4.1

The solar declination angle is given by the formula:

\[ \delta_d = 0.41 \sin \left( \frac{2\pi}{365} (d - 81) \right) \]

Since the sine function is bounded between -1 and 1, this implies that:

\[-0.41 \leq \delta_d \leq 0.41\]

In other words, the declination angle is always restricted between these two values regardless of the day of the year for this specific latitude. At solar noon the altitude angle is given by:

\[ \beta(0)_d = \frac{\pi}{2} - \ell - \delta_d = \frac{\pi}{2} - \frac{\pi}{6} - \delta_d = \frac{\pi}{3} + \delta_d \]

Since the declination angle is bounded by the values above, the solar noon altitude angle is similarly bounded. Consequently,

\[ \frac{\pi}{3} - 0.41 \leq \beta(0)_d \leq \frac{\pi}{3} + 0.41 \]

and therefore:

\[ x_1 \geq \frac{2}{\tan \left( \beta(0)_d \right)} = \frac{2}{\tan \left( \frac{\pi}{3} - 0.41 \right)} = 2.7 \text{ ft} \]

\[ x_2 \geq \frac{4}{\tan \left( \beta(0)_d \right)} = \frac{4}{\tan \left( \frac{\pi}{3} - 0.41 \right)} = 5.4 \text{ ft} \]
4.3

a) For June solstice:

\[ d = 172 \Rightarrow \delta_{d=172} = 0.41 \sin \left( \frac{2\pi}{365} (172 - 81) \right) = 0.41 \text{ rads} \]

Therefore,

\[ \beta(0)_{d=172} = \frac{\pi}{2} - 0.698 + 0.41 = 1.28 \text{ rads} \]

\[ \Rightarrow P \geq \frac{8}{\tan(1.28)} = 2.37 \text{ ft} \]

b) For winter solstice:

\[ \delta_d = -0.41 \text{ rads} \]

Similar to question a) we get:

\[ \beta(0)_d = \frac{\pi}{2} - 0.697 - 0.41 = 0.463 \Rightarrow \]

\[ Y = P \tan(0.463) = 2.37 \tan(0.463) = 1.183 \text{ ft} \]

c) Skip this part
4.7

a) The azimuth angle of sunrise relative to due south is equal to the hour angle of sunrise for summer solstice, so from equation 4.17, the hour angle is

\[ H_{SR} = \cos^{-1}(-\tan L \tan \delta) \]

Where \( L \) is the latitude and angle and \( \delta \) is the declination angle. We have already calculated the declination angle for the summer solstice in problem 4.3, part a, so we will use that number and obtain

\[ H_{SR} = \cos^{-1}(-\tan(\frac{2\pi}{360} \times 47.63) \tan(0.41)) = 2.067 \text{ rads} \]

b) Using equation 4.18, the hour angle can be converted to time of sunrise by

\[ 12:00 - \frac{2.067}{\pi} = 4:06 \text{ a.m. solar time} \]

c) Sunrise actual time: 4:11 a.m.

The difference is usually because the term ‘sunrise’ is defined a little differently in different sources. Most sources use it to mean the time when the very top of the sun becomes visible. The geometric definition is based on the center of the sun crossing the horizon. Atmospheric refraction also causes the sunrise to appear at a time different that that indicated by geometric modeling.
4.9

a) First, begin by calculating the latitude and the declination angle.

\[
latitude = \frac{40\pi}{180} = 0.7 \text{rad}
\]

\[
delta_{d=1} = 0.41 \sin \left( \frac{2\pi}{365} (1 - 81) \right) = -0.40 \text{ rad}
\]

Then, the altitude angle is given by:

\[
\beta_{d=1} = \frac{\pi}{2} - 0.7 - 0.40 = 0.47 \text{ rad}
\]

Therefore the angle of incidence is given by:

\[
\cos(\theta) = \cos(0.47) \cos(0 - 0) \sin(0.7) + \sin(0.47) \cos(0.7) = 0.4 \text{ rad}
\]

The apparent solar irradiation is given by:

\[
A = 1160 + 75 \sin \left( \frac{360}{365} (1 - 275) \right) = 1107 \frac{W}{m^2}
\]

Now, evaluate the optical depth for the given day.

\[
k_{d=1} = 0.174 + 0.035 \sin \left( \frac{360}{365} (1 - 100) \right) = 0.152
\]

