UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 6

Fall 2014

Assigned: Thursday, October 16, 2014 Due: Friday, October 23, 2014

Reading: *Fundamentals of Signal Processing* by Minh Do, Sections 1-3 and 1-4

1 Frequency Response

Do **one** of the following two problems.

Problem 6.1.1

Consider the FIR filter $h[n] = 0.25\delta[n+2] + 0.5\delta[n+1] + \sqrt{3}\delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2]$.

(a) Calculate the frequency response, $H_d(\omega)$, of this filter. Note that, because it is symmetric in the time domain, the frequency response of this filter can be written as the sum of three cosines, and that is a useful way to write it.

(b) Calculate and sketch the magnitude and phase of this filter (hint: find $|H_d(\omega)|$ and $\angle H_d(\omega)$ for a few magic angles like $\omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$. Assume that the values at other frequencies are smoothly interpolated between those values).

(c) Is it a lowpass, highpass, bandpass, or bandstop filter?

Problem 6.1.2

(a) Use Zeno's paradox ($\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$) to compute the DTFT of

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Notice that your answer is only valid for $\omega \neq 0$, because the DTFT goes to infinity at $\omega = 0$.

(b) Use your answer to part (a), together with the time-shift property of the DTFT, to find the DTFT $R(\omega)$ of the rectangular window $r[n]$, where

$$r[n] = u[n+M] - u[n-(M+1)] = \begin{cases} 1 & -M \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

Notice that $r[n]$ is symmetric in time, so your answer should be real-valued!

(c) What is the first null of $R(\omega)$ (the first frequency at which $R(\omega) = 0$)?

(d) The first side-lobe of $R(\omega)$ is at the frequency $\omega = \frac{3\pi}{2M+1}$. What is its value, $R\left(\frac{3\pi}{2M+1}\right)$?