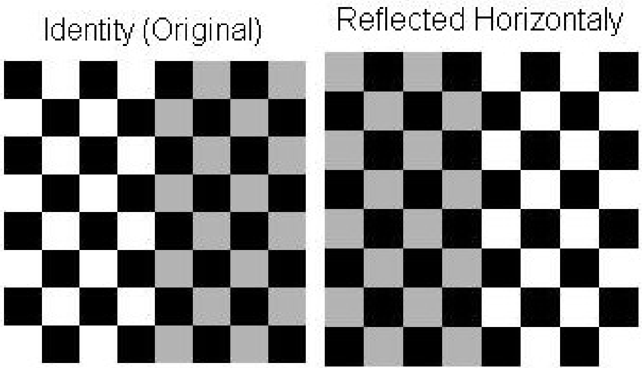
Affine Transforms

Notice that the affine transformation has 6 degrees of freedom: (a, b, c, d, e, f). Therefore, you can accmplish 6 different types of transformation:

- Shift the image left↔right (using f)
- Shift the image up↔down (using c)
- Scale the image horizontally (using e)
- Scale the image vertically (using a)
- Rotate the image (using a, b, d, e)
- Shear the image horizontally (using d)

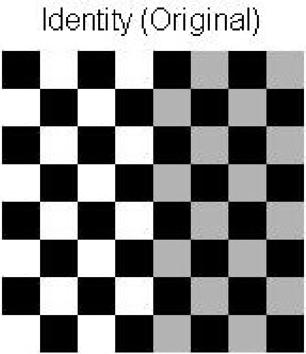
Vertical shear (using b) is a combination of horizontal shear + rotation.

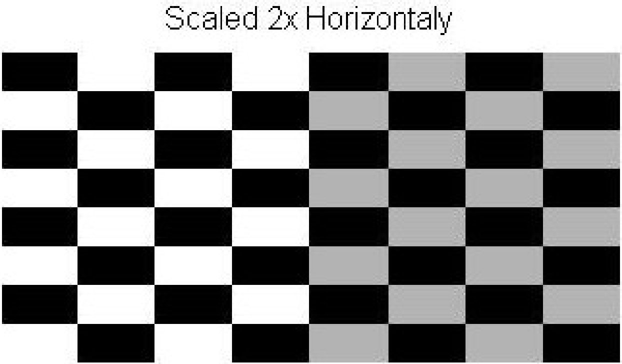
Example: Reflection



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

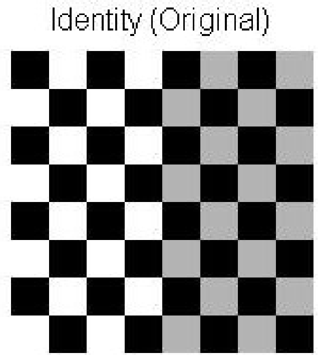
Example: Scale

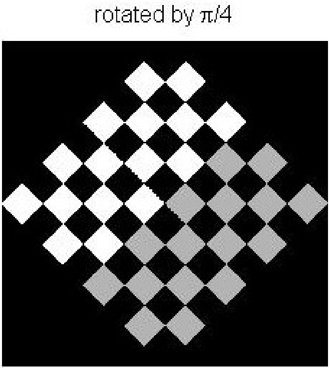




$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

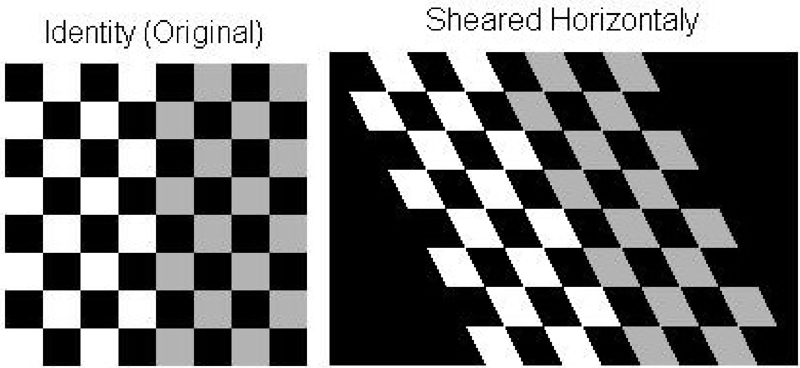
Example: Rotation





$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example: Shear



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$


Affine Transformations
Affine Transformations
* Combines linear transformations,
and Translations
TECH
TECH
the ones we looked at, that were
the you know the rotation scaling and
0:26 / 1:19

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Conclusions

- You can generate an output image $J[x, y]$ by warping an input image $I(u, v)$.
- If (u, v) are not integers, you can compute the value of $I(u, v)$ by interpolating among $I[m, n]$, where $[m, n]$ are integers.
- Shift, scale, rotation and shear are affine transformations, given by

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$