UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 1

Fall 2014

Assigned: Thursday, August 28, 2014 Due: Thursday, September 4, 2014

Reading: Jason Starck, *All About Circuits* chapter 2: Complex numbers,
`http://www.allaboutcircuits.com/vol_2/chpt_2/`

General Instructions:

Each week, there will be two or three topics assigned. In each topic, there will be three problems available. By the due date, you should do *one problem per topic*, for a total of two problems, which you will turn in on the due date. If you turn in more than two problems, only the first one per topic will be graded.

Grading:

- One point per problem is given for doing the problem on time. You will receive that point only if you seem to have made a reasonable effort to solve the problem. If special circumstances keep you from handing in your solution on time, contact the instructor by the start of class (earlier is better, in case the instructor thinks your reasons are not valid). Tell himn why you need an extension, and how much time you need. If he considers your reason valid, he can give you an extension.
- Five points per topic are given for correct answers. If you don't receive all five points, then you can hand in one of the other problems for the same topic, at any time prior to the last minute of the last scheduled lecture of the semester. If your second try is better, its score will replace the first. This process can be repeated a third time if you wish.

1 Continuous and Discrete-Time Signals

Do **one** of the following three problems.

Problem 1.1.1

A particular acoustic signal is given by the following air pressure, in Pascals, at the microphone:

$$x_c(t) = \cos(2\pi 2000t) + 2\sin(2\pi 4000t) + 3\sin(2\pi 2000t) + 4\sin(2\pi 7000t) + 5\cos(2\pi 8000t)$$

$x_c(t)$ is sampled by an ideal A/D at 8000 samples/second, to create DT signal $x[n]$. $x[n]$ is then passed into an ideal D/A, at 10000 samples/second, to construct CT signal $y_c(t)$.

Find $x[n]$ as a function of n, and $y_c(t)$ as a function of t. Simplify each term to a constant if possible, otherwise simplify each term to $Af(2\pi\phi n)$ or $Af(2\pi\phi t)$, where f is either `sin` or `cos`, and where both A and ϕ are either integers or irreducible fractions (irreducible means that the numerator and denominator are integers with no common divisors).

Problem 1.1.2

A particular acoustic signal is given by the following air pressure, in Pascals, at the microphone:

$$x_c(t) = \cos(2\pi 500t) + 2\cos(2\pi 16,000t) + 3\sin(2\pi 9000t) + 4\cos(2\pi 10,000t) + 5\sin(2\pi 8000t)$$

$x_c(t)$ is sampled by an ideal A/D at 16,000 samples/second, to create DT signal $x[n]$. $x[n]$ is then passed into an ideal D/A, at 8000 samples/second, to construct CT signal $y_c(t)$.

Find $x[n]$ as a function of n, and $y_c(t)$ as a function of t. Simplify each term to a constant if possible, otherwise simplify each term to $Af(2\pi\phi n)$ or $Af(2\pi\phi t)$, where f is either `sin` or `cos`, and where both A and ϕ are either integers or irreducible fractions (irreducible means that the numerator and denominator are integers with no common divisors).

Problem 1.1.3

A particular acoustic signal is given by the following air pressure, in Pascals, at the microphone:

$$x_c(t) = \cos(2\pi 11,000t) + 2\cos(2\pi 10,000t) + 3\sin(2\pi 9000t) + 4\sin(2\pi 5500t) + 5\sin(2\pi 500t)$$

$x_c(t)$ is sampled by an ideal A/D at 11,000 samples/second, to create DT signal $x[n]$. $x[n]$ is then passed into an ideal D/A, at 44,000 samples/second, to construct CT signal $y_c(t)$.

Find $x[n]$ as a function of n, and $y_c(t)$ as a function of t. Simplify each term to a constant if possible, otherwise simplify each term to $Af(2\pi\phi n)$ or $Af(2\pi\phi t)$, where f is either `sin` or `cos`, and where both A and ϕ are either integers or irreducible fractions (irreducible means that the numerator and denominator are integers with no common divisors).

2 Period and Frequency

Do **one** of the following three problems.

Problem 1.2.1

An acoustic signal has a pressure, at the microphone, of

$$x_c(t) = 5\sin(2\pi 220t)\cos(2\pi 660t) \text{ Pa}$$

(a) Write this signal as $x_c(t) = A_1\cos(2\pi f_1 t + \theta_1) + A_2\cos(2\pi f_2 t + \theta_2)$ for some scalar real numbers $A_1, A_2, f_1, f_2, \theta_1, \theta_2$.

(b) At what two frequencies (in Hertz) does this signal have energy?

(c) What musical notes (between A1 and E7) are the two frequencies you specified in part (b)?

(d) What is the fundamental frequency of this signal (in Hertz)?

Problem 1.2.2

An acoustic signal has a pressure, at the microphone, of

$$x_c(t) = 3\sin(2\pi 64t)\sin(2\pi 192t) \text{ Pa}$$

(a) Write this signal as $x_c(t) = A_1\sin(2\pi f_1 t + \theta_1) + A_2\sin(2\pi f_2 t + \theta_2)$ for some scalar real numbers $A_1, A_2, f_1, f_2, \theta_1, \theta_2$.

(b) At what two frequencies (in Hertz) does this signal have energy?

(c) What musical notes (between A1 and E7) are the two frequencies you specified in part (b) (round to the nearest semitone)?

(d) What is the fundamental frequency of this signal (in Hertz)?

Problem 1.2.3

An acoustic signal has a pressure, at the microphone, of

$$x_c(t) = 5\cos(2\pi 768t)\cos(2\pi 256t) \text{ Pa}$$

(a) Write this signal as $x_c(t) = A_1 \sin(2\pi f_1 t + \theta_1) + A_2 \sin(2\pi f_2 t + \theta_2)$ for some scalar real numbers $A_1, A_2, f_1, f_2, \theta_1, \theta_2$.

(b) At what two frequencies (in Hertz) does this signal have energy?

(c) What musical notes (between A1 and E7) are the two frequencies you specified in part (b) (round to the nearest semitone)?

(d) What is the fundamental frequency of this signal (in Hertz)?