11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3

Lecture 10: Exam 1 Sample Problems

Mark Hasegawa-Johnson All content CC-SA 4.0 unless otherwise specified.

ECE 401: Signal and Image Analysis, Fall 2021

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11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Outline

🚺 Fall 2011 Exam 1 Problem 1

11×1p1	11×1p4a	11×1p5b	11x3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Question

Calculate the Fourier series coefficients X_0 and X_k for the periodic signal x(t) = x(t+8):

$$x(t) = \left\{egin{array}{cc} 1, & 0 \leq t < 1\ -1, & 1 \leq t \leq 3\ 0, & 3 < t < 8 \end{array}
ight.$$

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Calculate the Fourier series coefficients X_0 and X_k for the periodic signal x(t) = x(t+8):

$$X_{0} = \frac{1}{8} \int_{0}^{8} x(t) dt$$
$$= \frac{1}{8} \left(\int_{0}^{1} dt - \int_{1}^{3} dt \right)$$
$$= -\frac{1}{8}$$

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11x1p1 11x1p4 11x1p5 11x3p6 13x1p1 13x1p2 13x3p1 13x3p2 13x3p8 14x1p1 14x1p2 14x1p3 14x3p3 000 00 00 00

Calculate the Fourier series coefficients X_0 and X_k for the periodic signal x(t) = x(t+8):

$$\begin{aligned} X_k &= \frac{1}{8} \int_0^8 x(t) e^{-j2\pi kt/8} dt \\ &= \frac{1}{8} \left(\int_0^1 e^{-j2\pi kt/8} dt - \int_1^3 e^{-j2\pi kt/8} dt \right) \\ &= \frac{1}{8} \left(\frac{1}{-j2\pi k/8} \right) \left(\left[e^{-j2\pi kt/8} \right]_0^1 - \left[e^{-j2\pi kt/8} \right]_1^3 \right) \\ &= \left(\frac{1}{-j2\pi k} \right) \left(2e^{-j2\pi k/8} - 1 - e^{-j6\pi k/8} \right) \end{aligned}$$

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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Outline

2 Fall 2011 Exam 1 Problem 4(a)

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Question

Suppose that we have a signal bandlimited to 5kHz. What is the minimum F_s necessary to avoid aliasing?

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Answer

10kHz

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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Outline

3 Fall 2011 Exam 1 Problem 5(b)

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14×1p2	14x1p3	14x3p3
		00											

Question

Assume that $x[n] = x_c(nT)$, where 1/T = 10,000 samples/second. Find x[n] and its spectrum if

$$x_c(t) = \cos(7000\pi t)$$

Answer

$$x[n] = \cos\left(\frac{7000\pi n}{10,000}\right)$$

... and the spectrum is

$$\left\{\left(-\frac{7000\pi}{10000},\frac{1}{2}\right),\left(\frac{7000\pi}{10000},\frac{1}{2}\right)\right\}$$

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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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Part (a)

Consider the signal $x(t) = -2 + \sin(40\pi t)$. Determine and list all of the analog frequencies in the signal x(t). Include negative frequencies.

Answer

 $\{-20,0,20\}$

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Part (b)

 $x(t) = -2 + \sin(40\pi t)$. What is the lowest possible sampling frequency that would avoid aliasing?

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Answer

 $F_s > 2f = 40$

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Part (c)

What is the corresponding Nyquist frequency for the sampling rate you found in part (b)?

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Answer

$$F_N = \frac{F_s}{2} > 20$$

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
			000000										

Part (d)

 $x(t) = -2 + \sin(40\pi t)$. For a sampling frequency of $F_s = 100$ Hz, find x[n].

Answer

$$x[n] = -2 + \sin\left(\frac{40\pi n}{100}\right)$$

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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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Part (e)

 $x(t) = -2 + \sin(40\pi t)$, $F_s = 100$ Hz. Determine and list all of the frequencies ω , $-\pi < \omega \leq \pi$, present in the discrete-time signal x[n]. Include negative frequencies.

