GREEN ELECTRIC ENERGY

Homework 6 Solutions

4.1

The solar declination angle is given by the formula:

\[ \delta_d = 0.41 \sin \left( \frac{2\pi}{365} (d - 81) \right) \]

which implies that: \[-0.41 \leq \delta_d \leq 0.41\]

At solar noon the altitude angle is given by:

\[ \beta(0)_d = \frac{\pi}{2} - \ell - \delta_d = \frac{\pi}{2} - \frac{\pi}{6} - \delta_d = \frac{\pi}{3} + \delta_d \]

Consequently, \[ \frac{\pi}{3} - 0.41 \leq \beta(0)_d \leq \frac{\pi}{3} + 0.41 \] and therefore:

\[ x_1 \geq \frac{2}{\tan \left( \beta(0)_d \right)} = \frac{2}{\tan \left( \frac{\pi}{3} - 0.41 \right)} \approx 2.7 \text{ ft} \]

\[ x_2 \geq \frac{4}{\tan \left( \beta(0)_d \right)} = \frac{4}{\tan \left( \frac{\pi}{3} - 0.41 \right)} \approx 5.4 \text{ ft} \]

4.3

a) For June solstice: \[ d = 172 \Rightarrow \delta_d = 0.41 \sin \left( \frac{2\pi}{365} (172 - 81) \right) = 0.41 \text{ rads} \]
Therefore, \( \beta(0)_{d=172} = \frac{\pi}{2} - 0.698 + 0.41 = 1.28 \text{ rads} \)

\[ \Rightarrow P \geq \frac{8}{\tan(1.28)} = 2.37 \text{ ft} \]

b) For winter solstice: \( \delta|_d = -0.41 \text{ rads} \)
Similarly with question a) we get:

\[ \beta(0)|_d = \frac{\pi}{2} - 0.697 - 0.41 = 0.463 \Rightarrow \]

\[ Y = P \tan(0.463) = 2.37 \tan(0.463) = 1.183 \text{ ft} \]

c) Skip this part

4.5

a)

For June 21, \( d = 172 \Rightarrow \delta|_{d=172} = 0.41 \text{ rads} \)

\[ \beta(h=1)|_{d=172} = \sin^{-1} \left( \cos \left( \frac{32\pi}{180} \right) \cos(0.41) \cos \frac{\pi}{12} + \sin \left( \frac{32\pi}{180} \right) \sin(0.41) \right) = 1.286 \text{ rads} \]

Therefore the height of the tree: \( H = 30 \tan(1.286) = 106.56 \text{ ft} \)

b)

See last page of this solution sheet

c) Skip this part

4.7

a)
The azimuth angle of sunrise relative to due south is equal to the hour angle of sunrise for summer solstice:

\[
\kappa_+|_{d=172} = \cos^{-1}\left(-\tan\left(\frac{47.63\pi}{180}\right)\tan(0.41)\right) = 2.067 \text{ rads}
\]

b)

\[
12:00 - \frac{2.067}{\pi} \cdot \frac{\pi}{12} = 4:06 \text{ a.m. solar time}
\]

c) Sunrise actual time: 4:11 a.m.

4.8

a)

\[
\kappa_+|_{d=172} = \left(12:00 - 4:11\right)\frac{\pi}{12} = \left(12 - \left(4 + \frac{11}{60}\right)\right)\frac{\pi}{12} = 2.0453 \text{ rads}
\]

\[
\kappa_-|_{d=172} = \left(12:00 - 20:11\right)\frac{\pi}{12} = -2.141 \text{ rads}
\]

Therefore, clock time for solar noon is \(\frac{4:11 + 20:11}{2} = 12:11 \text{ a.m.}\)

b)

\[
\theta|_{d=172} = \frac{2\pi}{364} (172 - 81) = 1.57 \text{ rads}
\]

\[
\Rightarrow \epsilon|_{d=172} = -1.5 \text{ minutes}
\]

Therefore,

\[
solar \ noon - noon = 12:00 - 12:11 = -11 \text{ minutes}
\]

\[
\Rightarrow -11 = \epsilon|_{d=172} + 4\left(local \ time \ meridian - local \ longitude\right)
\]
where local time meridian and local longitude are given in degrees.

