12. The Solar Energy Resource

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Solar energy is the most abundant renewable energy source and is very clean.

Solar energy is harnessed for many applications, including electricity generation, lighting and steam and hot water production.
SOLAR RESOURCE

- The solar energy source
- Extraterrestrial solar irradiation
- Analysis of solar position in the sky and its application to the determination of
  - optimal tilt angle design for a solar panel
  - sun path diagram for shading analysis
  - solar time and civil time relationship
UNDERLYING BASIS: THE SUN IS A LIMITLESS ENERGY SOURCE
SOLAR ENERGY

- The *thermonuclear reactions* – hydrogen atoms fuse together to form helium – in the sun are the source of solar energy.

- In every second, roughly 4 billion kg of mass are converted into energy, as described by Einstein’s well-known *mass-energy equation* \( E = mc^2 \).

- This immense energy generated is so large that it keeps the sun at very high temperatures.
SOLAR ENERGY

- The plentiful solar energy during the past 4 or 5 billion years is expected to continue in the future.

- Every object emits radiant energy in an amount that is a function of its temperature; the sun emits solar energy into space via radiation.

- Insolation or solar irradiation stated in units of \( \frac{W}{m^2} \) measures the power density of the solar energy.
PLANCK’S LAW

- In general, we use the theoretical concept of a blackbody – defined to be a perfect emitter, as well as a perfect absorber – to discuss radiation.

- The emissive power intensity of a blackbody is a function of its wavelength $\lambda$ and temperature $\tau$ as expressed by Planck’s law.
PLANCK'S LAW

emissive power intensity
$W / m^2 - \mu m$

$$\rho_\lambda(\tau) = \frac{3.74 \times 10^8}{\lambda^5 \left[ \exp \left( \frac{14,400}{\lambda \tau} \right) - 1 \right]}$$
An important feature of blackbody radiation is given by Wien’s displacement rule, which determines the wavelength $\lambda_{\text{max}}$ at which the emissive power intensity reaches its peak value.

$$\lambda_{\text{max}} = \frac{2,898}{\tau} \mu m$$
EXTRATERRESTRIAL SOLAR SPECTRUM

The total area under the curve is the extraterrestrial solar irradiation.

\[ \lambda_{\text{max}} = \frac{2,898}{5,800} = 0.5 \mu m \]

Source: [http://www.ioccg.org/groups/mueller.html](http://www.ioccg.org/groups/mueller.html)
THE SOLAR IRRADIATION

- The sun’s surface temperature is estimated at 5,800 K and its power density is assumed to be 1.37 kW/m² – the value of insolation or solar irradiation just outside the earth’s atmosphere.

- The sun emits maximum energy at the wavelength

\[ \lambda_{\text{max}}_{\text{sun}} = \frac{2,898}{5,800} = 0.5 \, \mu m \]
**Extraterrestrial Solar Irradiation**

*Extraterrestrial solar irradiation* is defined as the solar irradiation that strikes an imaginary surface at the top of the earth’s atmosphere, which is perpendicular to the line from the earth’s center to the sun’s center.
STEFAN–BOLTZMANN LAW OF RADIATION

- The total area under the power intensity curve is the **blackbody** radiant power density emitted over all the wavelengths.

- The **Stefan-Boltzmann law of radiation** states that

  \[ p_{\text{blackbody}} = \sigma A \tau^4 \]

  **Stefan-Boltzmann constant:** \( 5.67 \times 10^{-8} \text{ W} / \text{m}^2 \cdot \text{K} \)
The earth’s radiation

- We consider the earth to be a blackbody with average surface temperature $15^\circ C$ and area equal to $5.1 \times 10^{14} \ m^2$.

- The Stefan-Boltzmann law of radiation states that the earth radiates...
THE EARTH’S RADIATION

\[ p_{\text{earth}} = \sigma A \tau^4 \]

\[ = (5.67 \times 10^{-8})(5.1 \times 10^{14})(15 + 273)^4 \]

\[ = 2 \times 10^{17} \text{W} \]

The wavelength at which the maximum power is emitted is given by Wien’s displacement rule

\[ \lambda_{\text{max}} \bigg|_{\text{earth}} = \frac{2,898}{288} = 10.1 \mu\text{m} \]
THE SPECTRAL EMISSIVE POWER INTENSITY OF A 288 - K BLACKBODY

\[ \lambda_{\text{max}} \bigg|_{\text{earth}} = 10.1 \, \mu m \]

Source: http://www.ioccg.org/groups/mueller.html
EARTH’S ORBIT OVER ITS YEARLY REVOLUTION AROUND THE SUN

- **Winter Solstice**
- **Summer Solstice**
- **Autumnal Equinox**
- **Vernal Equinox**

Source: http://scijinks.nasa.gov
In the analysis of all solar issues, we use *solar time*, which is based on the sun’s position with respect to the earth, instead of *clock* or *civil time*.

