Problem 27.1

Suppose an RNN has input $x[n]$, and output $y[n] = \sigma(bx[n] + ay[n-1])$. Suppose the error is a binary cross-entropy between output $y[N]$ and target $t[N]$ at time $N$, i.e.,

$$E = t[N] \ln y[N] + (1 - t[N]) \ln(1 - y[N])$$

Define

$$\delta[n] = \frac{\partial E}{\partial y[n]}$$

1. Find $\delta[N]$.

2. Find a recursive formula for $\delta[n]$ in terms of itself, and in terms of $\dot{y}[n] = \dot{\sigma}(bx[n] + ay[n-1])$, where $\dot{\sigma}$ is the derivative of the logistic function.

3. Find $\frac{\partial E}{\partial b}$ in terms of $\delta[n]$, $\dot{y}[n]$, and $x[n]$.

Problem 27.2

Suppose a GRU has input $x[n]$, and output $y[n] = i[n]x[n] + f[n]y[n-1]$. Define

$$\delta[n] = \frac{\partial E}{\partial y[n]}$$

Find a recursive formula for $\delta[n]$ in terms of $i[n]$ and/or $f[n]$.