UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 8

Fall 2014

Assigned: Thursday, November 20, 2014 Due: Wednesday, Decmeber 4, 2014

Reading:

1 Upsampling and Downsampling

Do **one** of the following three problems.

Problem 8.1.1

Suppose you have a signal $x[n]$, and you want to delay it by half a sample. This is possible, e.g., using the following sequence of operations:

$$x[n] \rightarrow \boxed{\uparrow 2} \rightarrow \boxed{g[n]} \overset{y[n]}{\rightarrow} \boxed{z^{-1}} \overset{y[n-1]}{\rightarrow} \boxed{\downarrow 2} \rightarrow z[n]$$

In words: upsample $x[n]$ by a factor of two and then interpolate by $g[n]$ to create $y[n]$, delay $y[n]$ by one sample, then downsample $y[n-1]$ by a factor of two to create $z[n]$. Recall that upsampling+interpolation means that

$$y[n] = \sum_{m=-\infty}^{\infty} x[2m]g[n-2m]$$

Suppose the signal is

$$x[n] = \begin{cases} \cos(\pi n/4) & -2 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

thus $x[-2] = 0$, $x[-1] = 0.707$, $x[0] = 1$, $x[1] = 0.707$, $x[2] = 0$.

(a) Find the value of $z[n]$, for $-1 \leq n \leq 2$, if interpolation is performed using zero-order hold (PWC interpolation).

(b) Find the value of $z[n]$, $-1 \leq n \leq 2$, if interpolation is performed using first-order hold (PWL interpolation).

(c) Find the value of $z[n]$, $-1 \leq n \leq 2$, if sinc interpolation is used. You may find it useful to know that

$$\text{sinc}(0) = 1,\ \text{sinc}\left(\frac{\pi}{2}\right) = 0.6366, \quad \text{sinc}\left(\pi\right) = 0, \quad \text{sinc}\left(\frac{3\pi}{2}\right) = -0.2122$$

$$\text{sinc}\left(2\pi\right) = 0, \quad \text{sinc}\left(\frac{5\pi}{2}\right) = 0.12732, \quad \text{sinc}\left(3\pi\right) = 0, \quad \text{sinc}\left(\frac{7\pi}{2}\right) = -0.09095$$

(d) Find the desired ideal value of $z[n]$, $z[n] = \cos\left(\frac{\pi(n-0.5)}{4}\right)$, for $-1 \leq n \leq 1$.

Problem 8.1.2

Same as problem 8.1, but now

$$x[n] = \begin{cases} n^2 & -2 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

thus $x[-2] = 4$, $x[-1] = 1$, $x[0] = 0$, $x[1] = 1$, $x[2] = 4$.

Problem 8.1.3

Same as problem 8.1, but now

$$x[n] = \begin{cases} n^3 & -2 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

thus $x[-2] = -8$, $x[-1] = -1$, $x[0] = 0$, $x[1] = 1$, $x[2] = 8$.