UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Spring 2016

EXAM 1

Thursday, February 25, 2016

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of hand-written notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ____

Possibly Useful Formulas

Z transform/DTFT

$$X(z) = \mathcal{Z}\left\{x[n]\right\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}, \qquad x[n] = \mathcal{Z}^{-1}\left\{X(z)\right\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}d\omega$$

Convolution

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

DFT

$$X[k] = \text{DFT}\left\{x[n]\right\} = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \qquad x[n] = \text{DFT}^{-1}\left\{X[k]\right\} = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N}$$

Frequency Conversion: Hertz (f) to Mel (m)

$$m = G\ln(1+f/700), \quad G \equiv \frac{1000}{\ln(1+1000/700)}$$

Z-Transform/DTFT Pairs	
$h[n]$	$H(e^{j\omega})$
$\frac{\sin\omega_c n}{\pi n}$	$H(\omega) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \text{otherwise} \end{cases}$
$u[n] - u[n-N]$	$e^{-j\frac{\omega(N-1)}{2}}\frac{\sin(\omega N/2)}{\sin(\omega/2)}$
$\delta[n-\tau]$	$e^{-j\omega\tau}$
$e^{j\alpha n}$	$2\pi\delta(\omega-\alpha)$
$\sum_{\ell=-\infty}^{\infty}\delta[n-\ell T_0]$	$\left(\frac{2\pi}{T_0}\right)\sum_{k=1}^{T_0-1}\delta\left(\omega - \frac{2\pi k}{T_0}\right)$

Useful Angles			
θ	$\cos\theta$	$\sin\theta$	$e^{j\theta}$
0	1	0	1
$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/2 + j/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2 + j\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$	$1/2 + j\sqrt{3}/2$
$\pi/2$	0	1	j
π	-1	0	-1
$3\pi/2$	1	-1	$-j$
2π	1	0	1

Problem 1 (15 points)

A signal $s[n]$ is corrupted by filtering through a reverberant channel, $h[n]$, producing

$$x[n] = h[n] * s[n]$$

Suppose you don't know the channel, but you do know its cepstrum, defined as

$$\hat{h}[n] = \mathcal{Z}^{-1}\left\{\ln\left(\mathcal{Z}\left\{h[n]\right\}\right)\right\}$$

where $\mathcal{Z}\{\}$ is the Z transform, and $\mathcal{Z}^{-1}\{\}$ is the inverse Z transform. Draw the block diagram of a system that takes the following two inputs, and produces the following output. For each block in your block diagram, write an equation specifying its input-output relationship.

- **INPUT:** $x[n]$, the reverberant signal
- **INPUT:** $\hat{h}[n]$, the cepstrum of the channel
- **OUTPUT:** $s[n]$, the clean signal

Problem 2 (15 points)

Suppose that

$$x[n] = s[n] + 0.9s[n-80]$$

Define the cepstrum to be $\hat{x}[n] = \mathcal{Z}^{-1}\left\{\ln\left(\mathcal{Z}\left\{x[n]\right\}\right)\right\}$. Find the relationship between $\hat{x}[n]$ and $\hat{s}[n]$. HINT: You might find it useful to know that

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$$

Problem 3 (20 points)

A speech signal, $x[n]$, has been sampled at F_s samples/second. Suppose the signal is only N samples long, so we can compute its N-sample DFT, $X[k]$. The mel-frequency spectrum is then defined to be

$$\tilde{X}[m] = \sum_{k=0}^{N/2} H_m[k] \cdot |X[k]|, \quad 1 \leq m \leq M$$

where the filters are given by

$$H_m[k] = \begin{cases} \frac{k-k_{m-1}}{k_m-k_{m-1}} & k_{m-1} \leq k \leq k_m \\ \frac{k_{m+1}-k}{k_{m+1}-k_m} & k_m \leq k \leq k_{m+1} \\ 0 & \text{otherwise} \end{cases}$$

where $k_0 = 0$ and $k_{M+1} = N/2$.

Find a formula for k_m, $1 \leq m \leq M$. Your formula should depend on m, M, N, and/or F_s, and it should make use of a formula from the "Possibly Useful Formulas" page of this exam.

Problem 4 (30 points)

Suppose that you have M different D-dimensional vectorized face images, $\vec{\Gamma}_m = [\gamma_{1m}, \ldots, \gamma_{Dm}]^T$, whose mean is $\vec{\Psi} = [\psi_1, \ldots, \psi_D]^T$. Define the data matrix to be $A = [\vec{\Gamma}_1 - \vec{\Psi}, \ldots, \vec{\Gamma}_M - \vec{\Psi}]$, and suppose that the eigenvectors and eigenvalues of $A^T A$ are given by $U = [\vec{u}_1, \ldots, \vec{u}_M]$ and $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_M)$.

(a) Find the numerical value of the vector $U^T \vec{u}_3$.

(b) Your goal is to find a $(D \times M)$ matrix $V = [\vec{v}_1, \ldots, \vec{v}_M]$ so that $\vec{\Omega}_m = V^T(\vec{\Gamma}_m - \vec{\Psi})$ is a vector containing the first M principal components of the image $\vec{\Gamma}_m$. Write an equation showing how V can be computed from $\vec{\Psi}$, A, U, and/or Λ.

Problem 5 (20 points)

Suppose that you have M different D-dimensional vectorized face images, $\vec{\Gamma}_m = [\gamma_{1m}, \ldots, \gamma_{Dm}]^T$, whose mean is $\vec{\Psi} = [\psi_1, \ldots, \psi_D]^T$. Define the scatter matrix to be

$$S = \sum_{m=1}^{M} (\vec{\Gamma}_m - \vec{\Psi})(\vec{\Gamma}_m - \vec{\Psi})^T$$

Suppose that the eigenvectors and eigenvalues of S are $V = [\vec{v}_1, \ldots, \vec{v}_D]$ and $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_D)$. You want to find a value of K such that the K-dimensional PCA projection $\vec{\Omega}_m = [\vec{v}_1, \ldots, \vec{v}_K]^T(\vec{\Gamma}_m - \vec{\Psi})$ has the following property:

$$\sum_{m=1}^{M} \left|\vec{\Omega}_m\right|^2 = (0.95) \sum_{m=1}^{M} \left|\vec{\Gamma}_m - \vec{\Psi}\right|^2 \tag{1}$$

Specify an equation that, if satisfied, will guarantee the truth of Eq. **??**. Your equation should only include the scalars M, D, K, and/or the eigenvalues λ_d $(1 \leq d \leq D)$; your equation should not include $\vec{\Gamma}_m$ or $\vec{\Psi}$.