UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Spring 2016

EXAM 3

Thursday, May 12, 2016, 7:00-10:00pm

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of hand-written notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ____

Possibly Useful Formulas

One-Layer Neural Network

$$f_k(\vec{x}_i, W) = g\left(\sum_{j=1}^{q} w_{kj} x_{ji}\right)$$

Barycentric Coordinates

$$\begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

Bilinear Interpolation

$$i(m+e, n+f) = (1-e)(1-f)i(m,n) + (1-e)fi(m,n+1) + e(1-f)i(m+1,n) + efi(m+1,n+1)$$

Integral Image

$$ii(x,y) = \sum_{x' \le x} \sum_{y' \le y} i(x', y')$$

AdaBoost

$$\begin{aligned}
p_{it} &\leftarrow \frac{w_{it}}{\sum_{j=1}^{n} w_{jt}}, \quad 1 \le t \le T,\ 1 \le i \le n \\
P_E(t) &= \sum_{i=1}^{n} p_{it} \left[y_i \ne h_t(\vec{x}_i) \right], \quad 1 \le t \le T,\ 1 \le i \le n \\
\beta_t &= P_E(t)/(1 - P_E(t)), \quad 1 \le t \le T \\
w_{i,t+1} &\leftarrow \begin{cases} w_{it}\beta_t & h_t(\vec{x}_i) = y_i \\ w_{it} & h_t(\vec{x}_i) \ne y_i \end{cases}, \quad 1 \le t \le T,\ 1 \le i \le n \\
\alpha_t &= -\ln \beta_t, \quad 1 \le t \le T
\end{aligned}$$

Useful Angles			
θ	$\cos\theta$	$\sin\theta$	$e^{j\theta}$
0	1	0	1
$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/2 + j/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2 + j\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$	$1/2 + j\sqrt{3}/2$
$\pi/2$	0	1	j
π	-1	0	-1
$3\pi/2$	1	-1	$-j$
2π	1	0	1

Problem 1 (20 points)

Your training database contains matched pairs $\{(\vec{x}_1, \vec{y}_1), \ldots, (\vec{x}_n, \vec{y}_n)\}$ where $\vec{x}_i$ is the i^{th} observation vector, and $\vec{y}_i$ is the i^{th} label vector. For some initial weight matrix $W = \begin{bmatrix} w_{11} & \cdots \\ \cdots & w_{qp} \end{bmatrix}$, you have already computed the following two quantities:

$$f_\ell(\vec{x}_i, W) \qquad 1 \le \ell \le r,\ 1 \le i \le n \tag{1}$$

$$\frac{\partial f_\ell(\vec{x}_i, W)}{\partial w_{kj}} \qquad 1 \le \ell \le r,\ 1 \le k \le q,\ 1 \le j \le p,\ 1 \le i \le n \tag{2}$$

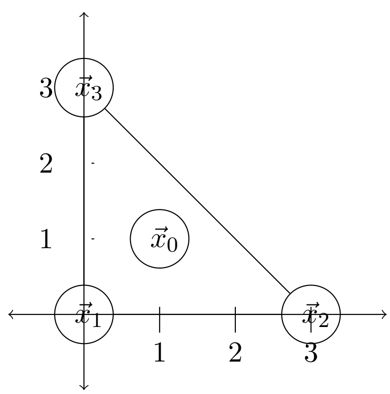
You want to find a new matrix $W' = \begin{bmatrix} w'_{11} & \cdots \\ \cdots & w'_{qp} \end{bmatrix}$ such that $\mathcal{J}(W') \ge \mathcal{J}(W)$ (that is, you want to **maximize** $\mathcal{J}$), where

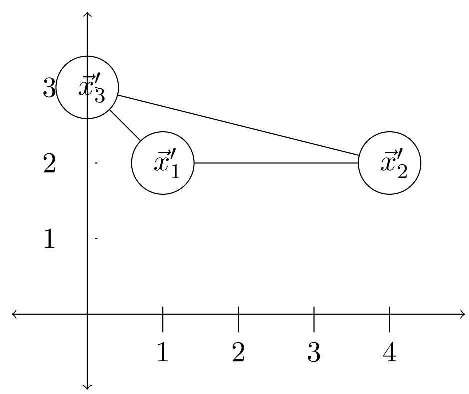
$$\mathcal{J}(W) = \sum_{i=1}^{n} \sum_{\ell=1}^{r} y_{\ell r} \ln\left(f_\ell(\vec{x}_i, W)\right)$$

Give a formula for w'_{kj} in terms of w_{kj}, $f_\ell(\vec{x}_i, W)$, $\frac{\partial f_\ell(\vec{x}_i, W)}{\partial w_{kj}}$, and in terms of a step size, η, such that for suitable values of η, $\mathcal{J}(W') \ge \mathcal{J}(W)$.