Extraterrestrial solar irradiation depends on the distance between the earth and the sun and therefore is a function of the day of the year.
THE ANNUAL EXTRATERRESTRIAL SOLAR IRRADIATION

extraterrestrial solar irradiation (kW/m²)

Source: http://solarat.uoregon.edu/SolarRadiationBasics.html/
The extraterrestrial solar irradiation change over a day is negligibly small and so we assume that its value is constant as the earth rotates each day.

We use the approximation for $i_0 |_d$ given by:

$$i_0 |_d = 1.367 \left[ 1 + 0.034 \cos \left( 2 \pi \frac{d}{365} \right) \right] \text{ W/m}^2 \quad d = 1, 2, \ldots, 365/366$$
EXTRATERRESTRIAL SOLAR IRRADIATION

- We consider the quantification of extraterrestrial solar irradiation on January 1: \( d = 1 \)

\[
i_0 \bigg|_1 = 1,367 \left[ 1 + 0.034 \cos \left( 2\pi \frac{1}{365} \right) \right] = 1,413 \frac{W}{m^2}
\]

- Now, for August 1, \( d = 213 \) and the extraterrestrial solar irradiation is

\[
i_0 \bigg|_{213} = 1,367 \left[ 1 + 0.034 \cos \left( 2\pi \frac{213}{365} \right) \right] = 1,326 \frac{W}{m^2}
\]
EXTRATERRESTRIAL SOLAR IRRADIATION

- We observe that in the Northern hemisphere, the extraterrestrial solar irradiation is higher on a cold winter day than on a hot summer day.

- This phenomenon results from the fact that the sunlight enters into the atmosphere with different incident angles; these angles impact greatly the...
fraction of extraterrestrial solar irradiation received on the earth’s surface at different times of the year

As such, at a specified geographic location, we need to determine the solar position in the sky to evaluate the effective amount of solar irradiation at that location.
The solar position in the sky varies as a function of:

- the specific geographic location of interest;
- the time of day due to the earth's rotation around its tilted axis; and,
- the day of the year that the earth is on its orbital revolution around the sun.
LATITUDE AND LONGITUDE

- A geographic location on earth is specified fully by the local *latitude* and *longitude*.

- The *latitude* and *longitude pair* of geographic coordinates specify the North–South and the East–West positions of a location on the earth's surface; the coordinates are expressed in *degrees or radians*.
LATITUDE AND LONGITUDE

Each parallel is an imaginary east-west circle that passes through all the locations that are at the corresponding latitude.

Tropic of Cancer at 23.45°

Tropic of Capricorn at −23.45°

Latitude and longitude

parallels

Tropic of Cancer

Tropic of Capricorn

Latitude

Longitudes

0°

30°

60°

90°
LATITUDE AND LONGITUDE

A meridian is an imaginary arc on the earth's surface that connects the North and South poles.
THE SOLAR IRRADIATION VARIES BY THE GEOGRAPHIC LOCATION
EARTH’S ROTATION
EARTH’S ROTATION

- Although the sun’s position is fixed in the space, earth’s rotation around its tilted axis results in the “movement” of sun from east to west during each day’s sunrise–to–sunset period.

- The “movement” of the sun’s position in the sky causes the variations in the solar irradiation at the specified location on the earth’s surface.
THE SOLAR IRRADIATION VARIATY VERS BY THE TIME OF A DAY

- East
- West
- North
- South
- Altitude angle
- Azimuth angle

Diagram showing solar irradiation variations with time of day.
The solar position in the sky at any time of the day – sunrise–to–sunset period – is expressed in terms of the altitude angle and the solar azimuth angle.

The altitude angle is defined as the angle between the sun and the local horizon, which depends on the location’s latitude, solar declination angle and solar hour angle.
The solar declination angle refers to the angle between the plane of the equator and an imaginary line from the center of the sun to the center of the earth.

