Problem 6.1

Consider the difference equation:

\[ y[n] = x[n] - \frac{1}{2}x[n - 1] + \frac{1}{4}x[n - 2] \]

Find the frequencies, \( \omega = \angle z_1 \) and \( \omega = \angle z_2 \), of the two zeros.

**Solution:** The transfer function is

\[ H(z) = 1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} \]

which has zeros at

\[ z = \frac{1}{4} \pm j\frac{\sqrt{3}}{4}. \]

The frequencies of these two zeros are

\[ \omega = \pm \frac{\pi}{3} \text{ radians/sample} \]

Problem 6.2

A particular filter has the difference equation

\[ y[n] = x[n] - 1.2e^{j3\pi/5}x[n - 1] + 0.8e^{j2\pi/5}y[n - 1] \]

Express the frequency response of this filter as

\[ H(\omega) = \frac{e^{j\omega} - z_1}{e^{j\omega} - p_1} \]

for some zero \( z_1 \) and pole \( p_1 \).

**Solution:**

\[ H(\omega) = \frac{e^{j\omega} - 1.2e^{j3\pi/5}}{e^{j\omega} - 0.8e^{j2\pi/5}} \]

Problem 6.3
Homework 6

Remember that
\[ G(z) = \frac{1}{1 - 0.8z^{-1}} \leftrightarrow g[n] = (0.8)^n u[n] \]

Use the linearity and time-shift properties of the Z-transform to find \( h[n] \), where
\[ H(z) = \frac{1 - 0.3z^{-1}}{1 - 0.8z^{-1}} = \frac{1}{1 - 0.8z^{-1}} - 0.3z^{-1} \frac{1}{1 - 0.8z^{-1}} \]

**Solution:** The time-shift property is
\[ z^{-n_0}G(z) \leftrightarrow g[n - n_0] \]
so
\[ G(z) - 0.3z^{-1}G(z) \leftrightarrow g[n] - 0.3g[n - 1] \]
therefore
\[ h[n] = (0.8)^n u[n] - 0.3(0.8)^{n-1} u[n - 1] \]

**Problem 6.4**

What is \( h[n] \) if
\[ H(z) = \frac{1}{(1 - e^{j0.1\pi}z^{-1})(1 - e^{-j0.1\pi}z^{-1})} \]

**Solution:** Using PFE, we get
\[ H(z) = \frac{C_1}{1 - e^{j0.1\pi}z^{-1}} + \frac{C_1^*}{1 - e^{-j0.1\pi}z^{-1}} \]
Solving for \( C_1 \), we can find that \( C_1 = p_1/(p_1 - p_1^*) = e^{j0.1\pi}/(2j \sin(0.1\pi)) \), so
\[ h[n] = \left( \frac{e^{j0.1\pi(n+1)}}{2j \sin(0.1\pi)} - \frac{e^{-j0.1\pi(n+1)}}{2j \sin(0.1\pi)} \right) u[n] = \frac{\sin(0.1\pi(n + 1))}{\sin(0.1\pi)} u[n] \]