$w'_{kj} =$

Problem 2 (20 points)





After image warping, the triangle $\{\vec{x}_1, \vec{x}_2, \vec{x}_3\} = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}$ has moved to $\{\vec{x}'_1, \vec{x}'_2, \vec{x}'_3\} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}$, and $\vec{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is moved to $\vec{x}'_0$. Find $\vec{x}'_0$.

$\vec{x}'_0 =$

Problem 3 (20 points)

Image warping has moved input pixel $i(4.6, 8.2)$ to output pixel $i'(15, 7)$. Input pixel $i(4.6, 8.2)$ is unknown, but you know that $i(4, 8) = a$, $i(4, 9) = b$, $i(5, 8) = c$, and $i(5, 9) = d$. Use bilinear interpolation to estimate $i(4.6, 8.2)$ in terms of a, b, c, and d.

$i(4.6, 8.2) \approx$

Problem 4 (20 points)

An integral image is computed from image $i(x, y)$ according to

$$ii(x, y) = \sum_{x' \leq x} \sum_{y' \leq y} i(x', y')$$

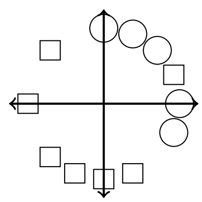
You want to compute the following feature:

$$f(i) = \sum_{y=101}^{200} \left(\sum_{x=51}^{100} i(x, y) - \sum_{x=101}^{150} i(x, y) + \sum_{x=151}^{200} i(x, y) \right)$$

Suppose that $ii(x, y)$ has already been computed. Find a formula for $f(i)$, in terms of $i(x, y)$ and/or $ii(x, y)$, that requires no more than seven additions (a sum with no more than eight terms).

$f(i) =$

Problem 5 (20 points)



Sun Tzu is surrounded by 12 armies. If Sun Tzu's position is $(0,0)$, the position of the i^{th} army is given by $(\cos\phi_i, \sin\phi_i)$, where

$$\{\phi_1,\ldots,\phi_{12}\} = \left\{-\frac{\pi}{8}, 0, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{\pi}{8}, \frac{3\pi}{4}, \pi, -\frac{3\pi}{4}, -\frac{5\pi}{8}, -\frac{\pi}{2}, -\frac{3\pi}{8}\right\}$$

The armies shown as circles, in the figure above, are allies; those shown as squares are enemies. Let's label allies as $y_i = 0$, and enemies as $y_i = 1$, so that

$$\{y_1,\ldots,y_{12}\} = \{0,0,0,0,0,1,1,1,1,1,1,1\}$$

Sun Tzu has two tripod-mounted motion detectors, called $h_1(\phi)$ and $h_2(\phi)$. Each motion detector outputs a 1 whenever an army in its field of view moves. The field of view is exatly π radians, therefore if the army at ϕ_i moves, and if the t^{th} motion director is pointed in the direction θ_t, then

$$h_t(\phi_i) = \left\{\begin{array}{ll} 1 & \cos(\theta_t - \phi_i) > 0 \\ 0 & \cos(\theta_t - \phi_i) \le 0 \end{array}\right.$$

Unfortunately, when a motion detector goes off, there's no way to tell which of the armies in its field of view has moved. Sun Tzu therefore wants to average the motion detectors in such a way that the average output is positive if and only if an enemy army moves. In other words, the goal is to find θ_1, θ_2, α_1 and α_2 in order to maximize the number of armies for which $\text{sign}(h(x)) = \text{sign}(2y_i - 1)$, where

$$h(x) = \left(\sum_{t=1}^{2} \alpha_t \left(2h_t(\phi_i) - 1\right)\right) \tag{3}$$

Use the AdaBoost algorithm to find values of θ_1, θ_2, α_1 and α_2 that maximize the accuracy of Eq. 3. (Note: θ_1 and θ_2 should be real-valued fractions of π; α_1 and α_2 should each be the logarithm of a real number.)

$\theta_1 =$	$\theta_2 =$

$\alpha_1 =$	$\alpha_1 =$