1 Packaging the Data

Suppose we start out with N real voltages $\hat{x}[0], \ldots, \hat{x}[N-1]$. These are the transmit voltages on the N subchannels using the separate communication architecture (they are generated by efficient coding techniques—such as LDPC codes—for the AWGN channel). Let us suppose that N is an even number. This will simplify our notations. We generate the first and middle components of the data vector as

$$\tilde{d}_0 \stackrel{\text{def}}{=} \hat{x}[0]$$
$$\tilde{d}_{N/2} \stackrel{\text{def}}{=} \hat{x}[1]$$

The remainder of the first half of the data vector $\tilde{\mathbf{d}}$ is as follows:

$$\Re\left[\tilde{d}_n\right] \stackrel{\text{def}}{=} \hat{x}[2n],\ n = 1, \ldots, \frac{N}{2} - 1$$
$$\Im\left[\tilde{d}_n\right] \stackrel{\text{def}}{=} \hat{x}[2n+1],\ n = 1, \ldots, \frac{N}{2} - 1.$$

The second half is simply conjugate symmetric of the first part (so as to respect the appropriate equation):

$$\tilde{d}_n = \tilde{d}^*_{N-n},\ n = \frac{N}{2} + 1, \ldots, N-1.$$

Since $\tilde{\mathbf{d}}$ is conjugate symmetric by construction, the inverse discrete Fourier transform (IDFT) vector $\mathbf{d}$ is composed only of real numbers. The cyclic prefix is then added on and the transmit voltages $x[m]$ are generated.

2 Unpacking the Data

At the output of the DFT of the received voltage vector $\mathbf{y}$ we have the complex vector $\tilde{\mathbf{y}}$. Taking complex conjugate operation on both sides of a previous equation:

$$\begin{aligned}\tilde{y}^*_n &= \tilde{h}^*_n \tilde{d}^*_n + \tilde{w}^*_n \\ &= \tilde{h}_{N-n}\tilde{d}_{N-n} + \tilde{w}_{N-n} \\ &= \tilde{y}_{N-n}.\end{aligned}$$

To recover $\tilde{d}$ from $\tilde{y}$, we poitwise divide by $\tilde{h}$ (the DFT of the channel taps, normalized by $\sqrt{N}$). After this step, the first and middle components are real-valued and they translate directly; among the rest, half the DFT outputs are redundant and can be discarded.

Using the first half, we arrive at the following N received voltages $\hat{y}[2], \ldots, \hat{y}[N-1]$:

$$\begin{aligned}\hat{y}[2n] &\stackrel{\text{def}}{=} \Re\left[\frac{\tilde{h}^*_n}{|\tilde{h}_n|}\tilde{y}_n\right] \\ &= |\tilde{h}_n|\ \hat{x}[2n] + \hat{w}[2n] \\ \hat{y}[2n+1] &\stackrel{\text{def}}{=} \Im\left[\frac{\tilde{h}^*_n}{|\tilde{h}_n|}\tilde{y}_n\right] \\ &= |\tilde{h}_n|\ \hat{x}[2n+1] + \hat{w}[2n+1],\ n = 1, \ldots, \frac{N}{2} = 1.\end{aligned}$$

Here $\hat{w}[\cdot]$ is also white Gaussian with zero mean, but the variances for 0 and $N/2$ will be different.

The unpackaging after division and decoding is just reading off the mapping from packaging.