UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Fall 2018

PRACTICE EXAM 2 SOLUTIONS

Wednesday, December 12, 2018

Problem 1 (16 points)

$$\begin{aligned}\delta_\ell &= \left((y_\ell - y_\ell^*) + \sum_{k=\ell+1}^{L} \delta_k w_{kl}\right)\sigma'(a_\ell)\\ &= \left((y_\ell - y_\ell^*) + \sum_{k=\ell+1}^{L} \delta_k w_{kl}\right) y_\ell(1-y_\ell)\end{aligned}$$

Problem 2 (17 points)

$$\delta[m_1,m_2] = \sum_{n_1}\sum_{n_2}\epsilon[n_1,n_2]u[m_1-n_1,m_2-n_2]$$

Problem 3 (16 points)

Use

$$A = \left[\begin{array}{ccc} a & b & c\\ d & e & f\\ 0 & 0 & 1\end{array}\right]$$

Define $\vec{du}_3 = \vec{u}_3 - \vec{u}_1$, $\vec{du}_4 = \vec{u}_4 - \vec{u}_2$, $\vec{dx}_3 = \vec{x}_3 - \vec{x}_1 = A\vec{du}_3$, $\vec{dx}_4 = \vec{x}_4 - \vec{x}_2 = A\vec{du}_4$. Then

$$\vec{dx}_3 = \left[\begin{array}{c} a\alpha\cos\theta + b\alpha\sin\theta\\ d\alpha\cos\theta + e\alpha\sin\theta\\ 0\end{array}\right],\quad \vec{dx}_4 = \left[\begin{array}{c} a\beta\cos\theta + b\beta\sin\theta\\ d\beta\cos\theta + e\beta\sin\theta\\ 0\end{array}\right]$$

Then the slopes are given by

$$\begin{aligned}\text{slope}(\bar{x_1 x_3}) &= \frac{d\alpha\cos\theta + e\alpha\sin\theta}{a\alpha\cos\theta + b\alpha\sin\theta} = \frac{d\cos\theta + e\sin\theta}{a\cos\theta + b\sin\theta}\\ \text{slope}(\bar{x_2 x_4}) &= \frac{d\beta\cos\theta + e\beta\sin\theta}{a\beta\cos\theta + b\beta\sin\theta} = \frac{d\cos\theta + e\sin\theta}{a\cos\theta + b\sin\theta}\end{aligned}$$

Problem 4 (17 points)

There are many, many acceptable solutions. Most are forms of E with two terms: the first term is reduced as y_{ik} gets more accurate, the second term is reduced as $z_{i\ell}$ gets more inaccurate. For example,

$$E = -\frac{1}{n}\sum_{i=1}^{n}\sum_{k=1}^{q} y_{ik}^* \ln y_{ik} + \frac{1}{n}\sum_{i=1}^{n}\sum_{\ell=1}^{r} z_{i\ell}^* \ln z_{i\ell}$$

Problem 5 (17 points)

$$\exists\alpha,\beta \text{ s.t. } \begin{array}{c} x_3 = \alpha x_1 + \beta x_2 \\ y_3 = \alpha y_1 + \beta y_2 \\ 1 = \alpha + \beta \end{array}$$

Problem 6 (17 points)

Define $\delta_{jk} = \left\{ \begin{array}{ll} 1 & j = k \\ 0 & \text{else} \end{array} \right.$. Define $\tilde{c}_j[n]$ to be the j^{th} element of $g\left(B_c\vec{x}[n] + A_c\vec{c}[n-1]\right)$, and

$$\frac{\partial c_j[n]}{\partial c_k[n-1]} = \frac{\partial c_j[n]}{\partial f_j[n]}\frac{\partial f_j[n]}{\partial c_k[n-1]} + f_j[n]\delta_{jk} + \frac{\partial c_j[n]}{\partial i_j[n]}\frac{\partial i_j[n]}{\partial c_k[n-1]} + i_j[n]\frac{\partial \tilde{c}_j[n]}{\partial c_k[n-1]}$$

Now define $\tilde{c}'_j[n]$ to be the j^{th} element of $g'\left(B_c\vec{x}[n] + A_c\vec{c}[n-1]\right)$. We could also define $i'_j[n]$ to be the j^{th} element of $\sigma'\left(B_i\vec{x}[n] + A_i\vec{c}[n-1]\right)$, but actually we don't need to, since the derivative of the logistic function is just $i_j[n](1-i_j[n])$. Define a^o_{mn} to be the $(m,n)^{\text{th}}$ element of the matrix A_o, and so on. Then

$$\frac{\partial c_j[n]}{\partial c_k[n-1]} = c_j[n-1]f_j[n](1-f_j[n])a^f_{jk} + f_j[n]\delta_{jk} + \tilde{c}_j[n]i_j[n](1-i_j[n])a^i_{jk} + i_j[n]\tilde{c}'_j[n]a^c_{jk}$$