ECE 333 – Renewable Energy Systems

5. Wind Power

George Gross
Department of Electrical and Computer Engineering
University of Illinois at Urbana–Champaign
OUTLINE

- The physics of rotors
- Evaluation of power in the wind
- The definition of specific power and its analysis
- Temperature and altitude variations in specific power
- The impacts of tower height on wind turbine output
We provide a brief introduction to how the rotor blades extract energy from the wind.

*Bernoulli’s principle* is the basis of the explanation of how an airfoil – be it an airplane wing or a wind turbine blade – obtains lift.
ROTOR BASICS

- Air that travels over the top of the airfoil must cover a longer distance before it rejoins the air that uses the shorter path under the foil.
- Air on top travels faster and so results in lower pressure than air under the airfoil.
- The difference between the two pressures creates the lifting force that holds an airplane up and that rotates the wind turbine blade.
The situation with a rotor is more complicated than that of an airplane wing for a number of reasons:

- Blade motion
- Lift
- Wind force
- Net resulting wind across blade
- Relative wind due to blade motion
a rotating blade experiences the air moving toward it from the wind and from the relative motion of the blade as it spins

the combined effect of the wind itself and the rotating blade results in a force that is at the appropriate angle so that the force is along the blade and can provide the lift that moves the rotor along
as the blade speed at the tip is faster than near the hub, the blade must be twisted along its length to keep the appropriate angle.

the angle between the wind and the airfoil is referred to as the \textit{angle of attack}.
as the angle of attack increases, the lift increases but so does the drag
too large of an angle of attack can lead to a stall phenomenon due to the resulting turbulence
wind turbines are equipped with a mechanism to shed some wind power so as to avoid damage to the generator
POWER IN THE WIND

- We wish to **analytically characterize** the level of power associated with wind.
- For this purpose, we view wind as a “packet” of air with mass $m$ moving at a constant speed $v$.
  
  Please note, this assumption represents a major simplification since air is a fluid; however, the simplified modeling is useful to explain the key concepts in wind generation.
POWER IN THE WIND

- The kinetic energy of wind is

\[ \varepsilon = \frac{1}{2} m v^2 \]

- Power is simply the rate of change in energy and so we view the power in the mass of air \( m \) moving at constant speed \( v \) through area \( a \) as the rate at which the mass \( m \) passes through area \( a \)
POWER IN THE WIND

mass of air \( m \) moving at constant speed \( v \)

\[
\text{power through area } a = \frac{d\varepsilon}{dt}
\]

\[
\frac{d\varepsilon}{dt} = \frac{d}{dt} \left( \frac{1}{2}mv^2 \right) = \frac{1}{2} \frac{dm}{dt} v^2
\]
The term $\frac{dm}{dt}$ is the rate of flow of the mass of air through area $a$ and is given by $\rho a v$ where $\rho$ is the air density, i.e., the mass per unit of volume.

The volume $w$ of mass $m$ is given by the area $a$ times the “length” of mass $m$.

Over time $dt$, the mass $m$ moves a distance $v dt$ resulting in the volume.
POWER IN THE WIND

\[ dw = a v \, dt \]

- Now

\[ \frac{dm}{dt} = \frac{dm}{dw} \cdot \frac{dw}{dt} = \frac{dm}{dw} \cdot a \, v \]

and

\[ \frac{dm}{dw} = \rho \quad \text{air density} \]

- Thus the power in the wind is

\[ p_w = \frac{1}{2} \rho \, a \, v^3 \]
We consider the units in

$$p_w = \frac{1}{2} \rho a v^3$$

where $W$ is the work, $\rho$ is the air density at $15^\circ C$ and 1 atm, $1.225 \frac{kg}{m^3}$, $a$ is the acceleration, $v$ is the velocity, and $m^2$ is the mass. The units are:

$$kg \left( \frac{m}{s} \right)^2 \frac{s}{s} = \frac{J}{s}$$
The power in wind is, typically, expressed in units per cross sectional area – \( \frac{W}{m^2} \).

