1. **[Inscribed Matter]**
   Many people are interested in communication by transporting inscribed matter. This has even been suggested by Rose and Wright as the best way to communicate with aliens. Some people call this approach the sneakernet. One intriguing terrestrial approach is using drones. Recently, Raff D'Andrea has put forth the following expression to describe the power consumption of drones:
   \[
   \frac{(m_p + m_v)v}{370\eta r} + p,
   \]
   where \(m_p\) is payload mass [kg], \(m_v\) is vehicle mass [kg], \(r\) is lift-to-drag ratio, \(\eta\) is power transfer efficiency for motor and propeller, \(p\) is power consumption of electronics [kW], and \(v\) is the cruising velocity [km/h]. Typical parameter values are: \(m_p = 2\), \(m_v = 4\), \(r = 3\), \(\eta = 0.5\), \(p = 0.1\), and \(v = 45\).
   
   (a) How much power does a typical system consume?
   (b) One portable 4TB hard drive was recently released that weighs 8.3 ounces. How much energy would it take for the drone to transport it 45 kilometers?
   (c) Let us say a communication system transports one bit-meter when one bit has been transported one meter toward its destination. How much energy is required to achieve one bit-meter with a drone carrying a hard drive?

2. **[Frequency Flat Fading]**
   Consider the \(L\)-tap discrete-time model for the wireless channel derived in the course notes:
   \[
   y_b[m] = \sum_{\ell=0}^{L-1} h_\ell x_b[m - \ell] + w_b[m]
   \]
   where
   \[
   h_\ell = \sum_i a_i e^{-j2\pi f_c \tau_i} \text{sinc} \left( \ell - \frac{\tau_i}{T} \right).
   \]
   Suppose \(T\) is much larger than the delay spread \((T_d = \tau_{\text{max}} - \tau_{\text{min}})\) of the channel.
   Show that in this case, \(L = 1\) and that
   \[
   h_0 \approx \sum_i a_i e^{-j2\pi f_c \tau_i} \triangleq h.
   \]
   Therefore
   \[
   y_b[m] \approx hx_b[m] + w_b[m].
   \]
   (This is the flat fading model that is used in Lecture 20 notes.)

3. **[Flat Fading]**
   We studied the performance of binary sequential signaling over a Rayleigh fading channel when the gain \(h\) is assumed known at the receiver. In particular, we saw that the error probability decreases like \(1/\text{SNR}\). A precise way of saying that \(P_e\) decays like \(1/\text{SNR}\) with increasing \(\text{SNR}\) is the following:
   \[
   \lim_{\text{SNR} \to \infty} P_e \text{SNR} = c.
   \]
   where \(c\) is a constant. Identify the value of \(c\) for the Rayleigh fading channel.
4. **[Quaternary communication on a slow flat Rayleigh fading channel]**
   Consider communicating two bits per symbol on a slow flat Rayleigh fading channel
   \[ y[m] = h x[m] + w[m] \]
   where
   \[ x[m] = x^I[m] \in \{-\sqrt{E}, -\sqrt{E}/3, \sqrt{E}/3, \sqrt{E}\}, \]
   \( h^I \) and \( h^Q \) are independent \( \mathcal{N}(0, A/2) \) random variables, and \( w^I[m] \) and \( w^Q[m] \) are independent \( \mathcal{N}(0, \sigma^2/2) \) random variables. Find the average error probability as a function of \( A \) and SNR, and its high SNR approximation.

5. **[Diversity and Outage]**
   Recall that we calculated the probability of outage for a Rayleigh fading channel is given by
   \[ P\{\text{outage}\} = P\{|h|^2 < a_{\text{th}} \text{SNR}\} \approx e^{-\frac{a_{\text{th}}}{\text{SNR}}} \]
   Now consider communication on three independent and identically distributed Rayleigh channels with channel parameters \(|h_1|^2, |h_2|^2, \text{ and } |h_3|^2\) being independent \( \text{exp}(A) \) random variables. Define the outage probability with outage as
   \[ P\{\text{outage}\} = P\{\max\{|h_1|^2, |h_2|^2, |h_3|^2\} < a_{\text{th}} \text{SNR}\}. \]
   Find \( P\{\text{outage}\} \) and its high SNR approximation.

6. **[Diversity and Error Probability]**
   Consider the real valued channel
   \[ y = ax + w \]
   where \( x = \pm \sqrt{E}, w \sim \mathcal{N}(0, \sigma^2) \), and \( a \) is random with
   \[ P\{a = 0\} = 0.1 \quad \text{and} \quad P\{a = 2\} = 0.9. \]
   (a) Determine the average probability of error \( \bar{P}_e \) for ML detection.
   (b) What value does \( \bar{P}_e \) approach as SNR approaches infinity
   (c) Suppose the same signal is transmitted on two statistically independent channels with gains \( a_1 \) and \( a_2 \), i.e.,
   \[ y_1 = a_1 x + w_1 \quad \text{and} \quad y_2 = a_2 x + w_2 \]
   where \( w_1 \) and \( w_2 \) are independent \( \mathcal{N}(0, \sigma^2) \) random variables and
   \[ P\{a_1 = 0\} = P\{a_2 = 0\} = 0.1 \quad \text{and} \quad P\{a_1 = 2\} = P\{a_2 = 2\} = 0.9. \]
   Determine \( \bar{P}_e \) in this case.
   (d) For part (c), what value does \( \bar{P}_e \) approach as SNR approaches infinity

7. **[Errors]**
   Please write down any errors you have seen in the course notes.