UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 11 Sample Problems

Problem 11.1

Assume that you have an image $x[n_1, n_2]$ that is well-defined for $(-\infty < n_1 < \infty, -\infty < n_2 < \infty)$, but whose pixels are all zero except in the range $(0 \le n_1 < N_1, 0 \le n_2 < N_2)$. Suppose that, for a particular object detection problem, you want to compute $f[n_1, n_2] = x[n_1, n_2] ** h[n_1, n_2]$ where

$$h[n_1, n_2] = \begin{cases} 1 & \left(0 \le n_1 < \frac{M_1}{2}, 0 \le n_2 < \frac{M_2}{2}\right) \text{ and } \left(\frac{M_1}{2} \le n_1 < M_1, \frac{M_2}{2} \le n_2 < M_2\right) \\ -1 & \left(0 \le n_1 < \frac{M_1}{2}, \frac{M_2}{2} \le n_2 < M_2\right), \text{ and } \left(\frac{M_1}{2} \le n_1 < M_1, 0 \le n_2 < \frac{M_2}{2}\right) \\ 0 & \text{otherwise} \end{cases}$$

You can assume that $N_1 \ge M_1$ and $N_2 \ge M_2$.

1. The standard way to implement 2D convolution is

$$f[n_1, n_2] = \sum_{m_1=\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} x[m_1, m_2] h[n_1 - m_1, n_2 - m_2] \tag{11.1-1}$$

   Suppose your goal is to compute the "full" convolution response. How many additions and subtractions (of terms other than zero) are required to perform this operation using Eq. 11.1-1? Express your answer in big-$\mathcal{O}$ notation, in terms of the unknown constants N_1, N_2, M_1, M_2.

2. Use the method of integral images, proposed by Viola & Jones, to devise an algorithm that generates $f[n_1, n_2]$ using no more than $\mathcal{O}\{N_1 N_2\}$ operations.