Problem 4.1

Consider this filter:

\[ y[n] = x[n] + x[n-1] \]

Show that the magnitude response of this filter is \( |H(\omega)| = 2\cos(\omega/2) \).

Problem 4.2

Suppose you have a filter whose frequency response is

\[ H(\omega) = 14e^{-j6\omega} \]

Show that, if \( x[n] = \cos(\omega n) \), the effect of convolving \( y[n] = x[n] * h[n] \) is to

(a) scale \( x[n] \) by a factor of 14, and

(b) delay it by 6 samples.

Problem 4.3

The signals \( x_1(t) \) and \( x_2(t) \) are cosines an octave apart (roughly C6 and C7):

\[ x_1(t) = \cos(2\pi 1000t) \]
\[ x_2(t) = \cos(2\pi 2000t) \]

The signals are sampled (at \( F_s = 16000 \) samples/second), then the resulting signals \( x_1[n] \) and \( x_2[n] \) are passed through a first-difference operator:

\[ y_1[n] = x_1[n] - x_1[n-1] \]
\[ y_2[n] = x_2[n] - x_2[n-1] \]

What are the amplitudes of the signals \( y_1[n] \) and \( y_2[n] \)?

Problem 4.4

The signals \( x_1(t) \) and \( x_2(t) \) are cosines an octave apart (roughly C6 and C7):

\[ x_1(t) = \cos(2\pi 1000t) \]
\[ x_2(t) = \cos(2\pi 2000t) \]
The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a seven-sample local average:

\[
y_1[n] = \frac{1}{7} \sum_{m=-3}^{3} x_1[n-m]
\]

\[
y_2[n] = \frac{1}{7} \sum_{m=-3}^{3} x_2[n-m]
\]

What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?