This is a CLOSED BOOK exam.

You are permitted one sheet of handwritten notes, 8.5x11.

Calculators and computers are not permitted.

Do not simplify explicit numerical expressions. The expression \( e^{-5 \cos(3)} \) is a MUCH better answer than \(-0.00667\).

If you’re taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.

There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.

You must SHOW YOUR WORK to get full credit.

Name: ________________________________
Convolution

\[ h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \]

Frequency Response

\[ H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \]

\[ h[n] * \cos(\omega n) = |H(\omega)| \cos(\omega n + \angle H(\omega)) \]

Rectangular Window and Ideal LPF

\[ w_R[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad W_R(\omega) = e^{-j(\omega(N-1)/2)} \frac{\sin(\omega N/2)}{\sin(\omega/2)} \]

\[ h_{\text{ideal}}[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n) \quad \leftrightarrow \quad H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases} \]

Hamming Window

\[ w_H[n] = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \]
1. (20 points) Consider the following filter:

\[ h[n] = \begin{cases} 1 & -6 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases} \]

Sketch \(|H(\omega)|\) as a function of \(\omega\), for \(-\pi < \omega < \pi\). Label the amplitude at \(\omega = 0\), and at least two of the frequencies at which \(|H(\omega)| = 0\).
2. (20 points) Consider the following filter:

\[ h[n] = \begin{cases} 
1 & \text{if } -6 \leq n \leq 2 \\
0 & \text{otherwise} 
\end{cases} \]

Design a filter \( g[n] \) such that \( F(\omega) \) is real valued (i.e., its imaginary part is zero, \( \Im\{F(\omega)\} = 0 \)), where

\[ f[n] = g[n] \ast h[n] \]

\[ g[n] = \]
3. (20 points) The signal \( x(t) = \cos(2\pi 800t) \) is sampled at \( F_s = 8000 \) samples/second, then processed by the filter

\[
y[n] = \frac{1}{7} \sum_{m=-3}^{3} x[n-m]
\]

The resulting signal is \( y[n] = A \cos(\omega n + \theta) \). What are \( A \), \( \omega \), and \( \theta \)?

\[
A = \\
\omega = \\
\theta =
\]
4. (20 points) Consider the system

$$y[n] = \frac{1}{4} x[n + 2] + \frac{1}{2} x[n + 1] + x[n] - x[n - 1] - \frac{1}{2} x[n - 2] - \frac{1}{4} x[n - 3]$$

Sketch the impulse response of this system as a function of $n$, for $-5 \leq n \leq 5$. Clearly label the values of every nonzero sample.
5. (20 points) You want to approximate the following ideal bandpass filter:

\[ H_{\text{ideal}}(\omega) = \begin{cases} 
1 & 0.3\pi < |\omega| < 0.4\pi \\
0 & \text{otherwise} 
\end{cases} \]

Design an FIR filter \( h[n] \) that is exactly 64 samples long, such that \( |H(\omega)| \approx |H_{\text{ideal}}(\omega)| \). Window \( h[n] \) in such a way that the first sidelobe has a level of less than -40dB relative to the mainlobe.

\[ h[n] = \]