UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 18 Sample Problem Solutions

Problem 18.1

Forward-Prop

$$
\begin{aligned}
a_i[n_1,n_2] &= u[n_1,n_2] * x_i[n_1,n_2] \\
&= \left\{\vec{0}, s[n_1,n_2]\right\}
\end{aligned}
$$

$$
y_i[n_1,n_2] = \left\{0, \max_{n_1}\max_{n_2} s[n_1,n_2]\right\}
$$

Let's define $s^* = \max s[n_1,n_2]$, and let's define n_1^* and n_2^* to be the image coordinates at which the maximum occurs.

$$
\begin{aligned}
y_i &= \{0, s^*\} \\
b_i &= \{0, s^*\} \\
z_i &= \left\{\frac{1}{2}, \frac{1}{2}\right\}
\end{aligned}
$$

... where in the last line, we took advantage of $\sigma(s[n_1,n_2]) \approx \frac{1}{2}$.

Back-Prop

If we define $E_i = \frac{1}{2}(z_i - \zeta_i)^2$, and $E = \sum_i E_i$, then

$$
\begin{aligned}
\epsilon_i &= \frac{\partial E_i}{\partial b_i} \\
&= (z_i - \zeta_i)\sigma'(b_i) \\
&= \left\{\frac{1}{8}, -\frac{1}{8}\right\}
\end{aligned}
$$

... where in the last line, we took advantage of $\sigma(s[n_1,n_2]) \approx \frac{1}{4}$.

$$
\begin{aligned}
\delta_i[n_1,n_2] &= \frac{\partial E_i}{\partial a_i[n_1,n_2]} \\
&= \left(\frac{\partial E_i}{\partial b_i}\right)\left(\frac{\partial b_i}{\partial y_i}\right)\left(\frac{\partial y_i}{\partial a_i[n_1,n_2]}\right) \\
&= \begin{cases} \epsilon_i & a_i[n_1,n_2] = \max_{m_1,m_2} a_i[m_1,m_2] \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

The last line tells us that $\delta_1[n_1,n_2]$ is 1/8 in all of the pixels where $a_i[m_1,m_2]$ has its maximum value—that's all of the pixels. $\delta_2[n_1,n_2]$ is an image that's all zeros except for the max-pixel, $n_1 = n_1^*$ and $n_2 = n_2^*$, where it has a value of

$$
\delta_2[n_1^*, n_2^*] = -\frac{1}{8}
$$

Then

$$\frac{\partial E_i}{\partial u[n_1,n_2]} = \delta_i[n_1,n_2] * x_[n_1,n_2]$$
$$= \left\{\vec{0}, -\frac{1}{8}s[n_1-n_1^*, n_2-n_2^*]\right\}$$

When we average these together, we get

$$\frac{\partial E}{\partial u[n_1,n_2]} = -\frac{1}{8}s[n_1-n_1^*, n_2-n_2^*]$$

Which means that the filter will be updated as

$$u[n_1,n_2] \leftarrow u[n_1,n_2] - \eta\frac{\partial E}{\partial u[n_1,n_2]} = \frac{\eta}{8}s[n_1-n_1^*, n_2-n_2^*]$$

. . . in other words, the filter becomes a scaled, shifted copy of the target signal image, kind of like a matched filter for the signal s.