Lecture 11 Sample Problem Solutions

Problem 11.1

\[ \Delta = \frac{1}{33} (m(8000) - m(0)) = \frac{2595}{33} \log_{10} \left( \frac{87}{7} \right) \]
\[ f_m = 700 \left( 10^{m\Delta/2595} - 1 \right) = 700 \left( \frac{87}{7} \right)^{\frac{m}{33}} - 1 \]
\[ k_m = \frac{N f_m}{F_s} = \left( \frac{7 \times 1024}{160} \right) \left( \frac{87}{7} \right)^{\frac{m}{33}} - 1 \]

Problem 11.2

The DCT is equivalent to the real-symmetric DFT of a real-symmetric function. So, under the assumption that \( c[0] = 0 \), computing the DCT of a 12-sample rectangular window is equivalent to computing the DFT of a \((2 \times 12 + 1)\)-sample, zero-centered window,

\[ w[n] = \begin{cases} 
1 & -12 \leq n \leq 12 \\
0 & \text{otherwise}
\end{cases} \]

...under the assumption that \( c[0] = 0 \). So if that assumption is valid, then

\[ W(e^{j\omega}) = \frac{\sin(\omega 25/2)}{25 \sin(\omega/2)} \]

which has its first null at \( \omega = \frac{2\pi}{25} \), corresponding to \( f = \frac{16000}{25} = 640 \) Hertz.

Problem 11.3

\( W \) is the DFT matrix, so

\[ w_{mn} = e^{-j2\pi mn} / N \]

\( V \) is the mel-frequency filterbank matrix, so

\[ v_{lm} = \begin{cases} 
m_{m-l-1} & m_l \geq m \geq m_{l-1} \\
m_{l+1}-m & m_{l+1} \geq m \geq m_l \\
0 & \text{otherwise}
\end{cases} \]

with band edges \( m_l \) as given in problem 1.

\( U \) is the (inverse) discrete cosine transform matrix, which is the symmetric real IDFT of a symmetric real sequence. It’s not totally clear what should be the length of the DFT, here, but a reasonable choice is \( 2L + 1 \), which would give:

\[ u_{kl} = \frac{2}{2L + 1} \cos \left( \frac{2\pi kl}{2L + 1} \right) \]