Problem 4.1

Consider the difference equation:

\[ y[n] = x[n] - \frac{1}{2} x[n-1] + \frac{1}{4} x[n-2] \]

Find the frequencies, \( \omega = \angle z_1 \) and \( \omega = \angle z_2 \), of the two zeros.

Solution: The transfer function is

\[ H(z) = 1 - \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2} \]

which has zeros at

\[ z = \frac{1}{4} \pm j \frac{\sqrt{3}}{4}. \]

The frequencies of these two zeros are

\[ \omega = \pm \frac{\pi}{3} \text{ radians/sample} \]

Problem 4.2

A particular filter has the impulse response

\[
 h[n] = \begin{cases} 
 0.5 & n = 0 \\
 0.75 & n = 1 \\
 0.3 & n = 2 \\
 0.1 & n = 3 \\
 0 & \text{otherwise} 
\end{cases}
\]

What is the transfer function, \( H(z) \)?

Solution:

\[ H(z) = 0.5 + 0.75z^{-1} + 0.3z^{-2} + 0.1z^{-3} \]
A particular filter has the difference equation
\[
y[n] = x[n] - 1.2e^{j3\pi n/5}x[n-1] + 0.8e^{j2\pi n/5}y[n-1]
\]
Express the frequency response of this filter as
\[
H(\omega) = \frac{e^{j\omega} - z_1}{e^{j\omega} - p_1}
\]
for some zero \(z_1\) and pole \(p_1\).

Solution:
\[
H(\omega) = \frac{e^{j\omega} - 1.2e^{j3\pi/5}}{e^{j\omega} - 0.8e^{j2\pi/5}}
\]

Problem 4.4

Remember that
\[
G(z) = \frac{1}{1 - 0.8z^{-1}} \leftrightarrow g[n] = (0.8)^nu[n]
\]
Use the linearity and time-shift properties of the Z-transform to find \(h[n]\), where
\[
H(z) = \frac{1 - 0.3z^{-1}}{1 - 0.8z^{-1}} = \frac{1}{1 - 0.8z^{-1}} - 0.3z^{-1} \frac{1}{1 - 0.8z^{-1}}
\]

Solution: The time-shift property is
\[
z^{-n_0}G(z) \leftrightarrow g[n - n_0],
\]
so
\[
G(z) - 0.3z^{-1}G(z) \leftrightarrow g[n] - 0.3g[n-1],
\]
therefore
\[
h[n] = (0.8)^nu[n] - 0.3(0.8)^{n-1}u[n-1]
\]