7.3 The analysis of a tidal power facility is similar to that for a normal wind turbine.
That is, we can still write \( P = \frac{1}{2} \rho A v^3 \) but now \( \rho = 1000 \text{ kg/m}^3 \) and \( v \) is the speed of water rushing toward the turbine. The following graphs assume sinusoidally varying water speed, with amplitude \( V_{\text{max}} \). We assume the turbine can accept flows in either direction (as the tide ebbs and floods) so it is only the magnitude of the tidal current that matters.

![Figure P 7.3](image)

For a sinusoidal tidal flow with \( V_{\text{max}} = 2 \text{ m/s} \),

a. What is the average power density (W/m\(^2\)) in the tidal current? A bit of calculus gives us the following helpful start:

\[
\left( \langle v^3 \rangle \right)_{\text{avg}} = \langle \frac{\rho}{3} V_{\text{max}} \sin v \rangle^3 = V_{\text{max}}^3 \frac{\int \sin^3 v \, dv}{\pi/2} = \frac{4}{3\pi} V_{\text{max}}^3
\]

b. If a 600-kW turbine with 20-m diameter blades has a system efficiency of 30%, how many kWh would it deliver per year in these tides?

\[
2) \quad \frac{P}{A} = \rho \frac{V_{\text{Avg}}^3}{A} = \rho \frac{4}{3\pi} V_{\text{Max}}^3
\]

\[
= 1000 \times \frac{4}{3\pi} (2)^3 = 1698 \frac{W}{m^2}
\]

\[
5) \quad A = \pi r^2 = \pi \left( \frac{D}{2} \right)^2 = \pi (10)^2 = 314.1 \text{ m}^2
\]

\[
P_{\text{Watt}} = \rho A V_{\text{Avg}}^3 = (314.1) (1698) \frac{W}{m^2} = 553.3 \text{ kW}
\]

\[
P_{\text{Turb}} = \eta P_{\text{Watt}} = 0.3 (553.3 \text{ kW}) = 165.99 \text{ kW}
\]

\[
E = P_{\text{Turb}} \times 365 \text{ days} \times 24 \text{ hr/day} \times \frac{1 \text{ kW h}}{1 \text{ kW}} = 14,544 \frac{\text{MWh}}{4 \text{ yr}} \text{ Ans.}
\]
7.4 Consider an 82-m (diameter), 1.65-MW wind turbine with a rated wind speed of 13 m/s.

a. At what rpm does the rotor turn when it operates with a TSR of 4.8 in 13 m/s winds? How many seconds per rotation is that?

b. What is the tip speed of the rotor in those winds (m/s and mph)?

c. What gear ratio is needed to match the rotor speed to an 1800 rpm generator when the wind is blowing at the rated wind speed?

d. What is the efficiency of the complete wind turbine (blades, gearbox, generator) in 13 m/s winds? (Masters 493-494)

\[ \text{TSR} = \frac{\text{BLADE TIP SPEED}}{\text{WIND SPEED}} = \frac{\text{BTS}}{\text{WS}} \]

\[ \text{BTS} = \text{TSR} \times \text{WS} = 4.8 \times 13 \text{ m/s} = 62.4 \frac{\text{m}}{\text{s}} \]

\[ \text{BLADE SWEET CIRCUMFERENCE} = \pi D = 3.14 \times 82 = 257.6 \text{ m} \]

\[ \frac{L}{\text{REV}} = \frac{257.6 \text{ m}}{62.4 \frac{\text{m}}{\text{s}}} = 4.125 \frac{\text{m}}{\text{rev}} \]

\[ \Rightarrow \frac{0.24 \times \text{ROTATION}}{\text{s}} \quad \text{ANS} \]

\[ \text{RPM} = \frac{0.24 \times \text{ROTATION}}{\text{s}} \times \frac{60 \text{ s}}{\text{min}} = 14.56 \text{ RPM} \quad \text{ANS.} \]

b. \[ \text{BLADE TIPSPEED} (\text{BTS}) = 62.4 \frac{\text{m}}{\text{s}} \quad \text{ANS.} \]

\[ \text{BTS} = 62.4 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ km}}{1 \text{ hr}} \times \frac{3600 \text{ s}}{1 \text{ hr}} \]

\[ = 139.9 \text{ MPH} \quad \text{ANS.} \]

c. \[ \text{GEAR RATIO} = \frac{\text{MOTOR}}{\text{TURBINE SHAFT}} = \frac{1800}{14.56} \Rightarrow 123.63 = 1 \quad \text{ANS.} \]

d. \[ \text{RATED PWR @ 13 m/s = 1.65 MW} \]

\[ P_{\text{WIND}} = \frac{1}{2} \rho A n^3 = 1.225 \times 10^3 \times \left( \frac{82}{2} \right)^2 \times (13)^3 \]

\[ \text{BLADE SWEET AREA.} \]

\[ = 7,106 \text{ MW} \]

\[ n = \frac{P_{\text{TURBINE}}}{P_{\text{WIND}}} = \frac{1.65 \text{ MW}}{7,106 \text{ MW}} \times 100 \% = \frac{23.2 \% \text{ EFFICIENT}}{\text{ANS}} \]
7.5 An early prototype 10-kW Makani Windpower system consisted of two 5-kW wind turbines mounted on a wing that flies in somewhat vertical circles (like a kite) several hundred meters above ground. A tether attached to the "kite" carries power from the turbines down to the ground. Since the speed of the kite-turbines moving through the air is much faster than the wind speed, much smaller turbine blades can be used than those on conventional ground-mounted wind turbines. Also with no need for a tower, the cost of materials is far lower than for a conventional system.

