UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 MULTIMEDIA SIGNAL PROCESSING
Fall 2017

EXAM 3 SOLUTIONS

Friday, December 15, 2017

Problem 1 (10 points)

A particular two-layer neural network accepts a two-dimensional input vector $\vec{x} = [x_1, x_2, 1]^T$, and generates an output $z = h(\vec{v}^T g(U\vec{x}))$. Choose network weights $\vec{v}$ and U, and element-wise scalar nonlinearities $h()$ and $g()$, that will generate the following output:

$$z = \begin{cases} 1 & |x_1| < 2 \text{ and } |x_2| < 2 \\ -1 & \text{otherwise} \end{cases}$$

Solution: Several solutions are possible. Here's one.

$$\begin{aligned} U &= \begin{bmatrix} -1 & 0 & 2 \\ 1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \\ g(a) &= u(a) \\ \vec{v}^T &= [1, 1, 1, 1, -3.5] \\ h(a) &= \text{sgn}(b) \end{aligned}$$

For this definition to exactly match the problem statement, it's necessary to define $u(0) = 0$. This is a trivial point.

Problem 2 (10 points)

A reference image $I_0(u, v)$ has the following pixel values:

$$I_0(u, v) = 1 + (-1)^{u+v}$$

The test image $I_1(x, y)$ is created by piece-wise affine transformation of the pixel locations in $I_0(u, v)$. In particular, the triangle U in $I_0(u, v)$ is moved to the triangle X in $I_1(x, y)$, where

$$U = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Find the reference coordinate $\vec{u} = [u, v, 1]^T$ that corresponds to the test coordinate $\vec{x} = [3, 2, 1]^T$.

<u>**Solution:**</u>

$$\vec{\lambda}^T = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right]$$
$$\vec{u}^T = [1.5, 2, 1]$$

(b) Use bilinear interpolation to find the value of the test pixel $I_1(3, 2)$.

<u>**Solution:**</u>

$$I_1(3, 2) = I_0(1.5, 2) = \frac{1}{2} I_0(1, 2) + \frac{1}{2} I_0(2, 2) = 1$$

Problem 3 (10 points)

A bimodal HMM uses a common state sequence, $Q = [q_1, \ldots, q_T]$, to explain two different observation sequences $X = [\vec{x}_1, \ldots, \vec{x}_T]$ and $Y = [\vec{y}_1, \ldots, \vec{y}_T]$. The HMM is parameterized by

$$\begin{aligned}
\pi_i &= p(q_1 = i) \\
a_{ij} &= p(q_t = j | q_{t-1} = i) \\
b_j(\vec{x}_t) &= p_X(\vec{x}_t | q_t = j) \\
c_j(\vec{y}_t) &= p_Y(\vec{y}_t | q_t = j)
\end{aligned}$$

Define

$$\begin{aligned}
\alpha_t(i) &= p(\vec{x}_1, \vec{y}_1, \ldots, \vec{x}_t, \vec{y}_t, q_t = i) \\
\beta_t(i) &= p(\vec{x}_{t+1}, \vec{y}_{t+1}, \ldots, \vec{x}_T, \vec{y}_T | q_t = i)
\end{aligned}$$

(a) Specify initialization formulas for $\alpha_1(i)$ and $\beta_T(i)$ in terms of π_i, a_{ij}, $b_j(\vec{x}_t)$, and $c_j(\vec{x}_t)$.

<u>**Solution:**</u>

$$\alpha_1(i) = \pi_i b_i(\vec{x}_1) c_i(\vec{y}_1)$$
$$\beta_T(i) = 1$$

(b) Specify iteration formulas for $\alpha_t(i)$ and $\beta_t(i)$ in terms of π_i, a_{ij}, $b_j(\vec{x}_t)$, $c_j(\vec{x}_t)$, $\alpha_{t-1}(j)$, and $\beta_{t+1}(j)$.

<u>**Solution:**</u>

$$\alpha_t(i) = \sum_{j=1}^{N} \alpha_{t-1}(j) a_{ji} b_i(\vec{x}_t) c_i(\vec{y}_t)$$
$$\beta_t(i) = \sum_{j=1}^{N} \beta_{t+1}(j) a_{ij} b_j(\vec{x}_{t+1}) c_j(\vec{y}_{t+1})$$

Problem 4 (10 points)

Suppose that you have a training database with three training vectors $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$ whose correct labels are y_1, y_2, and y_3. You also have a set of three weak classifiers h_1, h_2, and h_3, each of which is right for exactly two of the three training tokens, as follows:

$$h_t(\vec{x}_i) = \begin{cases} y_i & i \neq t \\ \text{incorrect} & i = t \end{cases}$$

Adaboost begins with the weights $w_{1,i} = \frac{1}{3}$, and runs for three iterations, resulting in the strong classifier

$$H(\vec{x}) = \sum_{t=1}^{3} \alpha_t h_t(\vec{x})$$

You may assume that the weak classifiers are selected in order: h_1 is selected in the first iteration of Adaboost, h_2 in the second iteration, and h_3 in the third iteration. Find α_1, α_2, and α_3.

Solution:

$$\begin{aligned}
\epsilon_1 &= \frac{1}{3} \\
\alpha_1 &= \log(2) \\
w_{2,i} &= \begin{cases} \frac{1}{2} & i = 1 \\ \frac{1}{4} & i = 2, 3 \end{cases} \\
\epsilon_2 &= \frac{1}{4} \\
\alpha_1 &= \log(3) \\
w_{3,i} &= \begin{cases} \frac{1}{3} & i = 1 \\ \frac{1}{2} & i = 2 \\ \frac{1}{6} & i = 3 \end{cases} \\
\epsilon_3 &= \frac{1}{6} \\
\alpha_3 &= \log(5)
\end{aligned}$$