1. Suppose a wind turbine has a cut-in wind speed of 5 m/s and a furling wind speed of 25 m/s. If the winds the turbine sees have Rayleigh statistics with an average wind speed of 9 m/s,

   a. (4 pts) For how many hours per year will the turbine be shut down because of wind speeds below cut-in?

   \[ P(\nu \leq 5) = 1 - e^{-\frac{5}{9} \left( \frac{5}{9} \right)^2} = 0.2153 \]

   Expected hour per year = 0.2153 x 8760 = 20.15 hrs/year

   b. (2 pts) Draw a picture showing the portion of the probability distribution function comprising your answer in pt a.

   ![Probability Distribution](image)

   c. (4 pts) If this is a 1-MW turbine, how much energy (kWh/yr) would be produced for winds blowing between 8 and 12 m/s?

   \[ P(\nu \leq 12) = 1 - e^{-\frac{12}{9} \left( \frac{12}{9} \right)^2} = 0.7525 \]

   \[ P(\nu \leq 8) = 1 - e^{-\frac{8}{9} \left( \frac{8}{9} \right)^2} = 0.4624 \]

   \[ P(8 \leq \nu \leq 12) = P(\nu \leq 12) - P(\nu \leq 8) = 0.2901 \]

   \[ E = 0.2901 \times 8760 \times 1 \text{ MW} = 2541.5 \text{ MWh/year} \]
2. Consider the design of a home-built wind turbine with a 350 W permanent magnet DC motor as a generator. The goal is to deliver 70kWh over 60 days.

a. (2.5 pts) What would the machine's capacity factor be?

\[
CF = \frac{70000 / 24.60}{350} = 0.139
\]

\[
\frac{d}{dt} \left( \frac{d\text{ (energy)}}{dt} \right) = 8760 \cdot 0.087 \cdot \bar{V} - \frac{2P_n}{D^2} = \text{Energy delivered}
\]

\[
CF = \frac{\text{Energy delivered}}{\text{Energy@full pwr}} = 0.087 \cdot \bar{V} - \frac{P_n}{D^2}
\]

\[
f(v) = \frac{\pi v}{2v^2} e^{-\frac{v^2}{a^2}}
\]

\[
CF = 0.0435 \cdot \bar{V}
\]

\[
p = p_o e^{-0.01412 \frac{T}{T}}
\]

\[
\rho = \frac{353.1 e^{-0.01412 \frac{T}{T}}}{pV} = nRT
\]

\[
\left( \frac{v}{v_o} \right) = \left( \frac{H}{H_o} \right)^{\frac{a}{\gamma}}
\]

\[
\left( \frac{v}{v_o} \right) = \frac{\ln \left( \frac{H}{H_o} \right)}{\ln \left( \frac{H_i}{H_o} \right)}
\]

\[
D^2 = \frac{\text{Prated}}{0.087 \cdot \bar{V} - CF} = \frac{0.350}{0.087 \times 7 - 0.139} = 0.7445 \left( \frac{P}{P_o} \right) = \left( \frac{H}{H_o} \right)^{3a}
\]

\[
D = 0.8604 m
\]

b. (2.5 pts) if the average wind speed is 7 m/s what should the rotor diameter be if the CF developed to estimate Rayleigh winds is used?

\[
CF = 0.087 \cdot \bar{V} - \frac{\text{Prated}}{D^2}
\]

\[
\rho = \frac{353.1 e^{-0.01412 \frac{T}{T}}}{pV} = \frac{nRT}{pV}
\]

\[
\frac{v}{v_o} = \left( \frac{H}{H_o} \right)^{\frac{a}{\gamma}}
\]

\[
\frac{v}{v_o} = \frac{\ln \left( \frac{H}{H_i} \right)}{\ln \left( \frac{H_o}{H_i} \right)}
\]

\[
D^2 = \frac{\text{Prated}}{0.087 \cdot \bar{V} - CF} = \frac{0.350}{0.087 \times 7 - 0.139} = 0.7445 \left( \frac{P}{P_o} \right) = \left( \frac{H}{H_o} \right)^{3a}
\]

\[
D = 0.8604 m
\]

c. (2.5 pts) How fast would the wind have to blow to cause the turbine to put out its full 0.35 kW if the machine is 20% efficient at that point?

\[
P_R = \eta \frac{1}{2} \rho A \bar{V}^3
\]

\[
V^3 = \frac{2 \cdot \frac{P_R}{\eta \rho A}}{0.2 \times 1.225 \times 0.5 \times 47} = 4.656.2
\]

\[
\therefore \, \bar{V} = 16.47 \text{ m/s}
\]

d. (2.5 pts) If the TSR is assumed to be 8, what gear ratio would be needed to match the rotor speed to the generator if the generator needs to turn at 600 rpm to deliver its rated 350 W?

\[
\text{Tip speed} = 8 \times 16.47 \text{ m/s} = 135.75 \text{ m/s}
\]

\[
\text{Tip sweep circumference} = 2 \pi R = \pi \times 0.8624 = 2.7107 \text{ m}
\]

\[
\text{BONUS (2 pts) Define leadership}
\]

\[
\frac{135.75}{2.7107} = 50.08 \text{ Hz}
\]

\[
\text{(2 pts) Define management}
\]

\[
\frac{50.08 \text{ RPM} \times \frac{60}{4} \text{ min}}{2 \text{ of 20 gear ratio} = \frac{\text{Gen RPM}}{\text{Rotor RPM}}} = \frac{600}{\text{3004.8}} \approx 0.2
\]