Answer

$$\left\{-\frac{40\pi}{100}, 0, \frac{40\pi}{100}\right\}$$

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Outline

5 Fall 2013 Exam 1 Problem 1

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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Question

$$\cos(\omega t) + \cos(\omega t + \frac{\pi}{3}) = m\cos(\omega t + \theta)$$

x and y such that $m = \sqrt{x^2 + y^2}$ and $\theta = \operatorname{atan2}(x, y)$, the

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Find two-argument arctangent of x and y.

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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Answer

$$\cos(\omega t) + \cos(\omega t + \frac{\pi}{3}) = \Re \left\{ (1 + e^{j\pi/3})e^{j\omega t} \right\}$$
$$= \Re \left\{ (1 + \cos(\pi/3) + j\sin(\pi/3)) e^{j\omega t} \right\}$$

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$$x = 1 + \cos(\pi/3), \quad y = \sin(\pi/3)$$

11×1p1	11×1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Outline

6 Fall 2013 Exam 1 Problem 2

11×1p1	11×1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Question

A signal $x(t) = \cos(2\pi 6000t)$ is sampled at $F_s = 8000$ samples/second to create y[n]. The digital signal y[n] is then played back through an ideal D/A at the same sampling rate, $F_s = 8000$ samples/second, to generate a signal z(t). Find z(t).

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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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Answer

$$\begin{aligned} x(t) &= \cos(2\pi 6000t) \\ y[n] &= \cos\left(\frac{2\pi 6000n}{8000}\right) \\ &= \cos\left(\frac{3\pi n}{2}\right) \\ &= \cos\left(\frac{\pi n}{2}\right) \\ z(t) &= \cos\left(\frac{\pi}{2}8000t\right) = \cos(4000\pi t) \end{aligned}$$

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Question

The signal x[n] is periodic with period $N_0 = 4$. Its values in each period are

$$x[n] = \begin{cases} 1 & n = 0 \\ -1 & n = 1, 2, 3 \end{cases}$$

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Find the Fourier series coefficients.

11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Answer

$$X_{k} = \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-j2\pi kn/4}$$
$$= \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-j\pi kn/2}$$
$$= \frac{1}{4} \left(1 - e^{-j\pi k/2} - e^{-j\pi k} - e^{-j\pi 3k/2} \right)$$

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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Question

$$\begin{split} & 6\cos\left(2\pi1000\left(t-\frac{1}{4000}\right)\right)+6\sin\left(2\pi1000\left(t-\frac{1}{4000}\right)\right)\\ & = A\cos(\Omega t+\phi) \end{split}$$

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Find A, Ω , and ϕ .

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Answer

$$\begin{aligned} 6\cos\left(2\pi 1000\left(t-\frac{1}{4000}\right)\right) + 6\sin\left(2\pi 1000\left(t-\frac{1}{4000}\right)\right) \\ &= 6\cos\left(2\pi 1000\left(t-\frac{1}{4000}\right)\right) + 6\cos\left(2\pi 1000\left(t-\frac{1}{4000}\right)-\frac{\pi}{2}\right) \\ &= 6\cos\left(2\pi 1000t-\frac{\pi}{2}\right) + 6\cos\left(2\pi 1000t-\frac{\pi}{2}-\frac{\pi}{2}\right) \\ &= \Re\left\{6(e^{-j\pi/2}+e^{-j\pi})e^{j2000\pi t}\right\} \\ &= \Re\left\{6(-j-1)e^{j2000\pi t}\right\} \\ &= \Re\left\{6\sqrt{2}e^{-j3\pi/4}e^{j2000\pi t}\right\} \end{aligned}$$

So
$$A = 6\sqrt{2}$$
, $\Omega = 2000\pi$, $\phi = -\frac{3\pi}{4}$.