The change of solar declination angle during a day is sufficiently small and so we assume it to remain constant and represent it as a function of \( d \) by \( \delta |_d \).
SOLAR DECLINATION ANGLE

\[ \delta \bigg|_d = 0.41 \sin \left[ \frac{2\pi}{365} (d - 81) \right] \text{ radians} \]

solar rays

solar declination angle

equator

solar declination angle
Solar noon is the time at which the solar position in the sky is vertically over the local meridian, i.e., the line of longitude; in other words, the sun is due South (North) of the location in the Northern (Southern) Hemisphere.

Solar hour angle $\theta(h)$ refers to the angular rotation in radians the earth must go through to reach the solar noon; $h$ is positive before the solar noon – ante meridiem – and negative after noon – post meridiem.
We consider the earth to rotate at \( \frac{2\pi}{24} \) per hour,

\[
\theta(h) = \frac{\pi}{12} h \text{ radians}
\]

At 11 a.m. in solar time

\[
\theta(1) = \frac{\pi}{12}
\]

and at 2 p.m. in solar time

\[
\theta(-2) = -\frac{\pi}{6}
\]
Then, the relation of altitude angle $\beta(h)|_d$ and the location’s latitude, solar declination angle and solar hour angle is given by

$$\sin\left(\beta(h)|_d\right) = \cos(\ell) \cos\left(\delta|_d\right) \cos(\theta(h)) + \sin(\ell) \sin\left(\delta|_d\right)$$

where $\ell$ is the local latitude.
EXAMPLE: ALTITUDE ANGLE AT CHAMPAIGN

- Champaign’s latitude is 0.7 radians

- For October 22, \( d = 295 \); the solar declination angle is computed to be

\[
\delta \bigg|_{295} = 0.41 \sin \left[ \frac{2\pi}{365} \left( 295 - 81 \right) \right] = -0.21 \text{ radians}
\]

- At 1 p.m. solar time, the hour angle is:

\[
\theta(-1) = \frac{\pi}{12} \cdot (-1) = -\frac{\pi}{12} \text{ radians}
\]
EXAMPLE: ALTITUDE ANGLE AT CHAMPAIGN

- We compute the *altitude angle* at Champaign from

\[
sin\left(\beta( - 1)\bigg|_{295}\right)
\]

\[
= \cos(0.7) \cos(-0.21) \cos\left(-\frac{\pi}{12}\right) + \sin(0.7) \sin(-0.21)
\]

\[
= 0.59
\]

and so

\[
\beta(-1)\bigg|_{295} = \sin^{-1}(0.59) = 0.623 \text{ radians}
\]
SPECIAL CASE: THE ALTITUDE ANGLE AT SOLAR NOON
The altitude angle at solar noon of day $d$ satisfies

$$\sin \left( \beta(0) \bigg|_d \right)$$

$$= \cos(\ell) \cos(\delta \bigg|_d) \cos(\theta(0)) + \sin(\ell) \sin(\delta \bigg|_d)$$

However, a more natural expression for $\beta(0) \bigg|_d$ is obtained from the geometric relation

$$\beta(0) \bigg|_d = \frac{\pi}{2} - \ell + \delta \bigg|_d \text{ radians}$$
EXAMPLE: ALTITUDE ANGLE AT SOLAR NOON

We determine the altitude angle for Champaign at

\[ \ell = 0.7 \text{ radians}, \text{ at solar noon on March 1 (d = 60)} \]

The solar declination angle is

\[ \delta \bigg|_{60} = 0.41 \sin \left[ \frac{2\pi}{365} (60 - 81) \right] = -0.15 \text{ radians} \]

The altitude angle at solar noon is

\[ \beta(0) \bigg|_{60} = \frac{\pi}{2} - \ell + \delta \bigg|_{60} = 0.72 \text{ radians} \]
THE SOLAR AZIMUTH ANGLE

- *The solar azimuth angle* $\phi$ is defined as the angle between a due South line in the Northern Hemisphere and the projection of the line of sight to the sun on the earth surface.

- We use the *convention* that $\phi$ is positive when the sun is in the East – before solar noon – and negative when the sun is in the West – after noon.
THE SOLAR AZIMUTH ANGLE

\[ \phi \]

before solar noon \( \phi > 0 \)

after solar noon \( \phi < 0 \)

sunrise

solar noon

sunset

\[ \beta(h) \]

\[ d \]

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The equation for the solar azimuth angle $\phi(h)_{|d}$ is determined from the relationship

$$\sin\left(\phi(h)_{|d}\right) = \frac{\cos\left(\delta_{|d}\right) \sin(\theta(h))}{\cos\left(\beta(h)_{|d}\right)}$$

