1 Windowed FIR Design

Do one of the following three problems.

Problem 7.1.1

Suppose you have a signal, \( x_c(t) \), that is sampled at \( F_s = 16,000 \) samples/second, creating a signal \( x[n] \). You would like to implement a discrete time highpass filter with a cutoff frequency of \( f_c = 4000 \text{Hz} \).

(a) What is the discrete-time cutoff frequency, \( \omega_c \), in radians/sample?

(b) Define \( D(\omega) = 1 \) for \( |\omega| > \omega_c \), \( D(\omega) = 0 \) otherwise. What is its inverse DTFT, \( d[n] \)?

(c) Use windowing to create \( h[n] \), a causal approximation to \( d[n] \). Suppose that you are willing to tolerate a stopband ripple of -20dB, therefore you are able to use a rectangular window. You want the separation between passband and stopband to be 800Hz, i.e., you want a passband ripple to peak at \( f = 4400 \text{Hz} \), while the first stopband ripple peaks at \( f = 3600 \text{Hz} \). How many nonzero samples does \( h[n] \) need to have?

(d) Same as part (c), but now you are only willing to tolerate a -50dB stopband ripple, so you will need to use a Hamming window.

(e) For part (d), write an explicit formula that would allow you to compute the values of every sample \( h[n] \), in terms of \( n \). There should be no variables other than \( n \) in your answer.

Problem 7.1.2

Suppose you have a signal, \( x_c(t) \), that is sampled at \( F_s = 16,000 \) samples/second, creating a signal \( x[n] \). You would like to implement a discrete time bandpass filter with a passband of \( f_1 \leq f \leq f_2 \), for \( f_1 = 2000 \text{Hz} \), \( f_2 = 6000 \text{Hz} \).

(a) What are the discrete-time cutoff frequencies, \( \omega_1 \) and \( \omega_2 \), in radians/sample?

(b) Define \( D(\omega) = 1 \) for \( \omega_1 < |\omega| < \omega_2 \), \( D(\omega) = 0 \) otherwise. What is its inverse DTFT, \( d[n] \)?

(c) Use windowing to create \( h[n] \), a causal approximation to \( d[n] \). Suppose that you are willing to tolerate a stopband ripple of -20dB, therefore you are able to use a rectangular window. You want the separation between passband and stopband to be 800Hz, i.e., you want passband ripples that peak at \( f = 2400 \text{Hz} \) and \( f = 5600 \text{Hz} \), and you want stopband ripples that peak at \( f = 1600 \text{Hz} \) and \( f = 6400 \text{Hz} \). How many nonzero samples does \( h[n] \) need to have?
(d) Same as part (c), but now you are only willing to tolerate a -50dB stopband ripple, so you will need to use a Hamming window.

(e) For part (d), write an explicit formula that would allow you to compute the values of every sample $h[n]$, in terms of $n$. There should be no variables other than $n$ in your answer.

Problem 7.1.3

Suppose you have a signal, $x_c(t)$, that is sampled at $F_s = 16,000$ samples/second, creating a signal $x[n]$. You would like to implement a discrete time stopband filter with stopband of $f_1 < f < f_2$ for $f_1 = 2000\text{Hz}$, $f_2 = 6000\text{Hz}$.

(a) What are the discrete-time cutoff frequencies, $\omega_1$ and $\omega_2$, in radians/sample?

(b) Define $D(\omega) = 0$ for $\omega_1 < |\omega| < \omega_2$, $D(\omega) = 1$ otherwise. What is its inverse DTFT, $d[n]$?

(c) Use windowing to create $h[n]$, a causal approximation to $d[n]$. Suppose that you are willing to tolerate a stopband ripple of -20dB, therefore you are able to use a rectangular window. You want the separation between passband and stopband to be 800Hz, i.e., you want passband ripples that peak at $f = 1600\text{Hz}$ and $f = 6400\text{Hz}$, and you want stopband ripples that peak at $f = 5600\text{Hz}$. How many nonzero samples does $h[n]$ need to have?

(d) Same as part (c), but now you are only willing to tolerate a -50dB stopband ripple, so you will need to use a Hamming window.

(e) For part (d), write an explicit formula that would allow you to compute the values of every sample $h[n]$, in terms of $n$. There should be no variables other than $n$ in your answer.