Then calculate the air mass ratio for this altitude angle

\[
m = \sqrt{(708 \cdot 0.453)^2 + 1417 - 708 \cdot 0.453} = 2.20
\]

Now we have all the values needed to calculate the clear sky direct beam radiation.

\[
\Rightarrow I_B_{d=1} = 1107 e^{-2.20(0.152)} = 792 \frac{W}{m^2}
\]

And the insolation striking the collector’s face is given by:

\[
I_{BC} = I_B \cos(\theta) = 792 \cos(0.4) = 729 \frac{W}{m^2}
\]
5.3

a) From Kirchoff’s current law, we know that the sum of the currents leaving any node is 0. So we have

\[ I_L = I_{SC} - I_d - \frac{V_D}{R_P} = I_{SC} - I_0(e^{38.9V_D} - 1) - \frac{V_D}{R_P} \]

Substituting the values from the problem, we get,

\[ I_L = 6.4 - 4 \times 10^{-11}(e^{38.9 \times 0.57} - 1) - \frac{0.57}{10} = 6.173A \]

Therefore the power is given by:

\[ P = V \times I = 6.173 \times 0.57 = 3.519 W \]

The efficiency is:

\[ \eta = \frac{\text{output}}{\text{input}} = \frac{3.519W}{0.017m^2 \times 1000W/m^2} = 0.207 = 20.7\% \]
5.8
Recall that the current adds when sources are in parallel and the voltage adds when sources are added in series. Here two modules are added in series, and the two such strings are connected in parallel.

**SOLN:** 480 W
6.1 
a) 

\[ p_{DC, stc} = (1 \text{kW/m}^2)(1 \text{m}^2)(0.15) = 150 \text{W} \]

b) First calculate the temperature derating:

\[ \text{derating} = 1 - 0.5\% \times (45 - 25)^\circ C = 0.90 \]

The inverter rating is given at 0.90. Therefore the total derating is 0.90x0.90 = 0.81

\[ \text{energy} = 6 \text{hours} \cdot (150 \text{W})(0.81) = 0.729 \text{kWh} \]

6.2 
a) 

\[ \chi' = \frac{\text{annual energy}}{p_{DC, stc} \times \left( \frac{\text{daily insolation}}{1 \text{kW/m}^2} \right) \times 365} \]

\[ = \frac{1,459 \text{kWh/yr}}{1 \times \left( \frac{5.56 \text{hr}}{1 \text{day}} \right) \times 365 \text{d/yr}} \]

\[ = 0.718 \]

b) 

\[ \text{temperature derate} = \frac{0.718}{0.77} = 0.932 \]

6.3 
a) 

\[ p_{DC, stc} = \frac{\text{annual energy}}{\chi' \times \left( \frac{\text{daily insolation}}{1 \text{kW/m}^2} \right) \times 365} = \frac{4,000}{0.72 \times \left( \frac{5.5}{1} \right) \times 365} = 2.76 \text{kW} \]
b) 

\[ \text{area} = \frac{p_{DC,\text{stc}}}{1 - \text{sun} \times \eta} = \frac{2.76}{1 \times 0.18} = 15.4 \, \text{m}^2 \]

Problem-6.6

Since some modules are connected in series to form a string with increased voltage output, we determine the value of the number of modules in a string so as to satisfy

\[ N_s \leq \min \left\{ \frac{v_{\text{inverter}}^M}{v_{\text{MPP}}^M}, \frac{v_{\text{MPPT}}^M}{v_{\text{MPP}}^M} \right\} = \min \left\{ \frac{600}{34}, \frac{550}{34} \right\} = 16.1 \]

\[ N_s \geq \frac{v_{\text{MPPT}}^m}{v_{\text{MPP}}^m} = \frac{250}{34} = 7.4 \]

For the modules connected in parallel so as to increase the current output, we determine \(N_p\) that satisfies:

\[ N_p \leq \frac{i_{\text{inverter}}^M}{i_{\text{MPP}}^M} = \frac{11}{\left(\frac{150}{34}\right)} = 2.5 \]

Thus (16S, 1P) and (8S, 2P) are feasible

Problem-6.8

For a single module

![Graph](image-url)
For (a)

![Graph showing a resistor with $v \approx 19$ V, $i \approx 1.9$ A.]

For (b) best with the maximum power output among the three resistors.

![Graph showing a resistor with $v = 20$ V, $i = 2$ A.]

For (c)

![Graph showing a resistor with $v = 10$ V, $i = 1$ A.]

v (V) vs. i (A) graphs.