1 Damped Sinusoids: CTFS, CTFT, DTFT, and DFT

Do one of the following three problems.

Problem 4.1.1

The vowel /a/ is characterized by formant frequencies at $F_1 = 900$ and $F_2 = 1100$ Hertz, and with bandwidths of roughly $B_1 = B_2 = 150$ Hertz. This problem will focus only on the positive-frequency part of the first formant ringing, and will ignore amplitude and phase, thus

$$x(t) = e^{-\left(\pi 150 - j 2 \pi 900\right)t} u(t)$$

(a) Find $X(\omega)$ and $|X(\omega)|^2$, the CTFT and its associated power spectrum.

(b) Suppose that $y(t)$ is the periodic repetition of $x(t)$, repeated once every 10ms. Find the Fourier series coefficients $Y_k$, and the associated power spectrum $|Y_k|^2$.

(c) Suppose that $f[n]$ is produced by sampling $x(t)$ once every 0.1ms ($F_s = 10,000$ samples/second). Find $F(\omega)$, the DTFT of $f[n]$, and its associated power spectrum $|F(\omega)|^2$.

(d) Suppose that $g[n]$ is produced by sampling $y(t)$ once every 0.1ms, for a total of exactly ten pitch periods (thus there are a total of $N = 1000$ samples). Let $G[k]$ be the 1000-point DFT of $g[n]$. Find $G[k]$, and its associated power spectrum $|G[k]|^2$.

Problem 4.1.2

The vowel /i/ is characterized by formant frequencies at $F_1 = 300$ and $F_2 = 2000$ Hertz, and with bandwidths of roughly $B_1 = 150$ and $B_2 = 300$ Hertz. This problem will focus only on the positive-frequency part of the first formant ringing, and will ignore amplitude and phase, thus

$$x(t) = e^{-\left(\pi 150 - j 2 \pi 300\right)t} u(t)$$

(a) Find $X(\omega)$ and $|X(\omega)|^2$, the CTFT and its associated power spectrum.

(b) Suppose that $y(t)$ is the periodic repetition of $x(t)$, repeated once every 10ms. Find the Fourier series coefficients $Y_k$, and the associated power spectrum $|Y_k|^2$. 
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(c) Suppose that \( f[n] \) is produced by sampling \( x(t) \) once every 0.1ms \((F_s = 10,000 \text{ samples/second})\). Find 
\( F(\omega) \), the DTFT of \( f[n] \), and its associated power spectrum \( |F(\omega)|^2 \).

(d) Suppose that \( g[n] \) is produced by sampling \( y(t) \) once every 0.1ms, for a total of exactly ten pitch periods (thus there are a total of \( N = 1000 \) samples). Let \( G[k] \) be the 1000-point DFT of \( g[n] \). Find \( G[k] \), and its associated power spectrum \( |G[k]|^2 \).

Problem 4.1.3

The vowel /e/ is characterized by formant frequencies at \( F_1 = 600 \) and \( F_2 = 1700 \) Hertz, and with bandwidths of roughly \( B_1 = 150 \) and \( B_2 = 250 \) Hertz. This problem will focus only on the positive-frequency part of the first formant ringing, and will ignore amplitude and phase, thus

\[ x(t) = e^{-(\pi 150 - j 2\pi 600) t} u(t) \]

(a) Find \( X(\omega) \) and \( |X(\omega)|^2 \), the CTFT and its associated power spectrum.

(b) Suppose that \( y(t) \) is the periodic repetition of \( x(t) \), repeated once every 10ms. Find the Fourier series coefficients \( Y_k \), and the associated power spectrum \( |Y_k|^2 \).

(c) Suppose that \( f[n] \) is produced by sampling \( x(t) \) once every 0.1ms \((F_s = 10,000 \text{ samples/second})\). Find 
\( F(\omega) \), the DTFT of \( f[n] \), and its associated power spectrum \( |F(\omega)|^2 \).

(d) Suppose that \( g[n] \) is produced by sampling \( y(t) \) once every 0.1ms, for a total of exactly ten pitch periods (thus there are a total of \( N = 1000 \) samples). Let \( G[k] \) be the 1000-point DFT of \( g[n] \). Find \( G[k] \), and its associated power spectrum \( |G[k]|^2 \).