Solving for the local longitude we get: \( \text{local longitude} = 122.375 \text{ deg} \)

c) 

Since, \( \cos(2.0453) = -\tan \ell \tan(0.41) \Rightarrow \ell = 0.81 \text{ rad} \)

4.9 

a) 

\[
\text{latitude} = \frac{40\pi}{180} = 0.7 \text{ rad}
\]

\[
\delta|_{d=1} = 0.41\sin\left(\frac{2\pi}{365}(1-81)\right) = -0.40 \text{ rad}
\]

\[
\beta|_{d=1} = \frac{\pi}{2} - 0.7 - 0.40 = 0.47 \text{ rad} \quad \text{Therefore the angle of incidence is given by:}
\]

\[
\cos(\theta) = \cos(0.47)\cos(0)\sin(0.7) + \sin(0.47)\cos(0.7) = 0.4 \text{ rad}
\]

\[
A = 1160 + 75\sin\left(\frac{360}{365}(1-275)\right) = 1107 \frac{W}{m^2}
\]

\[
k|_{d=1} = 0.174 + 0.035\sin\left(\frac{360}{365}(1-100)\right) = 0.152
\]

\[
m = \sqrt{(708 \cdot 0.453)^2 + 1417 - 708 \cdot 0.453} = 2.20
\]

\[
\Rightarrow I_B|_{d=1} = 1107e^{-2.20(0.152)} = 792 \frac{W}{m^2}
\]

And the insolation striking the collector’s face is given by:

\[
I_{BC} = I_B \cos(\theta) = 792 \cos(0.4) = 729 \frac{W}{m^2}
\]
Problem-6.1

(a)

\[ p_{DC, stc} = (1kW / m^2)(1m^2)(0.15) = 150W \]

(b)

\[ energy = \left( \frac{6}{1} \right)(150)(0.90)(1 - 0.005(45 - 25)) = 0.729kWh \]

Problem-6.2

(a)

\[ \chi' = \frac{annual energy}{p_{DC, stc} \times \left( \frac{daily\ insolation}{1\ kW / m^2} \right) \times 365} \]

\[ = \frac{1,459}{1 \times \left( \frac{5.56}{1} \right) \times 365} \]

\[ = 0.718 \]

(b)

\[ temperature\ derate = \frac{0.718}{0.77} = 0.932 \]

Problem-6.3

(a)

\[ p_{DC, stc} = \frac{annual energy}{\chi' \times \left( \frac{daily\ insolation}{1\ kW / m^2} \right) \times 365} = \frac{4,000}{0.72 \times \left( \frac{5.5}{1} \right) \times 365} = 2.76kW_p \]

(b)

\[ area = \frac{p_{DC, stc}}{1 - sun \times \eta} = \frac{2.76}{1 \times 0.18} = 15.4m^2 \]
Problem-6.6

Since some modules are connected in series to form a string with increased voltage output, we determine the value of the number of modules in a string so as to satisfy

\[ N_s \leq \min \left\{ \frac{v_{\text{inverter}}^M}{v_{\text{MPP}}} \cdot \frac{v_{\text{MPP}}^M}{v_{\text{MPP}}} \right\} = \min \left\{ \frac{600}{34}, \frac{550}{34} \right\} = 16.1 \]

\[ N_s \geq \frac{v_{\text{MPP}}^m}{v_{\text{MPP}}} = \frac{250}{34} = 7.4 \]

For the modules connected in parallel so as to increase the current output, we determine \( N_p \) that satisfies:

\[ N_p \leq \frac{i_{\text{inverter}}^M}{i_{\text{MPP}}} = \frac{11}{150} = 2.5 \]

Thus (16S, 1P) and (8S, 2P) are feasible

Problem-6.8

For a single module
For (a)

For (b) best with the maximum power output among the three

For (c)

$V = 19V$, $i = 1.9\, A$

$V = 20V$, $i = 2\, A$

$V = 10V$, $i = 1\, A$
Problem 4.5.b:
June 21, Northern Hemisphere \( \Rightarrow \delta = 23.45^\circ \)
1 hour before solar noon \( \Rightarrow H=15^\circ \)
L is given: \(32^\circ\) N

The solar altitude angle is given by
\[
\beta = \sin^{-1}(\cos L \cos \delta \cos H + \sin L \sin \delta) \approx 74.23^\circ
\]

The azimuth angle is given by
\[
\phi_s = \sin^{-1}\left(\frac{\cos \delta \sin H}{\cos \beta}\right) = < 60^\circ
\]

Since \(\cos H \approx 0.966 > 0.694 \approx \frac{\tan(\delta)}{\tan(L)}\) (Eq. 4.11)
we have that \(\phi_s = 60^\circ\)

You may also determine \(\phi_s\) using the solar altitude angle, the day and the time information along with the given sun path diagram.