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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11x1p1	11x1p4a	11×1 p5b	11x3p6	13x1p1	13x1p2	13x1p3	13x3p1	13x3p2	13x3p8	14x1p1	14x1p2	14x1p3	14x3p3
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A periodic signal x(t), with period T_0 , is given by

$$x(t) = \left\{egin{array}{cc} 1 & 0 \leq t \leq rac{3T_0}{4} \ 0 & rac{3T_0}{4} < t < T_0 \end{array}
ight.$$

The same signal can be expressed as a Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

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Find $|X_2|$, the amplitude of the second harmonic.

11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Answer

$$\begin{aligned} X_2 &= \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j2\pi 2t/T_0} dt \\ &= \frac{1}{T_0} \int_0^{3T_0/4} e^{-j4\pi t/T_0} dt \\ &= \frac{1}{T_0} \left(\frac{1}{-j4\pi/T_0}\right) \left[e^{-j4\pi t/T_0} \right]_0^{3T_0/4} \\ &= \left(\frac{1}{-j4\pi}\right) \left(e^{-j3\pi} - 1 \right) = \left(\frac{-2}{-j4\pi}\right) \end{aligned}$$

So $|X_2| = 1/2\pi$.

11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13×3p8	14×1p1	14x1p2	14×1p3	14x3p3
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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Question

An 8000Hz tone, $x(t) = \cos(2\pi 8000t)$, is sampled at $F_s = \frac{1}{T} = 10,000$ samples/second in order to create x[n] = x(nT). Sketch $X(\omega)$ for $0 \le \omega \le 2\pi$ (**note the domain!!**). Specify the frequencies at which $X(\omega) \ne 0$.

Answer

Answer should be a spectrum plot with spikes at $\omega = 8\pi/5$ and $\omega = 2\pi/5$, each labeled with a phasor of 1/2.

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11×1p1	11×1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14x1p1	14x1p2	14x1p3	14x3p3
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11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14x1p3	14x3p3
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Each of the following is sampled at $F_s = 10000$ samples/second, producing either x[n] = constant, or $x[n] = \cos \omega n$ for some value of ω . Specify the constant if possible; otherwise, specify ω such that $-\pi \leq \omega < \pi$.

$$x(t) = \cos\left(2\pi900t\right)$$

Answer

Solution: $\omega = \frac{1800\pi}{10,000}$

11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13x1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Each of the following is sampled at $F_s = 10000$ samples/second, producing either x[n] = constant, or $x[n] = \cos \omega n$ for some value of ω . Specify the constant if possible; otherwise, specify ω such that $-\pi \leq \omega < \pi$.

$$x(t) = \cos\left(2\pi 10000t\right)$$

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Answer

Solution: x[n] = 1

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Each of the following is sampled at $F_s = 10000$ samples/second, producing either x[n] = constant, or $x[n] = \cos \omega n$ for some value of ω . Specify the constant if possible; otherwise, specify ω such that $-\pi \leq \omega < \pi$.

$$x(t) = \cos\left(2\pi 11000t\right)$$

Answer	
Solution:	$\omega = \frac{22000\pi}{10000} - 2\pi = \frac{2000\pi}{10000}$

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11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13x1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Question

Consider the signal

$$x(t) = 2\cos(2\pi 440t) - 3\sin(2\pi 440t)$$

This signal can also be written as $x(t) = A\cos(\omega t + \theta)$ for some $A = \sqrt{M}$, ω , and $\theta = \operatorname{atan}(R)$. Find M, ω , and R.

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11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Answer

$$\begin{aligned} x(t) &= 2\cos(2\pi 440t) - 3\sin(2\pi 440t) \\ &= 2\cos(2\pi 440t) - 3\cos\left(2\pi 440t - \frac{\pi}{2}\right) \\ &= \Re\left\{(2 - 3e^{-j\pi/2})e^{j2\pi 440t}\right\} \\ &= \Re\left\{(2 + 3j)e^{j2\pi 440t}\right\} \\ &= \Re\left\{\sqrt{5}e^{j\operatorname{atan}(3/2)}e^{j2\pi 440t}\right\} \end{aligned}$$

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So $A = \sqrt{13}$, $\omega = 2\pi 440$, and $\theta = \operatorname{atan}(3/2)$.