We refer to the expression for \( p_w \) as *specific power* or *power density*.

We next consider \( p_w \) in more detail and analyze the impacts of temperature and altitude.
The energy produced by a wind turbine is dependent on the power in the wind; to maximize the energy we therefore need to maximize $p_w$.

In the equation

$$p_w = \frac{1}{2} \rho a v^3$$

$\rho$ is a fixed parameter which we cannot “control”; however, we can control the area $a$ in the wind turbine design and we have some control over the wind speed in terms of the wind farm siting.
The area $a$ is the swept area by the turbine rotor:

for a HAWT with a blade with diameter $d$

$$a = \pi \left( \frac{d}{2} \right)^2 = \frac{1}{4} \pi d^2$$

Clearly, there are economies of scale that are associated with larger wind turbines:

- cost of a turbine $\propto d$
- power output of a turbine $\propto d^2$

and so the larger rotors are more cost effective.
NATURE OF AIR DENSITY

- The air density \( \rho \) at 15° C and 1 atm pressure at sea level is 1.225 \( \frac{kg}{m^3} \), but the value changes as a function of temperature and altitude.

- We know that \( \rho \) decreases as temperature increases since in a warmer day the air becomes thinner; a similar thinning of the air occurs with an increase in altitude.
NATURE OF AIR DENSITY

- We need to return to elementary chemistry and physics to determine the value of $\rho$ for changes in temperature from 15° C and for altitudes above sea level.

- The governing relation is the ideal gas law

$$\hat{p}w = nRT$$

where $\hat{p}$ is the pressure in atm, $w$ is the volume in $m^3$, $n$ is the mass in mol, $T$ is the absolute
temperature in \( K \), and \( R \) is the \textit{Avogadro number}, the ideal gas constant \( 8.2056 \times 10^{-5} \text{ m}^3 \text{ atm} \text{ K}^{-1} \text{ mol}^{-1} \).

- The pressure in \textit{atm} is expressible in \textit{SI units} since

\[
1 \text{ atm} = 101.325 \text{ kPa}
\]

where \( Pa \) is the abbreviation for the Pascal unit

and

\[
1 \text{ Pa} = \frac{N}{m^2}
\]
TEMPERATURE VARIATION OF $\rho$

- We can restate the expression for $\rho$ in terms of the molecular weight of the gas, denoted by $M.W.$, expressed in $\frac{g}{mol}$, as

$$\rho\left(\frac{kg}{m^3}\right) = \frac{n(mol) \cdot M.W.\left(\frac{g}{mol}\right) \cdot 10^{-3}\left(\frac{kg}{g}\right)}{\omega(m^3)}$$

- Air is the mixture of 5 gases and the associated $M.W.$ of each are given in the table.
## TEMPERATURE VARIATION OF $\rho$

<table>
<thead>
<tr>
<th>gas</th>
<th>fraction (%)</th>
<th>M.W. (g/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>nitrogen</td>
<td>78.08</td>
<td>28.02</td>
</tr>
<tr>
<td>oxygen</td>
<td>20.95</td>
<td>32.00</td>
</tr>
<tr>
<td>argon</td>
<td>0.93</td>
<td>39.95</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>0.039</td>
<td>44.01</td>
</tr>
<tr>
<td>neon</td>
<td>0.0018</td>
<td>20.18</td>
</tr>
</tbody>
</table>

