UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Principles of Signal Analysis
Spring 2015

EXAM 1

Thursday, February 19, 2015

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of hand-written notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ____

Possibly Useful Formulas

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega \leftrightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} \leftrightarrow X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$h[n] = \frac{\sin\omega_c n}{\pi n} \leftrightarrow H(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

$$u[n] - u[n-N] \leftrightarrow e^{-j\frac{\omega(N-1)}{2}}\frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$\delta[n] \leftrightarrow 1$$

$$e^{j\alpha n} \leftrightarrow 2\pi\delta(\omega-\alpha)$$

$$\sum_{\ell=-\infty}^{\infty}\delta[n-\ell T_0] \leftrightarrow \left(\frac{2\pi}{T_0}\right)\sum_{k=1}^{T_0-1}\delta\left(\omega - \frac{2\pi k}{T_0}\right)$$

$$S = \sum_{k=1}^{n}(\vec{x}_k - \vec{m})(\vec{x}_k - \vec{m})^T$$

θ	$\cos\theta$	$\sin\theta$	$e^{j\theta}$
0	1	0	1
$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/2 + j/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2 + j\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$	$1/2 + j\sqrt{3}/2$
$\pi/2$	0	1	j
π	-1	0	-1
$3\pi/2$	1	-1	$-j$
2π	1	0	1

Problem 1 (25 points)

A particular speech signal has the following log magnitude DTFT:

$$\log|X(\omega)| = \frac{2\pi G}{T_0}\sum_{k=0}^{T_0-1}\delta\left(\omega - \frac{2\pi k}{T_0}\right) + \sum_{m=1}^{4} h_m \cos(m\omega)$$

for some constants G, T_0, and h_m; assume that $T_0 > 4$. Define

$$\begin{aligned}\hat{x}[n] &= \text{DTFT}^{-1}(\log|X(\omega)|)\\ \hat{g}[n] &= w[n]\hat{x}[n]\\ \log G_M(\omega) &= \text{DTFT}(\hat{g}[n])\end{aligned}$$

and

$$w[n] = \begin{cases} 1 & |n| \le M \\ 0 & \text{otherwise}\end{cases}$$

Specify the value of $\log G_M(\omega)$, as a simple function of both M and ω, in terms of G, T_0, and h_m, for **every possible positive value of** M

Problem 2 (20 points)

A particular dataset has three data,

$$\vec{x}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{x}_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Define $X = [\vec{x}_1, \vec{x}_2, \vec{x}_3]$ and $R = X^T X$. The matrix R is given by $R = V\Lambda V^T$ where

$$V = \begin{bmatrix} -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Find a matrix W such that $\vec{y}_i = W^T \vec{x}_i$, $\vec{y}_i$ is two-dimensional, and the elements of $\vec{y}_i$ are uncorrelated.

Problem 3 (15 points)

Let $a_i[n_1, n_2]$ denote the $(n_1, n_2)^{\text{th}}$ pixel of a grayscale image a_i, where $0 \leq n_1 \leq N_1 - 1$ and $0 \leq n_2 \leq N_2 - 1$, therefore the vectorized version of the same image, $\vec{x}_i$, is an $(N_1 N_2)$-dimensional vector. For this problem it does not matter whether you vectorize the image in row order or in column order.

Suppose that the first image in the training database is a fuzzy striped image, given by

$$a_1[n_1, n_2] = \frac{255}{2} + \frac{255}{2} \cos\left(\frac{2\pi n_2}{6}\right)$$

Suppose that pixel values are constrained to be non-negative, and to lie in the range $0 \leq a_i[n_1, n_2] \leq 255$.

Under these constraints, find the image $a_2[n_1, n_2]$ that maximizes $\|\vec{x}_2\|^2$ subject to the constraint $\vec{x}_1^T \vec{x}_2 = 0$. **Hint:** First figure out which pixels can be nonzero, then figure out what their values must be.

Problem 4 (15 points)

Suppose you have a 16000-sample audio waveform, $x[n]$, such that $x[n] \neq 0$ for $0 \leq n \leq 15999$. You want to chop this waveform into 400-sample frames, with 10% overlap between frames. How many nonzero samples are there in the last frame?

Problem 5 (25 points)

Suppose you have a database with three samples from class 0, and two samples from class 1, in other words, the data labels are

$$Y = [y_1, y_2, y_3, y_4, y_5] = [0, 0, 0, 1, 1]$$

Each observation is an $\Re^2$-vector, and they are given by

$$X = [\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \vec{x}_5] = \left[\begin{array}{ccccc} -1 & 1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

(a) Consider a nearest-neighbor (NN) classifier. In a two-dimensional vector space, show the boundary that separates class 0 from class 1. Label the coordinates of every discontinuity.

(b) Repeat part (a), but this time for a 3NN (3-nearest-neighbor) classifier.