UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 5
Fall 2014

Assigned: Thursday, October 9, 2014 Due: Friday, October 16, 2014

Reading: *Fundamentals of Signal Processing* by Minh Do, Sections 1-3 and 1-4

1 Linearity and Time-Invariance

Do **one** of the following two problems.

Problem 5.1.1

Consider the system

$$y[n] = x[n] - x[0]$$

(a) Determine whether or not the system is linear. If nonlinear, give an example of signals $x_1[n] \to y_1[n]$, $x_2[n] \to y_2[n]$, and $x_3[n] \to y_3[n]$ such that $x_3[n] = x_1[n] + x_2[n]$ but $y_3[n] \neq y_1[n] + y_2[n]$.

(b) Determine whether or not the system is time-invariant. If time-varying, give an example of signals $x_1[n] \to y_1[n]$ and $x_2[n] \to y_2[n]$ such that $x_2[n] = x_1[n-d]$ but $y_2[n] \neq y_1[n-d]$.

Problem 5.1.2

Consider the system

$$y[n] = x[n] - 1$$

(a) Determine whether or not the system is linear. If nonlinear, give an example of signals $x_1[n] \to y_1[n]$, $x_2[n] \to y_2[n]$, and $x_3[n] \to y_3[n]$ such that $x_3[n] = x_1[n] + x_2[n]$ but $y_3[n] \neq y_1[n] + y_2[n]$.

(b) Determine whether or not the system is time-invariant. If time-varying, give an example of signals $x_1[n] \to y_1[n]$ and $x_2[n] \to y_2[n]$ such that $x_2[n] = x_1[n-d]$ but $y_2[n] \neq y_1[n-d]$.

2 Convolution and Impulse Response

Do **one** of the following two problems.

Problem 5.2.1

Find the convolution $y[n] = \text{conv}(x[n], h[n])$, for

$$x[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$h[n] = \begin{cases} 1 & n = 0 \\ 0.5 & n = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 5.2.2

Find $y[n] = x[n] * h[n]$, where

$$h[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$x[n] = \begin{cases} 1 & 0 \leq n \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

3 Frequency Response

Do **one** of the following three problems.

Problem 5.3.1

Consider the system

$$y[n] = x[n] - 0.5y[n-1]$$

The input to the system is $x[n] = \cos(\omega n)$ for some frequency ω. Find the output of the system. You should express the output as as $y[n] = A\cos(\omega n + \theta)$, where A and θ are terms that depend on ω.

Problem 5.3.2

Consider the system

$$y[n] = x[n] + 0.6x[n-1] + 0.4y[n-1]$$

The input to the system is $x[n] = \cos(\omega n)$ for some frequency ω. Find the output of the system. You should express the output as as $y[n] = A\cos(\omega n + \theta)$, where A and θ are terms that depend on ω.

Problem 5.3.3

Consider the system

$$y[n] = 0.5x[n] + 0.6x[n-1] - 0.3y[n-1]$$

The input to the system is $x[n] = \cos(\omega n)$ for some frequency ω. Find the output of the system. You should express the output as as $y[n] = A\cos(\omega n + \theta)$, where A and θ are terms that depend on ω.