Thus,

$$M.W. \text{ (air)} = (0.7808)(28.02) + (0.2095)(32.00) +$$

$$(0.0093)(39.95) + (0.039)(44.01) + (0.0018)(20.18)$$

$$= 28.97 \frac{g}{mol}$$
TEMPERATURE VARIATION OF $\rho$

The ideal gas law for the air $M.W.$ value obtains

$$\rho = \frac{\hat{p}(atm) \cdot M.W. \left( \frac{g}{mol} \right)}{RT}$$

$$= \frac{\hat{p}(atm) \cdot (28.97) \left( \frac{g}{mol} \right) \cdot 10^{-3} \left( \frac{kg}{g} \right)}{T(K) \cdot (8.2056 \times 10^{-5}) \left( \frac{m^3 \cdot atm}{K \cdot mol} \right)}$$

$$\rho \left( \frac{kg}{m^3} \right) = 353.1 \frac{\hat{p}}{T} \left( \frac{atm}{K} \right)$$
TEMPERATURE VARIATION OF $\rho$

- Then, at $30^\circ C$ at 1 atm

$$\rho(30^\circ C) = \frac{(353.1)(1)}{30 + 273.15} = 1.165 \frac{kg}{m^3}$$

while at $45^\circ C$ at 1 atm

$$\rho(45^\circ C) = \frac{(353.1)(1)}{45 + 273.15} = 1.110 \frac{kg}{m^3}$$

- Note that the doubling (tripling) of the $15^\circ C$ temperature results in a 5% (9%) decrease in air density; these reductions, in turn, translate in the same % reductions in power.
ALTITUDE VARIATION OF $\rho$

- A change in altitude brings about a change in air pressure; we evaluate the ramifications of such a change.

- We consider a static column of air with cross-sectional area $a$ and we examine a horizontal slice in that column of thickness $dz$ with air density $\rho$ so that its mass is $\rho a dz$. 
We examine the pressures at the altitudes $z + dz$ and $z$ due to the weight of the air above those altitudes:

$$\hat{p}(z) = \hat{p}(z + dz) + g \frac{\rho a}{a} dz$$

where, $g = 9.806 \frac{m}{s^2}$ is the gravitational constant.
ALTITUDE VARIATION OF $\rho$

- We rewrite the difference in $\hat{p}$ at the two altitudes as:

$$dp = \hat{p}(z + dz) - \hat{p}(z) = -g \rho \, dz$$

and so:

$$\frac{dp}{dz} = -g \rho$$

- Note that:

$$\rho = 353.1 \frac{\hat{p}}{T} \left( \frac{atm}{K} \right)$$
We need to make use of several conversion factors to get useful expressions

\[ \frac{dp}{dz} = - \left( \frac{353.1}{T} \right) \left( \frac{\text{kg}}{m^3} \right) \times \]

\[ (9.806) \left( \frac{m}{s^2} \right) \left( \frac{1\ \text{atm}}{101.325\ Pa} \right) \left( \frac{1\ Pa}{N/m^2} \right) \left( \frac{1\ N}{\text{kg}\ m/s^2} \right) \hat{p} (\text{atm}) \]

\[ = - 0.0342 \ \frac{\hat{p}}{T} \]
The solution of this differential equation is complicated by the fact that the temperature also changes with altitude at the rate of 6.5° C drop for each km increase in altitude.

Under the simplifying assumption that $T$ remains constant, the solution of the differential equation is
ALTITUDE VARIATION OF $\rho$

$$\hat{p}(z) = \hat{p}_0 \exp\left(-0.0342 \frac{z}{T}\right) \quad \hat{p}_0 = 1 \text{ atm}$$

- It follows that

$$\rho\left(\frac{kg}{m^3}\right) = \frac{353.1}{T} \exp\left(-0.0342 \frac{z}{T}\right)$$

where $T$ is in $K$ and $z$ is in $m$
EXAMPLE: COMBINED TEMPERATURE AND ALTITUDE IMPACTS

We compare the value of $\rho$ at $25^\circ \text{C}$ at 2,000 m to that under the standard 1 atm $15^\circ \text{C}$ conditions.