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
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11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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A signal x(t) is periodic with $T_0 = 0.02$ seconds, and its values are specified by

$$\mathbf{x}(t) = \left\{ egin{array}{cc} -1 & 0 \leq t \leq 0.01 \ 0 & 0.01 < t < 0.02 \end{array}
ight.$$

Sketch x(t) as a function of t for $0 \le t \le 0.02$ seconds. Label at least one important tic mark, each, on the horizontal and vertical axes.

Answer

Sketch should show x(t) = -1 between 0 and 0.01, then x(t) = 0 between 0.01 and 0.02.

11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14×1p2	14×1p3	14x3p3
												000000	

A signal x(t) is periodic with $T_0 = 0.02$ seconds, and its values are specified by

$$\mathbf{x}(t) = \left\{ egin{array}{cc} -1 & 0 \leq t \leq 0.01 \ 0 & 0.01 < t < 0.02 \end{array}
ight.$$

What is F_0 ?

Answer $F_0 = \frac{1}{T_0} = \frac{1}{0.02}$

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11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13x1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
												000000	

A signal x(t) is periodic with $T_0 = 0.02$ seconds, and its values are specified by

$$\kappa(t) = \left\{ egin{array}{cc} -1 & 0 \leq t \leq 0.01 \ 0 & 0.01 < t < 0.02 \end{array}
ight.$$

Find X_0 without doing any integral.

Answer

x(t) is -1 for half a period, and 0 for half a period, so its average value is $X_0 = -\frac{1}{2}$.

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A signal x(t) is periodic with $T_0 = 0.02$ seconds, and its values are specified by

$$x(t) = \left\{egin{array}{cc} -1 & 0 \leq t \leq 0.01 \ 0 & 0.01 < t < 0.02 \end{array}
ight.$$

Find X_k for all the other values of k, i.e., for $k \neq 0$. Simplify; your answer should have no exponentials in it.

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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Answer

$$\begin{aligned} X_k &= \frac{1}{0.02} \int_0^{0.02} x(t) e^{-j2\pi kt/0.02} dt \\ &= \frac{1}{0.02} \int_0^{0.01} e^{-j2\pi kt/0.02} dt \\ &= \frac{1}{0.02} \left(\frac{1}{-j2\pi k/0.02} \right) \left[e^{-j2\pi kt/0.02} \right]_0^{0.01} \\ &= \left(\frac{1}{-j2\pi k} \right) \left(e^{-j2\pi k0.01/0.02} - 1 \right) \\ &= \left(\frac{1}{-j2\pi k} \right) \left(e^{-j\pi k} - 1 \right) \\ &= \left(\frac{1}{-j2\pi k} \right) \left((-1)^k - 1 \right) \end{aligned}$$

11×1p1	11x1p4a	11×1p5b	11×3p6	13×1p1	13×1p2	13×1p3	13x3p1	13x3p2	13x3p8	14×1p1	14x1p2	14×1p3	14x3p3
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In order to become a billionaire, you've decided you need to know what was the total value of the U.S. GDP every day of every year since 1901. Unfortunately, GDP figures are only published once per year (once per 365 days), so you need to interpolate them. Consider the following system:

$$d[n] = \sum_{m=-\infty}^{\infty} y[m]g[n - 365m]$$
(1)

where y[m] is the GDP in the m^{th} year, and d[n] is the estimated GDP in the n^{th} day.

Design the filter g[n] so that Eq. 1 implements **PIECE-WISE LINEAR** interpolation. (Draw a sketch of g[n] that specifies the values of all of its samples, or write a formula that does so).

11×1p1	11x1p4a	11×1p5b	11x3p6	13×1p1	13×1p2	13×1p3	13×3p1	13x3p2	13x3p8	14×1p1	14x1p2	14x1p3	14x3p3
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Answer

$$g[n] = \begin{cases} 1 - \frac{|n|}{365} & -365 \le n \le 365 \\ 0 & \text{otherwise} \end{cases}$$

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