1 Continuous and Discrete-Time Signals

Do one of the following three problems.

Problem 1.1.1

A particular acoustic signal is given by the following air pressure, in Pascals, at the microphone:

\[ x_c(t) = \cos(2\pi 2000t) + 2 \sin(2\pi 4000t) + 3 \sin(2\pi 2000t) + 4 \sin(2\pi 7000t) + 5 \cos(2\pi 8000t) \]

\( x_c(t) \) is sampled by an ideal A/D at 8000 samples/second, to create DT signal \( x[n] \). \( x[n] \) is then passed into an ideal D/A, at 10000 samples/second, to construct CT signal \( y_c(t) \).

Find \( x[n] \) as a function of \( n \), and \( y_c(t) \) as a function of \( t \). Simplify each term to a constant if possible, otherwise simplify each term to \( Af(2\pi f_n) \) or \( Af(2\pi f_t) \), where \( f \) is either \( \sin \) or \( \cos \), and where both \( A \) and \( \phi \) are either integers or irreducible fractions (irreducible means that the numerator and denominator are integers with no common divisors).

Problem 1.1.2

A particular acoustic signal is given by the following air pressure, in Pascals, at the microphone:

\[ x_c(t) = \cos(2\pi 500t) + 2 \cos(2\pi 16,000t) + 3 \sin(2\pi 9000t) + 4 \cos(2\pi 10,000t) + 5 \sin(2\pi 8000t) \]
Homework 1

$x_c(t)$ is sampled by an ideal A/D at 16,000 samples/second, to create DT signal $x[n]$. $x[n]$ is then passed into an ideal D/A, at 8000 samples/second, to construct CT signal $y_c(t)$.

Find $x[n]$ as a function of $n$, and $y_c(t)$ as a function of $t$. Simplify each term to a constant if possible, otherwise simplify each term to $Af(2\pi\phi n)$ or $Af(2\pi\phi t)$, where $f$ is either $\sin$ or $\cos$, and where both $A$ and $\phi$ are either integers or irreducible fractions (irreducible means that the numerator and denominator are integers with no common divisors).

Problem 1.1.3

A particular acoustic signal is given by the following air pressure, in Pascals, at the microphone:

$$x_c(t) = \cos(2\pi 11,000t) + 2 \cos(2\pi 10,000t) + 3 \sin(2\pi 9000t) + 4 \sin(2\pi 5500t) + 5 \sin(2\pi 500t)$$

$x_c(t)$ is sampled by an ideal A/D at 11,000 samples/second, to create DT signal $x[n]$. $x[n]$ is then passed into an ideal D/A, at 44,000 samples/second, to construct CT signal $y_c(t)$.

Find $x[n]$ as a function of $n$, and $y_c(t)$ as a function of $t$. Simplify each term to a constant if possible, otherwise simplify each term to $Af(2\pi\phi n)$ or $Af(2\pi\phi t)$, where $f$ is either $\sin$ or $\cos$, and where both $A$ and $\phi$ are either integers or irreducible fractions (irreducible means that the numerator and denominator are integers with no common divisors).

2 Period and Frequency

Do one of the following three problems.

Problem 1.2.1

An acoustic signal has a pressure, at the microphone, of

$$x_c(t) = 5 \sin(2\pi 220t) \cos(2\pi 660t) \text{ Pa}$$

(a) Write this signal as $x_c(t) = A_1 \cos(2\pi f_1 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2)$ for some scalar real numbers $A_1, A_2, f_1, f_2, \theta_1, \theta_2$.

(b) At what two frequencies (in Hertz) does this signal have energy?

(c) What musical notes (between A1 and E7) are the two frequencies you specified in part (b)?

(d) What is the fundamental frequency of this signal (in Hertz)?

Problem 1.2.2

An acoustic signal has a pressure, at the microphone, of

$$x_c(t) = 3 \sin(2\pi 64t) \sin(2\pi 192t) \text{ Pa}$$

(a) Write this signal as $x_c(t) = A_1 \sin(2\pi f_1 t + \theta_1) + A_2 \sin(2\pi f_2 t + \theta_2)$ for some scalar real numbers $A_1, A_2, f_1, f_2, \theta_1, \theta_2$.

(b) At what two frequencies (in Hertz) does this signal have energy?
Homework 1

(c) What musical notes (between A1 and E7) are the two frequencies you specified in part (b) (round to the nearest semitone)?

(d) What is the fundamental frequency of this signal (in Hertz)?

Problem 1.2.3

An acoustic signal has a pressure, at the microphone, of

\[ x_c(t) = 5 \cos(2\pi 768t) \cos(2\pi 256t) \text{ Pa} \]

(a) Write this signal as \( x_c(t) = A_1 \sin(2\pi f_1 t + \theta_1) + A_2 \sin(2\pi f_2 t + \theta_2) \) for some scalar real numbers \( A_1, A_2, f_1, f_2, \theta_1, \theta_2 \).

(b) At what two frequencies (in Hertz) does this signal have energy?

(c) What musical notes (between A1 and E7) are the two frequencies you specified in part (b) (round to the nearest semitone)?

(d) What is the fundamental frequency of this signal (in Hertz)?