We compute

$$\rho \bigg|_{25^\circ \text{C}, \ 2,000 \ m} = \frac{353.1}{298.15} \exp \left( -0.0342 \frac{2000}{298.15} \right) = 0.9415 \frac{\text{kg}}{\text{m}^3}$$

The $1.225 \frac{\text{kg}}{\text{m}^2}$ is thus reduced by 23% and thus results in a 23% decrease in power output – a rather substantial decrease.
THE DEPENDENCE ON TOWER HEIGHT

- The fact that power in the wind varies with $v^3$
  where, $v$ is the wind speed, implies that an increase in the wind speed has a pronounced effect on the wind output.

- Since for a given site, $v$ increases as the height of the tower is raised, we can generally increase the wind turbine output by mounting it on a taller tower.
THE DEPENDENCE ON TOWER HEIGHT

- A good approximation of the relationship between $v$ and tower height $h$ is expressed in terms of the Hellman exponent $\alpha$ – often called a friction coefficient – by the relationship

$$\left( \frac{v}{v_0} \right) = \left( \frac{h}{h_0} \right)^\alpha,$$

where, $h_0$ is the reference height with the corresponding wind speed $v_0$. 
THE DEPENDENCE ON TOWER HEIGHT

- The Hellman exponent $\alpha$ depends on the nature of the terrain at the site; a higher value of $\alpha$ indicates heavier friction – rougher terrain – and a lower value indicates low resistance faced by the wind.

- Typical values for $\alpha$ are tabulated for different terrains.

<table>
<thead>
<tr>
<th>Terrain Characteristics</th>
<th>Friction Coefficient $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth hard ground, calm water</td>
<td>0.10</td>
</tr>
<tr>
<td>Tall grass on level ground</td>
<td>0.15</td>
</tr>
<tr>
<td>High crops, hedges, and shrubs</td>
<td>0.20</td>
</tr>
<tr>
<td>Wooded countryside, many trees</td>
<td>0.25</td>
</tr>
<tr>
<td>Small town with trees and shrubs</td>
<td>0.30</td>
</tr>
<tr>
<td>Large city with tall buildings</td>
<td>0.40</td>
</tr>
</tbody>
</table>
THE DEPENDENCE ON TOWER HEIGHT

A typical value for $h_0$ is 10 m and the behavior of $\frac{v}{v_0}$ as a function of $\frac{h}{h_0}$ is

\[
\frac{v}{v_0} \quad \text{as a function of} \quad \frac{h}{h_0} \quad \text{is}
\]

\[
\left( \frac{v}{v_0} \right)
\]

\[
\begin{align*}
\alpha &= 0.1 \\
\alpha &= 0.2 \\
\alpha &= 0.3 \\
\alpha &= 0.4
\end{align*}
\]
THE DEPENDENCE ON TOWER HEIGHT

- We can also determine the ratio of $p_w(h)$ to $p_w(h_0)$

under the assumption that the air density $\rho$

remains unchanged over the range $[h_0, h]$ using

the relationship

$$\frac{p_w(h)}{p_w(h_0)} = \frac{1}{2} \rho a \nu^3 = \left(\frac{\nu}{\nu_0}\right)^3 = \left(\frac{h}{h_0}\right)^{3\alpha}$$
We can observe the dramatic change in the power output ratio as a function of height.
A key implication of the power ratio at different heights is the fact that the stress as the turbine blade moves through an entire revolution may be rather significant, particularly over rough terrain.

\[ p_w \left( h + \frac{d}{2} \right) \]

\[ p_w \left( h - \frac{d}{2} \right) \]
THE DEPENDENCE ON TOWER HEIGHT

\[ p_w \left( h - \frac{d}{2} \right) \]  

is the lowest value of wind output

\[ p_w \left( h + \frac{d}{2} \right) \]  

is the highest value of wind output

\[
\frac{p_w \left( h + \frac{d}{2} \right)}{p_w \left( h - \frac{d}{2} \right)} = \left( \frac{h + \frac{d}{2}}{h - \frac{d}{2}} \right)^{3\alpha}
\]