• This is a CLOSED BOOK exam.
• You are permitted two sheets of handwritten notes, 8.5x11.
• Calculators and computers are not permitted.
• Do not simplify explicit numerical expressions. The expression “$e^{-5 \cos(3)}$” is a MUCH better answer than “-0.00667”.
• If you’re taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
• There will be a total of 200 points in the exam. Each problem specifies its point total. Plan your work accordingly.
• You must SHOW YOUR WORK to get full credit.

Name: ________________________________

netid: ________________________________
Phasors

\[ A \cos(2\pi ft + \theta) = \Re \left\{ Ae^{j\theta} e^{j2\pi ft} \right\} = \frac{1}{2} e^{-j\theta} e^{-j2\pi ft} + \frac{1}{2} e^{j\theta} e^{j2\pi ft} \]

Fourier Series

Analysis: \( X_k = \frac{1}{T_0} \int_{0}^{T_0} x(t) e^{-j2\pi kt/T_0} dt \)

Synthesis: \( x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0} \)

Sampling and Interpolation:

\[ x[n] = x \left( t = \frac{n}{F_s} \right) \]
\[ f_a = \min \left( f \mod F_s, -f \mod F_s \right) \]
\[ z_a = \begin{cases} z & f \mod F_s < -f \mod F_s \\ z^* & f \mod F_s > -f \mod F_s \end{cases} \]

\[ y(t) = \sum_{n=-\infty}^{\infty} y[n] p(t-nT_s) \]

Convolution

\[ h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m] x[n-m] = \sum_{m=-\infty}^{\infty} x[m] h[n-m] \]

Frequency Response and DTFT

\[ H(\omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} \]

\[ h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega \]

\[ h[n] \cos(\omega n) = |H(\omega)| \cos (\omega n + \angle H(\omega)) \]

Rectangular & Hamming Windows; Ideal LPF

\[ w_R[n] = \begin{cases} 1 & 0 \leq n \leq N - 1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-j\omega(N-1)/2} \frac{\sin(\omega N/2)}{\sin(\omega/2)} \]

\[ w_H[n] = 0.54 - 0.46 \cos \left( \frac{2\pi n}{N-1} \right) \leftrightarrow W_H(\omega) = 0.54 W_R(\omega) - 0.23 W_R \left( \omega - \frac{2\pi}{N-1} \right) - 0.23 W_R \left( \omega + \frac{2\pi}{N-1} \right) \]

\[ h_{\text{ideal}}[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases} \]

Discrete Fourier Transform

Analysis: \( X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \)

Synthesis: \( x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N} \)
Z Transform Pairs

\[ b_k z^{-k} \leftrightarrow b_k \delta[n - k] \]
\[ \frac{1}{1 - az^{-1}} \leftrightarrow a^n u[n] \]
\[ \frac{1}{(1 - e^{-\sigma_1 - j\omega_1} z^{-1})(1 - e^{-\sigma_1 + j\omega_1} z^{-1})} \leftrightarrow \frac{1}{\sin(\omega_1)} e^{-\sigma_1 n} \sin (\omega_1 (n + 1)) u[n] \]