7.6:

a. the probability density function of wind speed is:

\[ f(v) = \begin{cases} \frac{k}{10} v & 0 \leq v \leq 10 \\ 0 & \text{otherwise} \end{cases} \]

For this to be a legitimate probability density function, it must satisfy the following condition:

\[ \int_{-\infty}^{\infty} f(v) \, dv = 1 \]

Thus

\[ \int_{-\infty}^{\infty} f(v) \, dv = \int_{0}^{10} \frac{k}{10} v \, dv = \frac{k}{20} (10^2 - 0^2) = 5k = 1 \quad k = 0.2 \]

b.

\[ E(P) = E\left(\frac{1}{2} \rho A V^3\right) = \frac{1}{2} \rho A \cdot E(V^3) = \frac{1}{2} \rho A \cdot \int_{-\infty}^{\infty} v^3 \cdot f(v) \, dv = \frac{1}{2} \rho A \cdot \int_{0}^{10} v^3 \cdot 0.02 v \, dv \]

\[ = \frac{1}{2} \cdot 1.225 A \cdot \frac{0.02}{5} \cdot (10^5 - 0^5) \]

\[ \frac{E(P)}{A} = 245 \text{ W/m}^2 \]
7.7:
For $k = 2$, the Weibull distribution is called the Rayleigh p.d.f.

The probability density function is:

$$f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2}$$

The average wind speed is given by:

$$\bar{v} = \int_0^\infty v f_v dv = 2 \int_0^\infty \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} dv = \frac{\sqrt{\pi}}{2} c$$

And the cumulative density function is:

$$F_v(V \leq v)_{Rayleigh} = 1 - e^{-\left[\frac{\pi v^2}{4 \bar{v}^2}\right]}$$

Since the average wind speed is 9 m/s

$$c = \frac{2}{\sqrt{\pi}} \bar{v} = \frac{2}{\sqrt{\pi}} 9 = 10.157$$

a. The probability that the wind speed is bigger than 25 m/s

$$F_v(V \geq 25)_{Rayleigh} = 1 - F_v(V \leq 25)_{Rayleigh} = e^{-\left[\frac{\pi (25)^2}{4 \bar{v}^2}\right]} = 2.34 \times 10^{-3}$$

The average hours over a year when the wind speed is bigger than 25 m/s

$$8760 \cdot F_v(V \geq 25)_{Rayleigh} = 20.51 \text{ hours}$$

Thus there are 20.51 hours per year in average when the turbine will be shut down because of excessively high-speed wind
b. The probability that the wind speed is smaller than 5 m/s

\[ F_V(V \leq 5)_{\text{Rayleigh}} = 1 - e^{-\left(\frac{5}{4}\right)} = 0.215 \]

The average hours over a year when the wind speed is smaller than 5 m/s

\[ 8760 \cdot F_V(V \leq 5)_{\text{Rayleigh}} = 1884.86 \text{ hours} \]

thus there are 1884.86 hours per year in average when the turbine will be shut down because the wind speed are too low

c. the probability that the wind speed is smaller than 25 m/s and bigger than 12 m/s

\[ F_V(V \leq 25)_{\text{Rayleigh}} - F_V(V \leq 12)_{\text{Rayleigh}} = e^{-\left(\frac{12}{4}\right)} - e^{-\left(\frac{25}{4}\right)} = 0.245 \]

the average hours over a year when the wind speed is smaller than 25 m/s and bigger than 12 m/s

\[ 8760 \times 0.245 = 2149.30 \text{ hours} \]

when the wind speed is smaller than (cut-off speed) 25 m/s and bigger than (rated wind speed) 12 m/s, the power output of the wind turbine is 1 MW. Thus the average energy produced by the wind at or above 12 m/s

\[ 2149.30 \text{ MWh} \]
Problem a

(i) if the wind blows continuously between 15 and 20 m/s all day, the power output is 1.25 MW all day. Thus the total energy is $1.25 \times 24 = 30$ MWh/day

(ii) No. Because the relation between power output and wind speed is non-linear, the average power obtained over time in a variable wind with a given average wind speed is not the same as the power obtained in a steady wind of the same speed.
Problem b

For \( k = 2 \), the Weibull distribution is called the Rayleigh p.d.f.

\[
f(v) = \frac{2v}{c^2} e^{-\left(\frac{v}{c}\right)^2}
\]

\[
\bar{v} = \int_{0}^{\infty} vf(v) dv = 2\int_{0}^{\infty} \left(\frac{v}{c}\right)^2 e^{-\left(\frac{v}{c}\right)^2} dv = \frac{\sqrt{\pi}}{2} c
\]

\[
F_V(V \leq v)_{\text{Rayleigh}} = 1 - e^{-\left[\frac{\pi}{4}\left(\frac{v}{c}\right)^2\right]}
\]

Since the average wind speed is 20 m/s

(i).

\[
c = \frac{2}{\sqrt{\pi}} \bar{v} = \frac{2}{\sqrt{\pi}} 20 = 22.53
\]

when temperature is 15 °C

\[
\rho_{10m,15^\circ C} = \frac{353.1}{T} \exp\left(-0.0342 \frac{z}{T}\right) = \frac{353.1}{273.15 + 15} \exp\left(-0.0342 \frac{10}{273.15 + 15}\right) = 1.224 \text{ kg/m}^3
\]

\[
E(p) = E\left(\frac{1}{2} \rho V^3\right) = \frac{1}{2} \cdot 1.224 \cdot E(V^3) = \frac{1}{2} \cdot 1.224 \cdot 1.91 \cdot \bar{V}^3 = \frac{1}{2} \cdot 1.224 \cdot 1.91 \cdot 20^3 = 9351 \text{ W/m}^2
\]

when temperature is -5 °C

\[
\rho_{10m,-5^\circ C} = \frac{353.1}{T} \exp\left(-0.0342 \frac{z}{T}\right) = \frac{353.1}{273.15 - 5} \exp\left(-0.0342 \frac{10}{273.15 - 5}\right) = 1.315 \text{ kg/m}^3
\]

\[
E(p) = E\left(\frac{1}{2} \rho V^3\right) = \frac{1}{2} \cdot 1.315 \cdot E(V^3) = \frac{1}{2} \cdot 1.315 \cdot 1.91 \cdot \bar{V}^3 = \frac{1}{2} \cdot 1.315 \cdot 1.91 \cdot 20^3 = 10047 \text{ W/m}^2
\]

(ii).

when temperature is 15 °C

\[
\text{annual energy} = 8760 \cdot \eta E(p) = 8760 \cdot 0.3 \cdot E(p) \cdot A = 8760 \cdot 0.3 \cdot 9351 \cdot \pi \left(\frac{60}{2}\right)^2 = 69447 \text{ MWh}
\]

when temperature is -5 °C

\[
\text{annual energy} = 8760 \cdot \eta E(p) = 8760 \cdot 0.3 \cdot E(p) \cdot A = 8760 \cdot 0.3 \cdot 10047 \cdot \pi \left(\frac{60}{2}\right)^2 = 74616 \text{ MWh}
\]