Since the sinusoidal function is given to ambiguity because $\sin x = \sin(\pi - x)$, we need to
THE SOLAR AZIMUTH ANGLE

determine whether the azimuth angle is greater or less than \( \frac{\pi}{2} \):

\[
\text{if } \cos(\theta(h)) \geq \frac{\tan(\delta_d)}{\tan(\ell)} \quad \text{then} \quad \left| \phi(h) \right|_d \leq \frac{\pi}{2}
\]

\[
\text{else} \quad \left| \phi(h) \right|_d > \frac{\pi}{2}
\]
EXAMPLE: WHERE IS THE SUN IN THE SKY

- Determine the *altitude* and the *solar azimuth* angles at 3 p.m. in Champaign with latitude $\ell = 0.7$ radians at the summer solstice – $d = 172$

- The solar declination is

$$\delta = \frac{\pi}{4}$$

- The hour angle at 3 p.m. is

$$\theta(-3) = -\frac{\pi}{4}$$
EXAMPLE: WHERE IS THE SUN IN THE SKY

Then we compute the altitude angle:

\[ \sin\left(\beta(-3)\bigg|_{172}\right) \]

\[ = \cos(0.7) \cos(0.41) \cos\left(-\frac{\pi}{4}\right) + \sin(0.7) \sin(0.41) \]

\[ = 0.75 \]

Then

\[ \beta(-3)\bigg|_{172} = 0.85 \text{ radians} \]
EXAMPLE: WHERE IS THE SUN IN THE SKY

The sine of the azimuth angle is obtained from

$$\theta = \sin^{-1} \left( \frac{s}{c} \right)$$

Two possible values for the azimuth angle are

$$\theta = \sin^{-1} (0.9848) = -0.9848$$

$$\theta = \sin^{-1} (-0.9848) = -1.4 \text{ radians}$$

or

$$\theta = \pi - \sin^{-1} (0.9848) = 4.54 \text{ radians}$$
EXAMPLE: WHERE IS THE SUN IN THE SKY

Since

\[ \cos\left(\theta \left( -3 \right)\right) = 0.707 \quad \text{and} \quad \frac{\tan\left(\delta \mid _{172}\right)}{\tan(\ell)} = 0.515 \]

Then we can determine

\[ \cos\left(\theta \left( -3 \right)\right) > \frac{\tan\left(\delta \mid _{172}\right)}{\tan(\ell)} \]

Thus

\[ \phi \left( -3 \right) \mid _{172} = -1.4 \text{ radians} \]
IMPORTANCE OF THE ANALYSIS ON SUN’S POSITION IN THE SKY

- We are now equipped to determine the sun’s position in the sky at any time and at any location.

- To effectively design and analyze solar plants, the sun’s position in the sky analysis has some highly significant applications, including to
  - build sun path diagram and do shading analysis
  - determine sunrise and sunset times
  - evaluate a solar panel’s optimal position
SUN PATH

- Sun’s path at the summer solstice
- Sun’s path at an equinox
- Sun’s path on winter solstice
The **sun path diagram** is a chart used to illustrate the continuous changes of sun’s location in the sky at a specified location.

The sun’s position in the sky is found for any **hour** of the specified day $d$ of the year by reading the **azimuth and altitude angles** in the **sun path diagram** corresponding to that **hour**.
SUNRISE AND SUNSET

- An important issue is the determination of the sunrise/sunset times since solar energy is only collected during the sunrise to sunset hours.

- We estimate the sunrise/sunset time from the equation used to compute the solar altitude angle, which is zero at sunrise and sunset.
SUNRISE AND SUNSET

\[ \sin \left( \beta(h) \bigg|_d \right) = 0 \]

The relationship for the solar angle results in:

\[ \cos (\theta(h)) = -\frac{\sin(\ell) \sin \left( \delta \bigg|_d \right)}{\cos(\ell) \cos \left( \delta \bigg|_d \right)} = - \frac{\tan(\ell) \tan \left( \delta \bigg|_d \right)}{} \]

Now we can determine the sunrise solar hour angle \( \kappa^+ \bigg|_d \) and the sunset hour angle \( \kappa^- \bigg|_d \) to be:
SUNRISE AND SUNSET

The corresponding sunrise and sunset angles are

\[ \kappa_+ \left|_d \right. = \cos^{-1}\left( - \tan(\ell) \tan(\delta \left|_d \right. ) \right) \]

\[ \kappa_- \left|_d \right. = - \cos^{-1}\left( - \tan(\ell) \tan(\delta \left|_d \right. ) \right) \]

so that the solar times for sunrise/sunset are at

\[ 12:00 - \frac{\kappa_+ \left|_d \right.}{\pi / 12} \quad \text{and} \quad 12:00 - \frac{\kappa_- \left|_d \right.}{\pi / 12} \]
SUNRISE TIME IN CHAMPAIGN