Figure P 7.5

Suppose each wing/turbine is moving through the air at 50 m/s and suppose the overall efficiency is half that of the Betz limit, what blade diameter would be required to deliver 5 kW of power per turbine. Don't bother to correct air density for this altitude.

\[
P = \frac{1}{2} \rho A V^3 C_p
\]

\[
\eta = \frac{2P}{\rho \pi R^2 \cdot 295} = \frac{2 \cdot 5000}{1.225 \pi \cdot (50)^3 \cdot 295} = 0.0705
\]

\[
R = \frac{0.265 \text{ M}}{\text{ANS.}}
\]

\[
\Rightarrow \text{BLADE DIAMETER} = 0.53 \text{ M ANS.}
\]

\[
\begin{align*}
P &= 5 \text{kW} \\
\rho &= 1.225 \text{ kg/m}^3 \\
C_p &= 0.5(0.59) (\text{Fig. 7.16}) \\
\Rightarrow \text{BETZ LIMIT} &= 0.295
\end{align*}
\]

\[
\frac{53 \text{ cm}}{12 \text{ in}} = 1.74 \text{ ft}
\]
7.6 Consider the following probability density function for wind speed:

\[ f(v) = \begin{cases} \frac{2}{1.02}v & \text{for } 0 \leq v \leq 10 \\ 0 & \text{otherwise} \end{cases} \]

Figure P 7.6

a. What is an appropriate value of \( k \) for this to be a legitimate pdf?

b. What is the average power in these winds (W/m²) under standard temperature and pressure conditions (1 atm, 15°C)?

2) For a probability density function, the area under the curve must be 1, \( \Rightarrow \quad \text{CDF} = \int_{0}^{x} f(v) \, dv = 1 \quad k \rightarrow \infty \)

\[ 1 = \frac{k}{2} (10) k \]

\[ k = \frac{1}{5} = \frac{2}{5} \quad \text{ans.} \]

b) \[ (U^3)_{AV} = \int_{0}^{10} U^3 f(U) \, dU = \int_{0}^{10} U^3 (102 - U) \, dU \]

\[ = \left. \frac{U^5}{5} \right|_{0}^{10} = \frac{2}{5} \cdot 10^5 = 400 \]

\[ \frac{P_{AV}}{A} = \frac{1}{2} \rho (U^3)_{AV} = \frac{1}{2} (1.225) (400) \]

\[ = 245 \text{ W/m}^2 \]

\[ \text{specific average power at 1 atm, 15°C} \]
7.7 Suppose a wind turbine has a cut-in wind speed of 5 m/s and a furling wind speed of 25 m/s. If the winds the turbine sees have Rayleigh statistics with an average wind speed of 9 m/s,

a. For how many hours per year will the turbine be shut down because of excessively high-speed winds?

b. For how many hours per year will the turbine be shut down because winds are too low?

c. If this is a 1-MW turbine, how much energy (kWh/yr) would be produced for winds blowing at or above the rated wind speed of 12 m/s?

\[ P(\nu > 25) = 1 - P(\nu \leq 25) = 1 - \left[1 - e^{-\frac{\nu}{9}}\right]^2 = e^{-\frac{\nu}{9}} \]

\[ P(\nu > 25) = 0.0023 \]

**Expected hours shutdown/yr**

\[ = 0.0023(8760) = 20.15 \text{ hours \ ANS.} \]

\[ P(\nu < 5 \text{ m/s}) = 1 - e^{-\frac{\nu}{9}} = 1 - 0.7847 = 0.2153 \]

**Expected hrs \ (\nu < 5 \text{ m/s})**

\[ = 0.2153(8760) = 1885.1 \text{ hours \ ANS.} \]

\[ P (12 \leq \nu \leq 25) \]

\[ P (\nu > 12) = 1 - P (\nu \leq 12) = 1 - (1 - e^{-\frac{12}{9}})^2 \]

\[ P (\nu > 12) = 0.2475 \]

\[ P (12 \leq \nu \leq 25) = 0.2475 - 0.0023 = 0.2452 \Rightarrow 1 \text{ MW} \times 24.5 = 8760 \]

\[ = 2148 \text{ MWh \ ANS.} \]
The table below shows a portion of a discretized estimate of the energy delivered by a Siemens 2300-kW, 101-m diameter wind turbine (Table 7.5) exposed to Rayleigh winds with average speed 6 m/s. For example, for winds blowing around 4 m/s (3.5 ≤ v ≤ 4.5), the turbine produces 100 kW of power and in a year’s time, a Rayleigh pdf predicts it delivers 107,841 kWh.

a. How many kWh/yr would be generated for winds blowing at 5 m/s winds? Do this by hand.

b. Create your own spreadsheet (similar to Example 7.9) and find the annual energy delivered by this turbine in these winds.

c. Compare your answer in (b) to the energy predicted by using the simplified capacity factor Equation 7.63.

### Masters Problem 7.8 (c) Solution

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<th>v avg</th>
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#### FIGURE P7.8

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<th>hi V</th>
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Cum Dist =

| Cum Hours / year =
| 1.000 | 8759.998 |

Cum Dist =

| Cum Hours / year =
| 1.000 | 8759.998 |
Masters Problem 7.8 (c) Solution

Histogram

Turbine Power curve

kWh/yr