Problem 5.1

Suppose \( s[n] = \sum_{m=0}^{P-1} S_m e^{jmn\omega_0} \), and \( s_f[n] = w[n]s[n + fK] \), where \( w[n] \) is the rectangular window,

\[
    w[n] = \begin{cases} 
        1 & 0 \leq n \leq N - 1 \\
        0 & \text{else} 
    \end{cases}
\]

Find \( S_0(e^{j\omega}) \), the DTFT of the zero'th frame, in terms of \( dsinc(\theta, L) = \frac{\sin(\theta L/2)}{L \sin(\theta/2)} \), \( S_m \), \( P \), and \( R = L/P \), which is the number of pitch periods per window. Notice that your answer doesn’t depend on \( R \) being an integer, or even a rational number.

Problem 5.2

Suppose \( s[n] = \sum_{m=0}^{P-1} S_m e^{jmn\omega_0} \), and \( s_f[n] = w[n]s[n + fK] \), where \( w[n] \) is any window. Notice that \( s[n + fK] = s[n] * \delta[n + fK] \), that is, shifting \( s[n] \) to the left in time is the same thing as convolving with an impulse at time \( n = -fK \). Using this observation, or using any other approach that you find convenient, find the DTFT of \( s_f[n] = s[n + fK]w[n] \) in terms of \( W(e^{j\omega}) \), \( P \), \( S_m \), and the fixed phase-shift terms \( \theta_{fm} = \frac{2\pi mfK}{P} \).

Problem 5.3

Suppose \( s[n] = \sum_{p=-\infty}^{\infty} h[n - pP] \). Find \( S_f(e^{j\omega}) \) in terms of \( \theta_{fm}, P, W(e^{j\omega}), \) and \( H(e^{j\omega}) \).