- Champaign is located at $\ell = 0.7$ radians

- On October 22, the solar declination angle is $-0.21$ radians and the sunrise solar hour angle is:

$$\kappa_+ \bigg|_{295} = \cos^{-1} \left( -\tan(0.7) \tan(-0.21) \right) = 1.39 \text{ radians}$$

- The sunrise expressed in solar time is at

$$12:00 - \frac{1.39}{\pi / 12} = 6:27 \text{ a.m.}$$
SOLAR TIME AND CIVIL TIME

- So far, we used exclusively *solar time* measured with reference to solar noon in all our analysis of insolation and its impacts.

- However, in our daily life we typically use *civil* or *clock time*, which measures the time to align with the earth’s daily rotation over exactly 24 *hours*.
The difference at a specified location on the earth surface between the solar time and the civil time arises from the earth’s uneven movement along its orbit of the annual revolution around the sun and the deviation of the local time meridian from the location longitude.

As such, two distinct adjustments must be made in order to convert between solar time and civil time.
SOLAR DAY AND 24-HOUR DAY

- We examine the difference between a solar day and the corresponding civil 24-hour day.

- A solar day is defined as the time elapsed between two successive solar noons.
HOW LONG IS A SOLAR DAY

sun

earth's orbit around the sun

one solar day later

earth
SOLAR DAY

- The earth’s elliptical orbit in its \textit{revolution around the sun} results in a different duration of each solar day.

- The difference between a solar day and a 24-h day is given by the deviation in minutes

\[ e_d = 9.87 \sin \left(2 \left(b_d \right)\right) - 7.53 \cos \left(b_d \right) - 1.5 \sin \left(b_d \right), \]

where,

\[ b_d = \frac{2\pi}{364} \left(d - 81\right) \text{ radians} \]
DIFFERENCE BETWEEN A SOLAR AND A 24-HOUR DAY OVER A YEAR

minutes

day

365

-20 -15 -10 -5 0 5 10 15 20

1
There are 24 time zones to cover the earth, each with its own time meridian with 15° longitude gap between the time meridians of two adjacent zones.

The second adjustment deals with the longitude correction for the fact that the clock time at any location within each time zone is defined by its local time meridian which differs from the time...
LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

For every degree of longitude difference, the solar time difference corresponds to

\[
\frac{24 \text{ hour} \cdot 60 \text{ m / hour}}{360^\circ} = 4 \frac{m}{\text{degree longitude}}
\]

The time adjustment due to the degree longitude difference between the specified location and the local time meridian is the product of 4 times the longitude difference expressed in minutes.
The sum of the adjustment $e|_d$ and the longitude correction results in:

$$ \text{solar time} = \text{clock time} + e|_d + 4 \times \frac{180}{3.14} \times (\text{local time meridian} - \text{local longitude}) $$

This relationship allows the conversion between solar time and civil time at any location on earth.
EXAMPLE: SOLAR TIME AND CLOCK TIME

- Find the clock time of *solar noon* in Springfield on July 1, the 182\textsuperscript{nd} day of the year.

- For $d = 182$, we have

$$b\bigg|_{182} = \frac{2\pi}{364} (182 - 81) = 1.72 \text{ radians}$$

$$e\bigg|_{182} = 9.87 \sin (2 \times 1.72) - 7.53 \cos (1.72) - 1.5 \sin (1.72)$$

$$= -3.51 \text{ mins}$$
EXAMPLE: SOLAR TIME TO CLOCK TIME

For Springfield, IL, with longitude 1.55 radians, the clock time in the central time zone is:

\[
\text{solar time} - e \mid_d - 4 \times \frac{180}{3.14} \times (\text{local time meridian} - \text{local longitude})
\]

\[
= \text{solar noon} - (-3.51) - 57 \left( ( -1.44 ) - ( -1.55 ) \right)
\]

\[
= 11:38 \text{ a.m.}
\]
five time zones span across China’s territory, but by government decree the entire country uses the time zone at the location of the capital as the single standard time.
CONCLUSION

- With the conversion scheme between the solar and clock times, the analysis of solar issues, *e.g.*, the expression of sunrise/sunset on civil time basis, makes the results far more meaningful for use in daily life.

- Such a translation renders the analysis results to be much more concrete for all applications.