UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 3 Sample Problem Solutions

Problem 3.1

$$\left(\frac{1}{2}\right)^{10}$$

Problem 3.2

$$E\left\{X[k]\right\} = \sum_{n=0}^{N-1} \mu[n] e^{-j2\pi kn/N}$$

Problem 3.3

$$\begin{aligned}
E\left\{|X[k]|^2\right\} &= E\left\{X[k]X^*[k]\right\} \\
&= E\left\{\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} x[m]x[n]W^{n-m}\right\}, \quad W = e^{-j\frac{2\pi k}{N}} \\
&= \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} E\left\{x[m]x[n]\right\} W^{n-m} \\
&= \sigma^2 \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \rho^{|m-n|} W^{n-m} \\
&= \sigma^2 \sum_{n=0}^{N-1}\left(\sum_{m=0}^{n-1} (\rho W)^{n-m} + \sum_{m-n=0}^{N-1-n} (\rho/W)^{m-n}\right) \\
&= \sigma^2 \sum_{n=0}^{N-1}\left(\frac{(\rho W)^n - 1}{1-(\rho W)^{-1}} + \frac{1-(\rho/W)^{N-n}}{1-(\rho/W)}\right) \\
&= \frac{\sigma^2}{1-(\rho W)^{-1}}\left(\frac{1-(\rho W)^N}{1-\rho W} - N\right) + \frac{\sigma^2}{1-(\rho/W)}\left(N - \frac{(\rho/W)^N - 1}{1-(\rho/W)^{-1}}\right) \\
&= N\sigma^2\left(\frac{\rho - 1/\rho}{2\cos(2\pi k/N) - (\rho + 1/\rho)}\right) + \sigma^2(A + A^*), \quad A = \frac{1-(\rho W)^N}{2-(\rho W)^{-1} - \rho W}
\end{aligned}$$

Contrary to what I believed when I first assigned this problem, there doesn't seem to be any really simple form for the solution. Either of the two lines above would be considered an adequate solution, since the summations are all resolved. The last line, above, is simplified enough to show that the answer is real-valued, which is a good thing to show, since it is a way to double-check your answer: $\rho^{|n|}$ is symmetric in time, so its Fourier transform should be real-valued.

Problem 3.4

If $x[n]$ is nonzero only in $n \in \left[0, \frac{N}{2}-1\right]$ then, for values of n in that domain,

$$\sum_{m=-\infty}^{\infty} x[m]x[m-n] = \sum_{m=n}^{\frac{N}{2}-1} x[m]x[m-n] = \sum_{m=n}^{\frac{N}{2}-1} x\left[m\right] x\left[< m-n >_N\right] = \sum_{m=0}^{N-1} x\left[m\right] x\left[< m-n >_N\right]$$