UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Fall 2017

CONFLICT EXAM 2

Monday, October 30, 2017

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of handwritten notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 40 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
Total	

Name: ____

Possibly Useful Formulas

Fourier Transforms

$$\textbf{DTFT:} \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega \leftrightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\textbf{DFT:} \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} \leftrightarrow X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$h[n] = \frac{\sin\omega_c n}{\pi n} \leftrightarrow H(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

$$u[n] - u[n-N] \leftrightarrow e^{-j\frac{\omega(N-1)}{2}}\frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$\delta[n] \leftrightarrow 1$$

$$e^{j\alpha n} \leftrightarrow 2\pi\delta(\omega-\alpha)$$

$$\sum_{\ell=-\infty}^{\infty}\delta[n-\ell T_0] \leftrightarrow \left(\frac{2\pi}{T_0}\right)\sum_{k=1}^{T_0-1}\delta\left(\omega - \frac{2\pi k}{T_0}\right)$$

Gaussians, Mahalanobis, and PCA

$$\mathcal{N}\left(\vec{x};\vec{\mu},\Sigma\right) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}}e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu})}$$

$$\begin{aligned} \Sigma &= U\Lambda U^T \\ \Sigma^{-1} &= U\Lambda^{-1}U^T \\ U^T\Sigma U &= \Lambda \\ U^T U &= I \end{aligned}$$

$$d_\Sigma^2(\vec{x},\vec{\mu}) = (\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu}) = \vec{y}^T\Lambda^{-1}\vec{y}$$

$$\vec{y} = U^T(\vec{x}-\vec{\mu})$$

Problem 1 (10 points)

Suppose that a particular speech signal has the following log-magnitude DTFT:

$$\log|S(\omega)| = (62 + 6\cos\omega) + \frac{2\pi}{10}\sum_{k=0}^{9}\delta\left(\omega - \frac{2\pi k}{10}\right)$$

Suppose that the spectrum is liftered as follows:

$$\hat{s}[n] = \text{DTFT}^{-1}\left\{\log|S(\omega)|\right\}, \quad \hat{y}[n] = w[n]\hat{s}[n], \quad \log|Y(\omega)| = \text{DTFT}\left\{\hat{y}[n]\right\}$$

Where $w[n] = 1$ for $|n| \leq 2$, and 0 otherwise. Find $\log|Y(\omega)|$.

Problem 2 (10 points)

Recall that the IDFT is defined to be

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N}$$

(a) For what types of inputs, $X[k]$, can the $e^{j2\pi kn/N}$ term be replaced by $\cos(2\pi kn/N)$ without changing the output?

(b) Specify a further condition on $X[k]$ that will result in $x[n]$ being real valued, and demonstrate that $X[k] = \ln|S[k]|$ satisfies this condition, where $S[k]$ is the DFT of any real-valued signal $s[n]$.

Problem 3 (10 points)

A scalar random variable X is distributed as

$$p_{X|Y}(x|y) = \sum_{k=0}^{1} c_{yk}\mathcal{N}(x;\mu_{yk},1) \tag{1}$$

where $\mathcal{N}(x;\mu_{yk},1)$ is a scalar Gaussian with unit variance, and with class-dependent cluster-dependent means

$$\mu_{00} = -1, \quad \mu_{01} = 1, \quad \mu_{10} = 0, \quad \mu_{11} = 2 \tag{2}$$

and mixture weights

$$c_{00} = \frac{1}{2}, \quad c_{01} = \frac{1}{2}, \quad c_{10} = 1, \quad c_{11} = 0 \tag{3}$$

(a) Based on Eqs. 1 through 3, design a classifier $\hat{y}(x)$ such that

$$\hat{y}(x) = \arg\max_{y} \ln p_{X|Y}(x|y)$$

For which values of x is $\hat{y}(x) = 1$? Hint: you may find it useful to apply the approximation $\ln(e^a + e^b) \approx \max(a, b)$.

(b) Suppose that you are given the following training data, all of which comes from class $y = 0$:

$$\{a_1, a_2, a_3\} = \{-2, -1, 1\} \tag{4}$$

Use the expectation-maximization algorithm to re-estimate the two Gaussian mean parameters μ_{00} and μ_{01}. Give numerical values for the new values of μ_{00} and μ_{01}; in order to do so, assume that the unit normal Gaussian pdf has the following values: $\mathcal{N}(0; 0, 1) \approx \frac{2}{5}$, $\mathcal{N}(1; 0, 1) \approx \frac{1}{4}$, $\mathcal{N}(2; 0, 1) \approx \frac{1}{20}$, and $\mathcal{N}(x; 0, 1) \approx 0$ for $x \geq 3$. Note: since you didn't bring a calculator to this exam, feel free to leave your answer in the form of a fraction of fractions.

Problem 4 (10 points)

Suppose you have a scalar random variable X, distributed according to a GMM with unit variance:

$$p_X(x) = \sum_{k=0}^{1} c_k \mathcal{N}(x; \mu_k, \sigma^2 = 1)$$

Remember that the γ-probability is defined as:

$$\gamma_k(x) = \frac{c_k \mathcal{N}(x; \mu_k, 1)}{\sum_{\ell=0}^{1} c_\ell \mathcal{N}(x; \mu_\ell, 1)}$$

Demonstrate that $\gamma_1(x)$ has the following form, and find the values of a and τ in terms of μ_0, μ_1, c_0, and c_1:

$$\gamma_1(x) = \frac{1}{1 + e^{-a(x-\tau)}}$$