Problem 4.1

Consider this filter:
\[ y[n] = x[n] + x[n - 1] \]
Show that the magnitude response of this filter is \( |H(\omega)| = 2 \cos(\omega/2) \).

Solution:
\[ h[n] = \begin{cases} 
1 & n = 0, 1 \\
0 & \text{otherwise} 
\end{cases} \]
\[ H(\omega) = \sum_n h[n]e^{-j\omega n} = 1 + e^{-j\omega} = e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2}) = e^{-j\omega/2}2 \cos(\omega/2) \]

Problem 4.2

Suppose you have a filter whose frequency response is
\[ H(\omega) = 14e^{-j6\omega} \]
Show that, if \( x[n] = \cos(\omega n) \), the effect of convolving \( y[n] = x[n] * h[n] \) is to
(a) scale \( x[n] \) by a factor of 14, and
(b) delay it by 6 samples.

Solution: Using the frequency response formula,
\[ y[n] = |H(\omega)| \cos (\omega n + \angle H(\omega)) \]
\[ = 14 \cos (\omega n - 6\omega) \]
\[ = 14 \cos (\omega (n - 6)) \]

Problem 4.3
The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):
\[
x_1(t) = \cos(2\pi1000t) \\
x_2(t) = \cos(2\pi2000t)
\]
The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a first-difference operator:
\[
y_1[n] = x_1[n] - x_1[n-1] \\
y_2[n] = x_2[n] - x_2[n-1]
\]
What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

**Solution:** In radians/second, the frequencies are
\[
\omega_1 = \frac{2\pi1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi2000}{16000} = \frac{\pi}{4}
\]
The frequency response of the first-difference operator is
\[
H(\omega) = 1 - e^{-j\omega} = 2je^{-j\omega/2} \sin(\omega/2)
\]
Which has these magnitude responses:
\[
\left|H\left(\frac{\pi}{8}\right)\right| = 2 \sin(\pi/16) \approx 0.39, \quad \left|H\left(\frac{\pi}{4}\right)\right| = 2 \sin(\pi/8) \approx 0.77
\]

**Problem 4.4**

The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):
\[
x_1(t) = \cos(2\pi1000t) \\
x_2(t) = \cos(2\pi2000t)
\]
The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a seven-sample local average:
\[
y_1[n] = \frac{1}{7} \sum_{m=-3}^{3} x_1[n-m] \\
y_2[n] = \frac{1}{7} \sum_{m=-3}^{3} x_2[n-m]
\]
What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

**Solution:** In radians/second, the frequencies are
\[
\omega_1 = \frac{2\pi1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi2000}{16000} = \frac{\pi}{4}
\]
The frequency response of the seven-sample central local average filter is
\[
H(\omega) = \frac{\sin(7\omega/2)}{7 \sin(\omega/2)}
\]
Which has these magnitude responses:
\[
\left|H\left(\frac{\pi}{8}\right)\right| = \frac{\sin(7\pi/16)}{7 \sin(\pi/16)} \approx 0.72, \quad \left|H\left(\frac{\pi}{4}\right)\right| = \frac{\sin(7\pi/8)}{7 \sin(\pi/8)} \approx 0.14
\]