ECE 361: Second Exam
Tuesday, 8 April 2014, 9:30 am – 10:50 am

Name: (in BLOCK CAPITALS) ________________________________

Signature: ________________________________

Instructions
This is a closed-book closed-notes examination except that one side of one 8.5” × 11” sheet of notes is permitted. Tables of integrals, calculators, laptop computers, PDAs, iPods, cell phones, microscopes, etc. are neither needed nor permitted.

SHOW YOUR WORK. If you need additional space, use the back of the previous page. Write your final answers in the spaces provided.

This exam contains four problems on six separate pages

Grading

1. 24 points ___________
2. 32 points ___________
3. 20 points ___________
4. 24 points ___________

Total (100 points) ___________
1. Consider a binary digital communication system. Using our standard discrete-time model, the channel output is:

\[ y[m] = x[m] - 0.2x[m - 1] + w[m] \]

where \( x[m] = \pm \sqrt{E} \) and \( w[m] \sim \mathcal{N}(0, \sigma^2) \). The receiver computes \( \hat{y}[m] = y[m] - 0.2y[m + 1] \) and then sets \( \hat{x}[m] = \text{sgn}(\hat{y}[m])\sqrt{E} \).

(a) This is a matched filter receiver. 

TRUE or FALSE?

(b) What is the minimum value that the function \( \phi(a_1, a_2) = 0.96 - 0.2a_1 + 0.2a_2 \) can take, when \( a_1 \) and \( a_2 \) are chosen from the set \( \pm 1 \)?

\[ \min \phi = \]

(c) What is \( P(E)_{\text{max}} \), the maximum conditional error probability over the choice of the message sequence \( x[m] \), expressed in terms of \( \sigma \), the \( Q \) function, and elementary functions?

\[ P(E)_{\text{max}} = \]
(d) What is $P(E)_{\min}$, the minimum conditional error probability over the choice of the message sequence $x[m]$, expressed in terms of $\sigma$, the $Q$ function, and elementary functions?

$P(E)_{\min} =$
2. Consider the 2-tap ISI channel with:

\[ y[m] = x[m] + 2x[m-1] + w[m], \]

where the \( w[m] \) are i.i.d. \( \sim \mathcal{N}(0, \sigma^2) \) and we define SNR = \( E/\sigma^2 \). Only two symbols \( x[1] \) and \( x[2] \) are sent on this channel, so assume that \( x[m] = 0 \) for \( m \neq 1, 2 \). Further, assume that \( x[1] \) and \( x[2] \) are i.i.d. with mean 0 and variance \( E \). The goal is to detect \( x[1] \) from \( y[1] \) and \( y[2] \), using a linear equalizer based on the sum:

\[ \hat{y}[1] = c_1 y[1] + c_2 y[2]. \]

(a) Find the coefficients \( c_1 \) and \( c_2 \) of the matched filter equalizer, and the corresponding effective SINR.

\[
\begin{align*}
(c_1, c_2)_{mf} = & \\
\text{SINR}_{mf} = & 
\end{align*}
\]

(b) Find the coefficients \( c_1 \) and \( c_2 \) of the zero-forcing equalizer, and the corresponding effective SINR.

\[
\begin{align*}
(c_1, c_2)_{zf} = & \\
\text{SINR}_{zf} = & 
\end{align*}
\]
(c) Find the coefficients $c_1$ and $c_2$ of the MMSE equalizer, and the corresponding effective SINR.

\[
\begin{align*}
(c_1, c_2)_{\text{mmse}} &= \quad \\
\text{SINR}_{\text{mmse}} &= 
\end{align*}
\]
3. Consider a binary sequential communication system employing Tomlinson-Harashima precoding, with basic voltage values $d[m] = \pm \sqrt{E}$, and an extended constellation with voltage values $(4k - 1)\sqrt{E}$ representing one symbol, and voltage values $(4k + 1)\sqrt{E}$ representing the other symbol. The channel noise is independent across time instances and is identically distributed as $\sim \mathcal{N}(0, \sigma^2)$.

(a) Draw the extended constellation diagram, taking care to label all relevant voltage levels.

(b) Due to some inconsistencies in the channel medium, it turns out the transmitter has biased information and believes the intersymbol interference is actually $\hat{I}[m] = I[m] - \alpha$ rather than the true value $I[m]$, where $\alpha \ll \sqrt{E}$. Find a good upper bound on probability of error of the communication system.

\[ P(E)_{\text{bias}} \leq \]

(c) Now due to some uncertainties in the channel medium, it turns out the transmitter has noisy information and believes the intersymbol interference is actually $\hat{I}[m] = I[m] + n[m]$ rather than the true value $I[m]$, where $n[m] \sim \mathcal{N}(0, \gamma^2)$ and $\gamma \ll \sigma$. Find a good upper bound on probability of error of the communication system.

\[ P(E)_{\text{noisy}} \leq \]
4. For each of the following statements, mark whether they are true or false. Justify your answer briefly.

(a) The successive interference cancellation equalizer works on removing any intersymbol interference voltages, irrespective of the coding and modulation scheme used.

TRUE or FALSE?

(b) The ideal interpolation filter \( g(t) = \text{sinc}(t/T) \) has finite bandwidth and infinite time extent, whereas the rectangular interpolation filter has infinite bandwidth and finite time extent.

TRUE or FALSE?

(c) Consider a continuous-time AWGN channel. In the large bandwidth limit, the Shannon capacity is proportional to the average power \( \bar{P} \).

TRUE or FALSE?

(d) Consider our standard intersymbol interference channel model with \( L \) taps. If an overall information packet is broken into \( L \) subpackets, these subpackets are coded and modulated, and the voltages corresponding to each subpacket are interleaved at the transmitter, then the probability distribution of the interference will be Gaussian.

TRUE or FALSE?